Umbrella Pricing and Cartel Size

Stefan Napel
Dept. of Economics,
University of Bayreuth
stefan.napel@uni-bayreuth.de
[*Corresponding Author*]

Dominik Welter
Dept. of Economics,
University of Bayreuth
dominik.welter@uni-bayreuth.de

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Abstract

It is generally assumed that bigger scale and scope of private antitrust enforcement promotes effective competition. This has motivated several North American and European courts to uphold redress claims not only from clients of a detected cartel but also plaintiffs who were exposed to 'umbrella pricing', i.e. equilibrium price increases by non-colluding competitors. The paper shows that the presumed deterrence effects of obliging infringing firms to compensate aggrieved customers of non-infringing firms can be dominated by adverse cartel size effects: liability for umbrella damages primarily constrains small partial cartels. It thereby improves the comparative profitability and stability of large ones. More encompassing cartels can form, prices rise, and welfare falls.

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1. Introduction

Victims of antitrust infringements have a right to act against a detected cartel and to reclaim damages. Forward-looking firms anticipate the applicable compensation obligations in their decisions to form cartels and to set prices. Private antitrust enforcement can thus affect both the distribution and creation of economic surplus in oligopoly markets.

Legal discussion of private antitrust action is widely presuming that redress on a greater part of total harm is desirable not only on principle and for reasons of justice but because it generates beneficial deterrence effects. This has been acknowledged explicitly in the 2014 decision by the EU Court of Justice that extended legal standing from the customers of cartel members to customers of non-members who were harmed indirectly by an increased industry price level:

> "The right of any individual to claim compensation for such a[n indirect] loss actually strengthens the working of the European Union competition rules, since it discourages agreements or practices, frequently covert, which are liable to restrict or distort competition, thereby making a significant contribution to the maintenance of effective competition in the European Union ... " (Case C-557/12 *Kone AG v ÖBB-Infrastruktur AG*, ECLI:EU:C:2014:1317, rec. 23).

Victims of 'umbrella effects', signifying detrimental equilibrium reactions by competing firms to cartel activities, are entitled to compensation also in Canada (as affirmed by the Supreme Court in 2019) and in the US depending on the competent court.[1] Leon Higginbotham Jr., former judge at the US Court of Appeals for the Third Circuit, noted in a well-cited case in 1979 (judgement 596 F.2d 573 3rd. Cir.): "Allowing standing [for customers of cartel outsiders] would also encourage enforcement, and thereby deter violation, of the antitrust laws."

Similar views are expressed in scholarly publications. For instance, Blair and Maurer (1982) hold that "[i]t is obvious that the prospect of recovery by purchasers

[1]The US Supreme Court has not yet ruled on the issue. Negative decisions include 600 F.2d 1148 5th Cir. 1979; positive ones 62 F. Supp. 2d 25 1999. See Blair and Durrance (2018), as well as Inderst et al. (2014) and Holler and Schinkel (2017). Laitenberger and Smuda (2015) show that umbrella losses constituted a big part of damages suffered by German customers of the European detergent cartel. Bos et al. (2019) find suggestive evidence for umbrella effects in some but not all considered industries.

from noncolluding competitors should have a greater deterrent effect than recovery limited to direct purchasers, assuming a constant probability of detection". Blair and Durrance (2018) affirm this conclusion.

Our analysis, in contrast, questions the pro-competitive merits of a regime in which customers of non-infringing firms have legal standing, i.e., the right to sue infringing firms for damages. Many former cartels involved a strict subset of suppliers[2] and it can seem very intuitive that redress obligations to the implied victims of umbrella pricing would have made collusion less attractive and encouraged compliance with antitrust law. However, affected firms might re-optimize their cartel operations rather than stop them. In particular, the compensation regime changes the comparative profitability of partial vs. all-encompassing cartels. Giving standing to non-cartel customers advantages the latter. This risks prompting previously well-behaved firms to infringe instead of making rogue firms comply.

We demonstrate that a pro-competitive assessment of extended legal standing is warranted only if a cartel's size can be taken as constant. Then an obligation to compensate umbrella losses reduces the expected profitability of cartels and optimal prices. But insofar as small cartels become unstable while large or all-encompassing ones – with few or no competitors whose customers could claim compensation – remain or newly become stable, extended rights for indirect cartel victims foster cartel growth. We demonstrate this so far unacknowledged effect for a non-negligible set of market configurations. A regime with umbrella compensation can even raise total unatoned overpayments, not just prices and deadweight losses.

Our formal statements draw on a model of cartel formation proposed by Bos and Harrington (2010) but the underlying economic argument is sensitive only to three features of this model: first, stable partial cartels can emerge in the default regime, i.e. without compensation of umbrella losses; second, the umbrella regime with compensation does not entirely prevent all collusion; third, bigger cartels set higher prices ceteris paribus.

[2] In the recent European truck cartel, for example, only 9 out of 10 trucks were produced by a cartel member. In the data set of Connor (2020), about 60% of the approximately 500 reported cartels to which a market share could be attributed were partial. There are good theoretical reasons to expect partial rather than industry-wide cartels. See, e.g., Bos and Harrington (2010), Gabszewicz et al. (2019), or de Roos and Smirnov (2021) for models featuring capacity constraints, differentiated goods, or imperfectly attentive customers.

Anticipated compensation claims by umbrella victims lower the prospective profits of partial cartels. This can render a given cartel downright unprofitable. Alternatively, it may become dynamically unsustainable or it will optimally adjust prices downwards because expected compensation costs increase in the markup. All this combines to a desirable deterrence effect.

However, cartels with few members and relatively low market coverage are disproportionately burdened by damage claims in the umbrella regime. They must pay compensation to many umbrella victims while a cartel that covers, say, 90% of the market closely resembles the industry-wide cartel, which by definition faces no umbrella victims. Being a member of a partial cartel hence loses attractiveness in the umbrella regime, while the profitability of an all-inclusive cartel is unchanged. The latter – or more generally bigger cartels – gain internal stability while the external stability of small cartels falls.

Whether the beneficial dynamic stability effects dominate and yield lower prices, or detrimental structural stability changes induce a cartel with more members and higher prices depends on the market parameters at hand. We derive sufficient conditions such that the structural incentives to form bigger cartels with greater margins prevail.

The blanket conclusion that compensation of umbrella damages promotes effective competition hence needs to be qualified.[3] Judges and policymakers should become aware that greater scope of private enforcement can have negative welfare implications. This matches findings on ambiguous effects of stricter public enforcement obtained by Bos and Harrington (2015). Adverse side effects of well-intentioned antitrust measures also have been identified by McCutcheon (1997), Andersson and Wengström (2007), Bageri et al. (2013) and Bos et al. (2015).

The rest of the paper is organized as follows: Section 2 describes our market model and derives general results regarding dynamic and structural cartel stability. Section 3 presents more specific findings for linear demand. We assess the robustness of our baseline analysis to several variations in Section 4. This includes the assumptions regarding fines and leniency options, more sophisticated criteria for cartel stability, asymmetric firms, and an alternative market model with differentiated goods. None

[3]No qualification is needed for markets that lack some of the listed features: a detrimental increase of cartel size is impossible if the umbrella regime makes *all* collusion unprofitable or dynamically unstable. When no partial cartel is stable in the default regime then there is no scope for umbrella losses to start with, and larger cartels are no worry if their prices are non-increasing in size.

of the variations changes the key message, which is summarized again in Section 5.

2. Compensation for Umbrella Losses and Stability of Cartels

We adopt a version of the Bertrand-Edgeworth competition model developed by Bos and Harrington (2010). It combines the market features that are necessary to observe detrimental size effects of compensating umbrella victims in a particularly tractable way. The original model entailed no antitrust enforcement but, in an extension, Bos and Harrington (2015) showed that adding a competition authority can raise or reduce stable cartel sizes depending on circumstance.[4] So public antitrust action can have similarly counterintuitive effects as requiring compensation of umbrella victims. A distinctive aspect of the latter is that it advantages more encompassing cartels by design. Surviving cartels hence are never smaller than before.

Our presentation focuses on the case of symmetric firms, which allows to study cartel stability as a function of the number rather than identity of infringing firms. We will discuss the implications of not making this simplification in Section 4 and will also vary many other baseline assumptions. We next introduce these assumptions (Section 2.1), compare the dynamic stability of collusion with and without umbrella compensation (Section 2.2), and then conduct structural stability analysis à la d'Aspremont et al. (1983) (Section 2.3). The investigation will leave aside any agency problems and transaction costs of coordination, personal criminal sanctions, or fairness and justice concerns.

2.1. Formal Setup

Let $n \geq 3$ symmetric firms repeatedly engage in simultaneous price setting for a homogeneous good. Each firm i faces an exogenous capacity constraint $q_i \leq k$ on period production and maximizes the present value of profits for a common discount factor $\delta \in (0, 1)$ with infinite time horizon.

Constant unit costs are denoted by $c \geq 0$. Prices must be integer multiples of

[4] Bos and Harrington focus on the widest possible range of stable cartels and emphasize comparative statics regarding the detection rate, which is assumed to increase discontinuously in cartel size. They show that stricter enforcement can cause the largest stable cartels to shrink while the smallest stable cartels may shrink or grow. They also identify sufficient conditions for unambiguous growth and increasing overall cartel prices in a symmetric quadropoly with linear demand.

a small unit of account $\varepsilon > 0$ and consumers buy at the lowest available price à la Bertrand. Market demand is described by a smooth function $D(p)$ with $D'(p) < 0$ such that $(p - c) \cdot D(p)$ is strictly concave with $D(c) =: a > 0$, i.e., marginal costs are not too high. Demand is rationed efficiently when a firm's capacity is exhausted and (residual) demand is split equally if several firms post identical prices.

We assume that the capacity k of each firm satisfies

$$\frac{a}{n-1} < k < q^m \tag{A1}$$

where $q^m > 0$ denotes the monopoly output associated with c and $D(p)$. The upper bound ensures positive residual demand for a cartel of $n - 1$ firms, which allows partial cartels to form for $\delta \approx 1$ at least in the default regime and also simplifies the analysis by making firms that undercut a cartel produce at capacity. The lower bound guarantees existence of a competitive static pure-strategy Nash equilibrium. In particular, it implies two symmetric equilibria in which all firms price approximately at cost, i.e., either $p = c$ or $p = c + \varepsilon \approx c$. We suppose cartel prices $p \geq c + 2\varepsilon$.

As Bos and Harrington (2010, 2015), we allow at most one cartel to operate. Its $2 \leq s \leq n$ members are assumed to use stationary strategies that do not condition on past behavior of non-cartel members but permanently revert to the static zero-profit equilibrium after a deviation (the harshest possible punishment). So non-members will at any point in time maximize their static period profits and undercut the anticipated uniform cartel price p by ε (cf. Bos and Harrington 2010). This leaves a residual demand of $D_s^R(p) = \max\left\{D(p) - (n - s)k, 0\right\}$ for the cartel.

In any period t of cartel operations, the infringement is detected with probability $\alpha \in (0, 1)$. As our baseline, we take α to be fixed, i.e., it depends neither on the legal regime as such, nor on a cartel's size or price choices (Katsoulacos et al. 2015, 2020).[5] In case of detection, each active cartel member must pay a fine of $\tau > 0$ times its period t profit and $\beta > 0$ times eligible overcharges.[6] This gives rise to an individual

[5]To have some ballpark figure: Bryant and Eckard (1991) estimated the annual probability of getting indicted by federal authorities in the US at between 13% and 17%. Combe et al. (2008) obtained comparable results for a European sample. Ormosi (2014) corroborates these estimates.

[6]Profit-based fines simplify the exposition by allowing to work with net demand functions $D_s^*(\cdot)$ and $D_s^{*u}(\cdot)$ below. Revenue-based fines will be discussed in Section 4. We focus on claims for overcharge damages since lost profits are rarely recovered in legal practice (see, e.g., Weber 2021, Laborde 2021). Basso and Ross (2010) discuss the extent to which overcharge damages are a good proxy for total harm.

expected profit for a cartel member of

$$\pi_s(p) = (p - c) \cdot \underbrace{(1 - \alpha(\beta + \tau))D_s^R(p)/s}_{:= D_s^*(p)} = (p - c)D_s^*(p) \tag{1}$$

in the default regime and

$$\pi_s^u(p) = (p - c) \cdot \underbrace{\Big((1 - \alpha\tau)D_s^R(p) - \alpha\beta D(p)\Big)/s}_{:= D_s^{*u}(p)} = (p - c)D_s^{*u}(p) \tag{2}$$

in the umbrella regime, where

$$D_s^*(p) = \frac{1 - \alpha(\beta + \tau)}{s}\Big[D(p) - (n - s)k\Big] \tag{3}$$

and

$$D_s^{*u}(p) = \frac{1 - \alpha(\beta + \tau)}{s}D(p) - \frac{1 - \alpha\tau}{s}(n - s)k \tag{4}$$

describe the *net demand* of a cartel member after subtracting units that cover expected fines and redress payments. Note that private antitrust enforcement with parameter β is equivalent to purely public action with an increased multiplier $\tau' = \beta + \tau$ in the default regime but not the umbrella case. We assume

$$1 - \alpha(\beta + \tau) > 0 \tag{A2}$$

to ensure that an all-encompassing cartel that chooses $p = c + 2\varepsilon$ would face positive net demand $nD_n^*(p) = nD_n^{*u}(p) \approx \Big(1 - \alpha(\beta + \tau)\Big)D(c)$ and thus be profitable.

Like Katsoulacos et al. (2015, 2020) we let a detected cartel resume its activities in period $t + 1$, provided the applicable profitability and stability conditions are met.[7] However, cartel activities are ended for good if a member deviates in any way from the agreed price in period t and all firms revert to $p \approx c$ from $t + 1$ on. The best deviation of a cartel member is then either to match the non-members' price $p - \varepsilon$ or to undercut them with $p - 2\varepsilon$, depending on outsiders' joint capacity. In both cases the dominant effect is to raise the respective firm's net demand from $D_s^*(p)$ or $D_s^{*u}(p)$ to k, and we equate the one-off deviation profit with approximately $(p - c)k$.

[7]One might also suppose that a detected cartel breaks down forever with probability $\gamma \in (0, 1]$, or else re-forms in $t + 1$. This would scale up the critical discount factors identified below by $\frac{1}{1-\alpha\gamma}$.

2.2. *Profitability and Dynamic Stability*

A cartel with s members is dynamically stable if the present value of profits that accrue from serving net demand $D_s^*(p)$ or $D_s^{*u}(p)$ in the default or umbrella regime, respectively, at cartel price $p \geq c + 2\varepsilon$ is at least as big as realizing the one-off deviation profit of approximately $(p - c)k$ and then reverting to competition. For any given p, this is equivalent to the requirement that firms' discount factor $\delta \in (0, 1)$ is not smaller than the critical discount factor

$$\delta_s(p) = 1 - \frac{D_s^*(p)}{k} \text{ or } \delta_s^u(p^u) = 1 - \frac{D_s^{*u}(p^u)}{k}. \tag{5}$$

$(p - c) \cdot D_s^*(p)$ and $(p - c) \cdot D_s^{*u}(p)$ inherit the strict concavity of monopoly profits and respective unconstrained cartel profit maximizers, denoted p_s^* and p_s^{*u}, are uniquely determined by

$$(p - c) \cdot D'(p) + D(p) = (n - s)k \tag{6}$$

in the default and

$$(p - c) \cdot D'(p) + D(p) = (n - s)k/\mu \tag{7}$$

in the umbrella regime. The constant $\mu := {}^{(1-\alpha(\beta+\tau))}/_{(1-\alpha\tau)} < 1$ will be referred to as the *umbrella coefficient*. It is smaller the greater the compensation multiplier β (e.g. triple vs. single damages), the detection probability α, and the fine multiplier τ.

First focus on the choice of p in the default regime. Adoption of the unconstrained profit maximizer p_s^* determined by (6) can be sustained only if $\delta \geq \bar{\delta}_s$ with

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1 - \alpha(\beta + \tau))D(p_s^*) - (1 - \alpha(\beta + \tau))(n - s)k}{sk}. \tag{8}$$

If otherwise $\delta < \bar{\delta}_s$, cartel members maximize $sD_s^*(p)(p - c)$ subject to the constraint $\delta_s(p) = \delta$. A cartel cannot be dynamically stable if $\delta < \underline{\delta}_s$ where

$$\underline{\delta}_s := \delta_s(c + 2\varepsilon) \approx \delta_s(c) = 1 - \frac{(1 - \alpha(\beta + \tau))(a - (n - s)k)}{sk}. \tag{9}$$

To ensure that $\underline{\delta}_s < 1$, it is necessary that the cartel of size s is profitable, i.e., that it can choose a price $p \geq c + 2\epsilon$ with $D_s^*(c + 2\epsilon) > 0$. Given that $D_s^*(c + 2\epsilon) > 0 \Leftrightarrow D_s^R(c + 2\epsilon) > 0$ and noting $D_s^R(c + 2\epsilon) \approx D_s^R(c) = a - (n - s)k$, the size of the smallest profitable cartel is

$\underline{s} := \lceil n - \frac{a}{k} \rceil$.[8] Condition (A1) implies $2 \leq \underline{s} \leq n-1$ and $\underline{\delta}_{n-1} < \bar{\delta}_{n-1} < 1$. Existence of some dynamically stable partial cartel is thus ensured if $\delta \approx 1$.

For $\delta \in (\underline{\delta}_s, \bar{\delta}_s)$ a cartel of $s \geq \underline{s}$ members can stabilize some prices $c + 2\varepsilon \leq p_s < p_s^*$. The corresponding constrained profit maximizer $p_s^\circ(\delta)$ is characterized by

$$\delta = \delta_s(p_s^\circ) = 1 - \frac{D_s^*(p_s^\circ)}{k} \qquad \Leftrightarrow \qquad p_s^\circ(\delta) = D_s^{*-1}\Big((1-\delta)k\Big) \tag{10}$$

and increases continuously to p_s^* as $\delta \to \bar{\delta}_s$. In summary, a cartel of s members is dynamically stable in the default regime iff $\delta \in (\underline{\delta}_s, 1)$ and the respective overall cartel price is

$$p_s^\circledast(\delta) = \begin{cases} p_s^\circ(\delta) & \text{if } \delta \in (\underline{\delta}_s, \bar{\delta}_s), \\ p_s^* & \text{if } \delta \in (\bar{\delta}_s, 1). \end{cases} \tag{11}$$

Cartel and non-cartel customers suffer a price overcharge of approximately $p_s^\circledast(\delta) - c$.

Analogously, we can derive

$$p_s^{\circledast u}(\delta) = \begin{cases} p_s^{\circ u}(\delta) & \text{if } \delta \in (\underline{\delta}_s^u, \bar{\delta}_s^u), \\ p_s^{*u} & \text{if } \delta \in (\bar{\delta}_s^u, 1), \end{cases} \tag{12}$$

in the umbrella regime with

$$\delta = \delta_s(p_s^{\circ u}) \qquad \Leftrightarrow \qquad p_s^{\circ u}(\delta) = D_s^{*u-1}\Big((1-\delta)k\Big) \tag{13}$$

as the constrained price and

$$\bar{\delta}_s^u := \delta_s^u(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))D(p_s^{*u}) - (1-\alpha\tau)(n-s)k}{sk}, \tag{14}$$

$$\underline{\delta}_s^u := \delta_s^u(c+2\varepsilon) \approx \delta_s^u(c) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{sk} \tag{15}$$

as the relevant critical discount factors. It is noteworthy that the size $\underline{s}^u := \lceil n - \mu \cdot \frac{a}{k} \rceil$ of the smallest cartel that satisfies dynamic stability for $\delta \approx 1$ in the umbrella regime is weakly greater than size $\underline{s}$ in the default regime because $\mu < 1$. Relatedly we find:

Proposition 1. *For a cartel of size $s < n$, extending legal standing to umbrella victims*

(i) increases the minimal discount factor that is needed to sustain the agreement, i.e., $\underline{\delta}_s^u > \underline{\delta}_s$;

(ii) decreases the overall cartel price to $p_s^{\circledast u}(\delta) < p_s^\circledast(\delta)$.

[8] $\lceil x \rceil$ denotes the smallest integer not smaller than x.

(iii) Thresholds $\underline{\delta}_s^u$ and $\underline{\delta}_s$ are decreasing and overall cartel prices $p_s^{\circledast}(\delta)$ and $p_s^{\circledast u}(\delta)$ are increasing in the cartel's size $s \in \{2, \ldots, n\}$.

Proof. Part (*i*) follows directly from (9), (15) and $\mu^{-1} > 1$. Part (*ii*) is implied by two observations. First, the LHS of equations (6) and (7) coincide and decrease in p, while the RHS of eq. (7) is greater than that of eq. (6) given $\mu < 1$. Hence $p_s^* > p_s^{*u}$. Second, because $D_s^{*u}(p) < D_s^*(p)$ for all relevant p, any given sales q go with lower prices $D_s^{*u-1}(q) < D_s^{*-1}(q)$ in the umbrella regime. So setting $q = (1-\delta)k$ implies $p_s^{\circ}(\delta) > p_s^{\circ u}(\delta)$ and we can, overall, conclude $p_s^{\circledast u}(\delta) < p_s^{\circledast}(\delta)$.[9]

Moreover, for $2 < s \leq n$ we have

$$\underline{\delta}_s - \underline{\delta}_{s-1} \approx \frac{(1-\alpha(\beta+\tau))(a-(n-s+1)k)}{(s-1)k} - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{sk} \tag{16}$$

$$= \underbrace{(a-nk)}_{<0} \cdot \underbrace{\frac{1-\alpha(\beta+\tau)}{s(s-1)k}}_{>0} < 0 \tag{17}$$

by equation (9) and assumptions (A1) and (A2). $\underline{\delta}_s^u - \underline{\delta}_{s-1}^u < 0$ in part (iii) follows similarly. That p_s^* (p_s^{*u}) is increasing in s follows since the RHS of eq. (6) (eq. (7)) is decreasing in s, whereas the LHS is independant of s. $p_s^{\circ}(\delta)$ ($p_s^{\circ u}(\delta)$) is increasing in s since $D_s^*(p)$ ($D_s^{*u}(p)$) is increasing in s given (A2). □

At an intuitive level, the reduction of prices observed in part (ii) follows from the cartel's expected compensation payments being lower, the lower its overcharge. The critical discount factor in part (i) rises in the temptation to deviate, which is increased when expected compensation of umbrella victims reduces profits from continued collusion. This profit reduction also implied $\underline{s}^u \geq \underline{s}$, i.e., the smallest cartel that is profitable is (weakly) larger in the umbrella regime. Turning to part (iii), a cartel's influence on margins and the implied prices increase in s because greater market coverage makes undercutting by capacity-constrained outsiders less of a concern. Greater market coverage also explains the wider range of discount factors that render larger cartels dynamically stable, i.e. $\underline{\delta}_s < \underline{\delta}_{s-1}$ and $\underline{\delta}_s^u < \underline{\delta}_{s-1}^u$: a cartel first becomes dynamically stable when it can charge $p = c + 2\varepsilon$ and discourage defections with fixed deviation profit εk. This discouragement gets easier the greater the respective

[9] If $\bar{\delta}_s < \bar{\delta}_s^u$ and $\delta \in (\bar{\delta}_s, \bar{\delta}_s^u)$, then $p_s^{\circledast u}(\delta) = p_s^{\circ u}(\delta) < p_s^{*u} < p_s^* = p_s^{\circledast}(\delta)$. If $\bar{\delta}_s^u < \bar{\delta}_s$ and $\delta \in (\bar{\delta}_s^u, \bar{\delta}_s)$, then $p_s^{\circledast u}(\delta) = p_s^{*u} = p_s^{\circ u}(\bar{\delta}_s^u) < p_s^{\circ}(\bar{\delta}_s^u) < p_s^{\circ}(\delta) = p_s^{\circledast}(\delta)$.

firm-specific collusion profit, which for $p = c + 2\varepsilon$ is proportional to per-firm demand and increases in s.

2.3. *Structural Stability*

Provided that partial cartels remain dynamically stable in the umbrella regime, their continued operation is conditional on firms' incentives for being a member or non-member in the long run. We follow Escrihuela-Villar (2008, 2009) or Bos and Harrington (2010, 2015) and apply structural stability analysis à la d'Aspremont et al. (1983).

Define $p_s^{\circledast}(\delta) = c$ for $s \in \{\underline{s} - 1, n + 1\}$. Then a dynamically stable cartel of size $\underline{s} \leq s \leq n$ is called *internally stable* in the default regime if

$$\mathcal{I}_s(\delta) := (p_s^{\circledast}(\delta) - c)D_s^*\Big(p_s^{\circledast}(\delta)\Big) - (p_{s-1}^{\circledast}(\delta) - c)k \geq 0 \tag{18}$$

i.e., each cartel member's per period profit is at least as high as that from becoming a non-member. It is *externally stable* if

$$\mathcal{E}_s(\delta) := (p_s^{\circledast}(\delta) - c)k - (p_{s+1}^{\circledast}(\delta) - c)D_{s+1}^*\Big(p_{s+1}^{\circledast}(\delta)\Big) \geq 0, \tag{19}$$

i.e., each non-member's profit weakly exceeds that achievable by becoming a member. Cartels of size $s = \underline{s}$ ($s = n$) are automatically internally (externally) stable, i.e. $\mathcal{I}_{\underline{s}}(\delta) > 0$ ($\mathcal{E}_n(\delta) > 0$). Analogous conditions $\mathcal{I}_s^u(\delta) \geq 0$ and $\mathcal{E}_s^u(\delta) \geq 0$ apply in the umbrella regime. Note that $\mathcal{E}_n^u(\delta) = \mathcal{E}_n(\delta)$ and also that eqs. (18) and (19) imply $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}_s^u(\delta) = -\mathcal{I}_{s+1}^u(\delta)$ for $s < n$.

A dynamically stable cartel of s members will be considered *structurally stable* or simply *stable* in a given regime if it is internally and externally stable. If a unique size s is compatible with structural stability, a corresponding cartel will be assumed to form. In case multiple sizes are compatible with structural stability, a stable cartel of maximal size is presumed to operate: the latter is not just focal in that it maximizes total profits subject to stability but it is of the generically unique size that makes coordinated switches from one to another stable cartel individually unprofitable.[10]

[10]To elaborate on this, let $\tilde{s}$ be the maximal size s such that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$. Then $\mathcal{E}_{\tilde{s}-1}(\delta) = -\mathcal{I}_{\tilde{s}}(\delta) < 0$ renders it impossible for an $\tilde{s} - 1$-sized cartel to be stable. But it is conceivable (only) with non-linear demand that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$ for some $\hat{s}$ with $\underline{s} \leq \hat{s} \leq \tilde{s} - 2$: no single member finds it profitable to leave a cartel of $\hat{s}$ members, nor does a single non-member find it profitable to join the

From these premises we can conclude:

Proposition 2. *Let firms be sufficiently patient so that some (partial) cartel is dynamically stable in the umbrella (default) regime. Extending legal standing to customers of non-infringing firms then lowers welfare if $\mathcal{I}_n(\delta) < 0$ and $\mathcal{I}_n^u(\delta) > 0$.*

Proof. If any cartel is dynamically stable in a given regime, so is the all-encompassing one (recalling that $\underline{\delta}_s^u$ and $\underline{\delta}_s$ decrease in s). With $\mathcal{E}_n^u(\delta) > 0$ holding automatically, $\mathcal{I}_n^u(\delta) > 0$ implies that the industry-wide cartel is structurally stable in the umbrella regime. It is also the largest stable cartel – so if cartels of smaller size should also be stable it is unique in not providing incentives to form a different stable cartel. Recalling that an industry-wide cartel faces no umbrella victims, the overall market price equals $p_n^{\circledast u}(\delta) = p_n^{\circledast}(\delta)$ in the umbrella regime.

In the default regime, by contrast, the industry-wide cartel is unstable if $\mathcal{I}_n(\delta) < 0$. Because a cartel of size $s = \underline{s}$ with $2 \leq \underline{s} \leq n-1$ would automatically be internally stable, there must exist a largest size $\hat{s}$ with $\underline{s} \leq \hat{s} < n$ such that $\mathcal{I}_{\hat{s}}(\delta) > 0$ and $\mathcal{I}_{\hat{s}+1}(\delta) = -\mathcal{E}_{\hat{s}}(\delta) < 0$. When this forms (or some smaller stable cartel in defiance of joint entry incentives), the corresponding price $p_{\hat{s}}^{\circledast}(\delta)$ is strictly below $p_n^{\circledast}(\delta)$ by Proposition 1. For well-behaved cost and demand functions, as in our model, this implies greater total surplus in the default regime than in the umbrella regime. □

The conditions on internal stability in Proposition 2 are not overly difficult to satisfy. It is sufficient that there are enough firms with a capacity somewhere between monopoly output q^m and fraction μ of competitive output $a = D(c)$:

Proposition 3. *For any parameters c and $\mu a < k < q^m$ there exists $\hat{n} > 2$ such that $\mathcal{I}_n(\delta) < 0$, $\mathcal{I}_n^u(\delta) > 0$ and welfare is lowered for all $n \geq \hat{n}$ and $\delta \in (\hat{\delta}_n, 1)$ where $\hat{\delta}_n := \max\{\underline{\delta}_{n-1}, \underline{\delta}_n^u\} < 1$.*

Proof. $\underline{\delta}_n^u < 1$ follows directly from eq. (14); $\underline{\delta}_{n-1} < 1$ is implied by eq. (9) and $k < q^m < a$. So interval $(\hat{\delta}_n, 1)$ is non-empty. An industry-wide cartel can sustain the price $p_n^{\circledast u}(\delta) \geq c + 2\varepsilon$ in the umbrella regime for any $\delta \in (\hat{\delta}_n, 1)$ and, similarly, a cartel of $n-1$

cartel. However, in view of $p_{\hat{s}}^{\circledast}(\delta) < p_{\tilde{s}-1}^{\circledast}(\delta)$ and fixed capacities k, profits of an outsider to an $\hat{s}$-sized cartel are smaller than those of an outsider to an $\tilde{s}-1$-sized cartel. The latter are smaller than those of an insider to an $\tilde{s}$-sized cartel by $\mathcal{I}_{\tilde{s}}(\delta) > 0$. So it would be profitable for $\tilde{s} - \hat{s}$ outsiders of an $\hat{s}$-sized cartel to simultaneously become cartel members. Hence an $\hat{s}$-sized cartel fails to be stable against coordinated entry of multiple outsiders (cf. Section 4.1), while both individual and joint exits by members of the $\tilde{s}$-sized cartel would be unprofitable.

members can sustain the choice of $p^{\circledast}_{n-1}(\delta) \geq c + 2\varepsilon$ in the default regime. The patience requirements in Proposition 2 are thus satisfied.

The price $p^{\circledast}_n(\delta)$ set by an industry-wide cartel in the default regime is bounded by p^*_n. The latter price is characterized by $(p - c)D'(p) = -D(p)$ and constant in n. So total profits of an all-encompassing cartel have a fixed upper bound and associated per-member profits of $(p^{\circledast}_n - c)(1 - \alpha(\beta + \tau))D(p^{\circledast}_n)/n$ vanish as $n \to \infty$.[11] A cartel of only $n - 1$ members also makes positive profits given that $\delta > \hat{\delta}_n$. This implies a supracompetitive profit for the single non-member of at least εk, which does *not* vanish as $n \to \infty$. Therefore, some $\hat{n} > 2$ exists such that for all $n \geq \hat{n}$ permanently undercutting a cartel of size $n - 1$ is more profitable than being member of a cartel of size n, i.e. $\mathcal{I}_n(\delta) < 0$.

For the umbrella regime note that $\mu a < k < q^m$ implies $\underline{s}^u = \lceil n - \mu \cdot \frac{a}{k} \rceil = n$ for all $n \geq 3$. So any partial cartel is unprofitable and a fortiori unstable. In contrast, an industry-wide cartel earns positive profit and can be sustained given $\delta \in (\hat{\delta}_n, 1)$. Therefore $\mathcal{I}^u_n(\delta) > 0$ for all $n \geq 3$. Proposition 2 then yields the claim. □

The proof of Proposition 3 draws on the trivial observation that per-firm rents are small when the monopoly output must be divided between many. This causes at least a few firms to prefer staying outside of a partial cartel in the default regime. However, benefits to being non-member of a partial cartel only exist if the latter is profitable and dynamically stable. When non-members have sufficiently many customers who require compensation in the umbrella regime – reflected by a non-member's capacity k exceeding fraction μ of competitive demand – this fails to be the case. Then permanent freeriding on a partial cartel is no option and collection even of small rents in an industry-wide cartel is the best a firm can do. In other words, the industry-wide cartel, which was unstable by default, is stabilized by partial cartels' expected costs of umbrella compensation.

The sufficient condition in Proposition 3 pertains to cases with potentially many patient firms of which most but not all collude in the default regime.[12] Umbrella compensation can have harmful overall effects also for cartels involving few firms and

[11]The model's key assumption (A1) on capacities is implied by $\mu a < k < q^m$ for all $n \geq 2$. The only caveat to considering large n is that minimal discount factors also become large because $\hat{\delta}_n \to 1$.

[12]The minimum number $\hat{n}$ and what 'many' means in practice will clearly depend on demand, technology and enforcement. The data set compiled by Connor (2020) involves an average number of 10.2 firms (a median of 4.0 firms), with an average (median) market coverage of 87% (92%).

relatively small discount factors. Demonstrating this however requires closed form expressions and concrete parameter values. We pursue this route in the next section assuming linear demand. The latter is sufficient to show that umbrella compensation strengthens the internal stability of *all* profitable cartels – implying that weakly greater cartels are formed. It also ensures that for any $\delta > \delta_n$ ($\delta > \delta_n^u$) there generically exists a unique size s (s^u) such that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$ ($\mathcal{I}^u_{s^u}(\delta) > 0$ and $\mathcal{E}^u_{s^u}(\delta) > 0$).

3. Linear Market Environments

Suppose now that demand is linear with $D(p) = a - bp$ and set marginal costs to zero.[13] Assumption (A1) simplifies to

$$\frac{a}{n-1} < k < \frac{a}{2} \tag{A1'}$$

and now requires $n \geq 4$.

The overall market price in the default regime evaluates to

$$p_s^{\circledast}(\delta) = \begin{cases} p_s^{\circ}(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b} & \text{if } \delta \in (\underline{\delta}_s, \bar{\delta}_s), \\ p_s^{*} = \frac{1}{2b}\Big(a-(n-s)k\Big) & \text{if } \delta \in (\bar{\delta}_s, 1), \end{cases} \tag{20}$$

with

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{2sk}, \tag{21}$$

and

$$\underline{\delta}_s := \delta_s(2\varepsilon) \approx \delta_s(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{sk}. \tag{22}$$

The respective price in the umbrella regime is

$$p_s^{\circledast u}(\delta) = \begin{cases} p_s^{\circ u}(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b} & \text{if } \delta \in (\underline{\delta}^u_s, \bar{\delta}^u_s), \\ p_s^{*u} = \frac{1}{2b}\Big(a-(n-s)k\cdot\mu^{-1}\Big) & \text{if } \delta \in (\bar{\delta}^u_s, 1), \end{cases} \tag{23}$$

with

$$\bar{\delta}^u_s := \delta^u_s(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{2sk}, \tag{24}$$

[13] For $c > 0$, reinterpret p as the price markup: consider prices $\tilde{p} = c + p$ and demand $\tilde{D}(\tilde{p}) = \tilde{a} - b\tilde{p} = a - bp$ with $a = \tilde{a} + bc$. Then maximizing $(\tilde{p} - c)\tilde{D}(\tilde{p})$ is equivalent to maximizing $p(a - bp)$.

and

$$\underline{\delta}_s^u := \delta_s^u(2\varepsilon) \approx \delta_s^u(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{sk}. \tag{25}$$

This implies $\bar{\delta}_s < \bar{\delta}_s^u$ for $s < n$ (whereas $\bar{\delta}_s \geq \bar{\delta}_s^u$ is possible for non-linear demand).

Linearity of demand makes $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ (and hence $\mathcal{E}_{s-1}(\delta)$ and $\mathcal{E}_{s-1}^u(\delta)$) piecewise polynomial in δ. Figure 1 illustrates the specific quadratic, linear and constant parts when $n = 5$, $a = 10$, $b = 1$, $k = 3$, $\alpha = 1/5$, $\beta = 1$, and $\tau = 1/2$, which represents a case that is neither covered by Proposition 2 nor Proposition 3 (as $\mathcal{I}_n^u(\delta) < 0$ and $\mu a > k$). Supposing δ is big enough to ensure that at least the industry-wide cartel is dynamically stable, internal stability $\mathcal{I}_5(\delta)$ increases on $(\underline{\delta}_5, \underline{\delta}_4)$: the constrained profit maximizer $p_5^{\circledast} = p_5^{\circ}$ rises linearly in δ, causing members' profits to rise quadratically, while an outsider to a cartel of only four firms could not earn positive profit because such cartel is dynamically unstable. The latter however becomes dynamically stable for $\delta \geq \underline{\delta}_4$. Then the linear increase of $p_4^{\circledast} = p_4^{\circ}$ raises outsider profits $p_4^{\circledast}k$ proportionally while negative quantity reactions dampen further increases of $p_5^{\circledast}D_5^*(p_5^{\circledast})$ – in total causing $\mathcal{I}_5(\delta)$ to decrease on $(\underline{\delta}_4, \bar{\delta}_5)$. The quadratic decrease becomes linear for $\delta \in (\bar{\delta}_5, \bar{\delta}_4)$ since the encompassing cartel charges the constant unconstrained profit maximizer $p_5^{\circledast} = p_5^*$ for $\delta > \bar{\delta}_5$ while $p_4^{\circledast}k = p_4^{\circ}k$ still increases linearly in δ until $\bar{\delta}_4$. Finally for $\delta \in (\bar{\delta}_4, 1)$, the price faced by outsiders to a cartel of four firms also becomes constant, and so does profit difference $\mathcal{I}_5(\delta)$. Analogous variation in δ applies to internal stability of partial cartels and in the umbrella regime.

Inspection of Figure 1 shows that the range of discount factors where dynamic stability of a cartel entails internal stability is widened by giving legal standing to umbrella victims. This holds irrespective of our specific parameter choices:

Proposition 4.
(i) Every dynamically stable cartel of size $s \geq \underline{s}^u$ that is internally stable in the default regime is also internally stable in the umbrella regime:

$$\mathcal{I}_s(\delta) > 0 \;\Rightarrow\; \mathcal{I}_s^u(\delta) > 0. \tag{26}$$

The reverse is not true and $\mathcal{I}_s^u(\delta) > \mathcal{I}_s(\delta)$ holds for $\delta \in (\underline{\delta}_{s-1}^u, 1)$.
(ii) Every dynamically stable cartel of size $s \geq \underline{s}^u$ that is externally stable in the umbrella

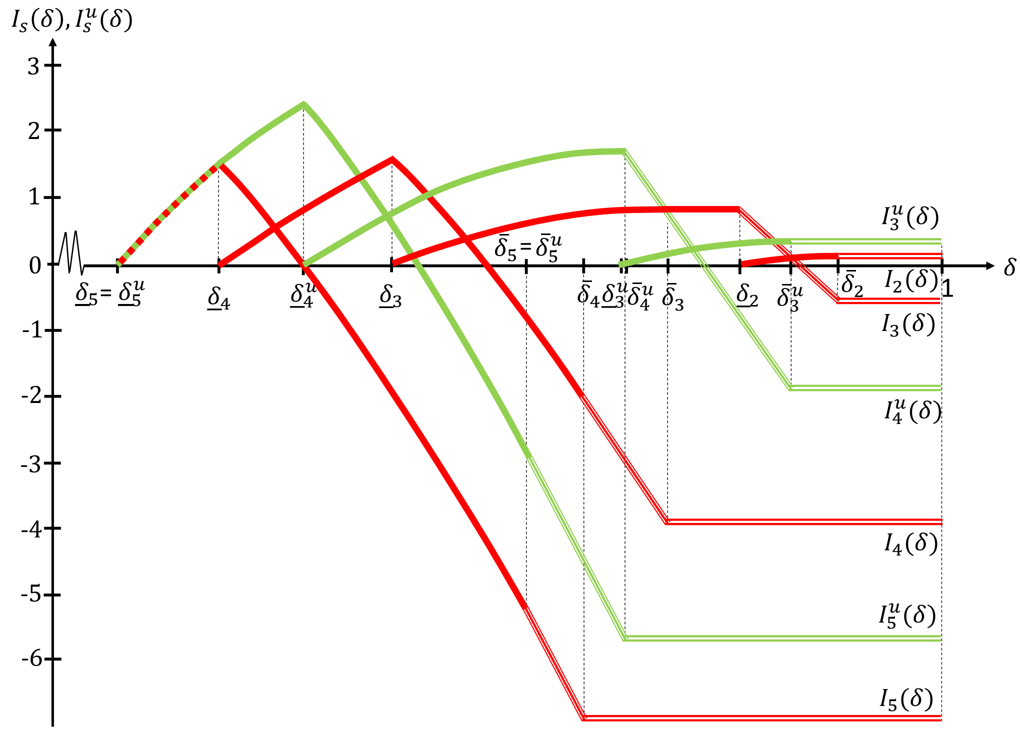


Figure 1: Internal stability measures $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ in default and umbrella regime

regime is also externally stable in the default regime:

$$\mathcal{E}_s^u(\delta) > 0 \;\Rightarrow\; \mathcal{E}_s(\delta) > 0. \tag{27}$$

The reverse is not true and $\mathcal{E}_s(\delta) \geq \mathcal{E}_s^u(\delta)$ holds for $\delta \in (\underline{\delta}_s^u, 1)$ with strict inequality for $s < n$.

Proof of Proposition 4 involves tedious case distinctions between constrained vs. unconstrained cartel prices and is provided in the Appendix. The observations rule out that a large or even all-encompassing cartel loses members because umbrella compensation is mandated. Whenever a cartel remains dynamically stable, its internal stability increases: freeriding on smaller cartels becomes less attractive as these cartels must lower prices the most to remain dynamically stable in the umbrella regime. The structural challenge is instead that non-members have incentives to join in. So operating cartels never shrink as legal standing is extended and rather tend to grow.

Consistent with findings for static market environments in, e.g., Donsimoni et al. (1986) or Shaffer (1995), the possibility that multiple cartel sizes are structurally stable in either regime can for linear demand be ruled out generically:

Proposition 5. *For any $\delta > \underline{\delta}_s$ ($\delta > \underline{\delta}_s^u$) there exists a generically unique stable cartel size $s^*(\delta)$ ($s^{*u}(\delta)$) in the default regime (umbrella regime) with $s^{*u}(\delta) \geq s^*(\delta)$.*

Proof. Leaving aside the null set of non-generic discount factors δ where $\mathcal{I}_s(\delta)$ or $\mathcal{I}_s^u(\delta)$ have zeros, internal (in)stability with s members rules out (implies) external stability with $s-1$ members. For any $\delta > \underline{\delta}_{\underline{s}}$, there is a largest $s \in \{\underline{s}, \ldots, n\}$ such that $\mathcal{I}_s(\delta) > 0$. This s uniquely combines internal stability of size s with external stability (= internal instability of size $s+1$). So the unique stable cartel size in the default regime is $s^*(\delta) := \max\left\{s \in \{\underline{s}, \ldots, n\} : \mathcal{I}_s(\delta) > 0\right\}$ and, analogously, that in the umbrella regime is $s^{*u}(\delta) := \max\left\{s \in \{\underline{s}^u, \ldots, n\} : \mathcal{I}_s^u(\delta) > 0\right\}$. $s^{*u}(\delta) \geq s^*(\delta)$ follows directly from Proposition 4. □

When inequality $s^{*u}(\delta) \geq s^*(\delta)$ is strict, i.e., detrimental structural stability effects of the umbrella regime induce a larger cartel, this is partly compensated by the beneficial dynamic stability effect captured by Proposition 1. Namely, profit maximizing prices are smaller in the umbrella than the default regime conditional on size (Prop. 1(ii)), even though a bigger cartel charges higher prices conditional on the compensation regime. It is conceivable that the cartel grows in the umbrella regime but still charges less than in the default situation. However, this requires the cartel in the umbrella regime to remain a partial one: if $s^{*u}(\delta) = n > s^*(\delta)$ then $p_n^{\circledast u}(\delta) = p_n^{\circledast}(\delta) > p_{s^*(\delta)}^{\circledast}(\delta)$ implies that prices rise, as exploited in the proof of Prop. 3.

Figure 2 plots prices in both regimes as a function of δ for the parameters considered in Figure 1 ($n = 5, a = 10, b = 1, k = 3, \alpha = 1/5, \beta = 1$, and $\tau = 1/2$). Endpoints of the three highlighted intervals reflect zeros of $\mathcal{I}_{s-1}(\delta)$, $\mathcal{I}_s(\delta)$, $\mathcal{I}_{s-1}^u(\delta)$ or $\mathcal{I}_s^u(\delta)$ (where the solution to $\mathcal{I}_n(\delta) = 0$ is close to $\underline{\delta}_4^u$). $p_{s+1}^{\circledast u}(\delta) > p_s^{\circledast}(\delta)$ holds for all $2 \leq s < n$. So welfare in the umbrella regime is lower than in the default regime for all δ such that $s^{*u}(\delta) > s^*(\delta)$. Prices fall instead and welfare rises if cartel size remains constant, i.e., $s^{*u}(\delta) = s^*(\delta)$.

For any discount factor inside the highlighted intervals, a regime change pushes up the size required for stability. Presuming that related transaction costs are second-order, the cartel grows by one member, the overall market price increases to $p_{s+1}^{\circledast u}(\delta) > p_s^{\circledast}(\delta)$, and welfare falls. For discount factors $\delta > \underline{\delta}_4^u$ outside of the highlighted intervals, a switch to the umbrella regime leaves the unique stable cartel size constant. The original cartel then continues its operations and lowers prices just enough to maintain dynamic stability.

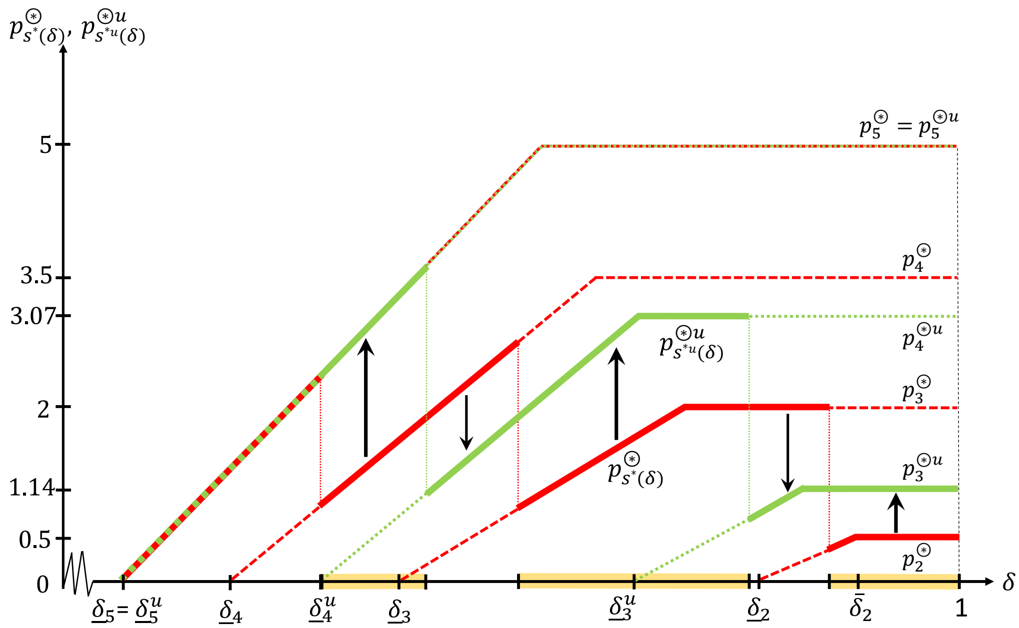


Figure 2: Optimal cartel prices for structurally stable cartel size $s^*(\delta)$ and $s^{*u}(\delta)$ in default and umbrella regime (δ s.t. $p^{\circledast u}_{s^{*u}(\delta)} > p^{\circledast}_{s^*(\delta)}$ highlighted)

The parametric example shows that price drops and welfare increases from extended legal standing are possible. Alas, they are restricted to two comparatively small intervals of δ. If we assumed that δ is a priori distributed uniformly on $(0, 1)$, like Katsoulacos et al. (2015), the average overcharge in the umbrella regime (equal to $\mathbf{E}p^{\circledast u}_{s^{*u}} = 1.01$ given $c = 0$) would be approximately 63% higher than in the default regime ($\mathbf{E}p^{\circledast}_{s^*} = 0.62$).[14] The associated consumer welfare in expectation falls from $44.33 + 0.52 = 44.85$ in the default to $42.61 + 1.29 = 43.90$ in the umbrella regime (standard consumer surplus + expected compensation payments), and more than 60% of the respective surplus accrues when cartels are unstable in either regime (i.e., for $\delta < \bar{\delta}_5 = \bar{\delta}^u_5 = 0.53$). Average profits net of expected compensation payments rise from 4.64 in the default to 5.16 in the umbrella regime.

It is noteworthy that even unatoned overpayments can be higher in the umbrella regime. For any $\delta > \bar{\delta}_2$, standing for umbrella victims raises the cartel size from $s = 2$ to $s' = 3$ and the overall market price from $p^{\circledast}_2 = 0.50$ to $p^{\circledast u}_3 = 1.14$. The expected uncompensated damage for customers in the default scenario comprises an overcharge of 0.50 for all $3k$ units purchased from outsiders, plus $D^R_2(0.50) = 0.50$ units from the cartel that remain uncompensated with probability $1 - \alpha = 4/5$ – which makes

[14]Numbers are rounded to two decimal places.

$0.50 \cdot (9 + 4/5 \cdot 0.50) = 4.70$ in total. In contrast, an overcharge of 1.14 on 8.86 units accrues in the umbrella scenario, yielding an expected uncompensated overcharge damage of $1.14 \cdot 4/5 \cdot 8.86 = 8.08$. It may seem paradoxical but total *uncompensated* overpayments increase by almost 72% after umbrella customers gain the right to be compensated.

4. Discussion

The finding that umbrella compensation increases deterrence and lowers prices for a given cartel size but can foster more encompassing cartels that raise prices is robust to a number of variations of above analysis. We will first consider two alternative criteria for cartel stability (Section 4.1). Then we discuss asymmetric production capacities (Section 4.2), endogenous risk of cartel detection (Section 4.3), fines that are based on revenue rather than excess profit (Section 4.4) as well as different liability and leniency assumptions (Section 4.5). We close with a numerical example in which firms face no capacity constraints but produce differentiated goods (Section 4.6).

4.1. Alternative Stability Conditions

The conception of structural stability à la d'Aspremont et al. (1983) in Section 2.3 is a common analytical default but assumes isolated myopic decisions to leave or enter a cartel. For instance, an all-inclusive cartel is deemed unstable if individual profits $\pi_n(p_n^{\circledast})$ are smaller than the profits $\tilde{\pi}(p_{n-1}^{\circledast})$ that would be achievable as a non-member of a cartel of $n-1$ firms. But suppose a member of the pertinent cartel with $n-1$ members could also earn higher profits by leaving and undercutting a stable cartel of $s' < n-1$ firms (after having triggered other exits perhaps). A forward-looking assessment of the stability of an all-inclusive cartel should then compare $\pi_n(p_n^{\circledast})$ to $\tilde{\pi}(p_{s'}^{\circledast})$ rather than to $\tilde{\pi}(p_{n-1}^{\circledast})$. Decisions (not) to exit or join a partial cartel may similarly be farsighted rather than focused on current size $s \pm 1$.

The challenge for a correspondingly refined notion of stability is its recursive nature: farsighted internal (external) stability of a cartel of s members depends on the stability of cartels with $s' < s$ ($s' > s$) members, which again depends on the stability of cartels with s members. Diamantoudi (2005) resolved this issue – drawing on von Neumann and Morgenstern's (1953, Sec. 30) concept of "stable sets" – by jointly characterizing a set σ of farsighted stable cartels: all cartels in σ are (i) *farsighted*

Cartel size s	$p_s^{\circledast}$	$\pi_s(p_s^{\circledast})$	$\tilde{\pi}(p_s^{\circledast})$	$p_s^{\circledast u}$	$\pi_s(p_s^{\circledast u})$	$\tilde{\pi}(p_s^{\circledast u})$
1	0	0	0	0	0	0
2	0.5	0.09	1.5	0	0	0
3	2	0.93	6	1.14	0.31	3.43
4	3.5	2.14	10.5	3.07	1.65	9.21
5	5	3.5	n.a.	5	3.5	n.a.

Table 1: Price, member and non-member profits for $\delta \in (\bar{\delta}_2, 1)$ (rounded)

internally stable, i.e., there exists no sequence of ultimately profitable individual cartel exits that leads to a smaller cartel inside σ; (ii) *farsighted externally stable*, i.e., there is no sequence of ultimately profitable individual cartel entries that leads to a larger cartel in σ; and (iii) every cartel not in σ violates farsighted internal or external stability. Checking this can be cumbersome and details of our findings change. But umbrella compensation can still cause cartel growth and raise prices.

To see this, consider our numerical example with linear demand again (Section 3). Table 1 summarizes prices and profits for $\delta > \bar{\delta}_2$ (cf. Figures 1 and 2). Only a cartel of size $s = 2$ ($s' = 3$) is myopically stable à la d'Aspremont et al. in the default (umbrella) regime. By contrast the sets of farsighted stable cartel sizes à la Diamantoudi are $\sigma = \{2, 4\}$ without and $\sigma' = \{3, 5\}$ with compensation for umbrella damages.[15] The uniqueness established in Proposition 5 is lost and we cannot conclude for the example that a stable cartel *must* be bigger in the umbrella regime. However, stable cartel sizes are uniformly shifted up. Cartel growth is hence not only possible but – assuming the same practical forces select between the elements of σ and σ' (e.g., profitability of joint entry, which favors $4 \in \sigma$ and $5 \in \sigma'$) – also likely.

For non-linear demand, several cartel sizes may satisfy myopic stability. The respective set of cartels must include the smallest element of the non-empty set of farsighted stable cartels (cf. Diamantoudi 2005, Theorem 5). This implies that conditions which ensure stability of a partial and instability of the all-inclusive cartel in the default regime also imply existence of some farsighted stable partial cartel. Moreover, conditions under that only the all-inclusive cartel is stable in the umbrella

[15]We use, e.g., "4" to denote any cartel with 4 members. 4 is farsighted internally stable in the default regime because (i) exit by a single firm leads to $3 \notin \sigma$ and (ii) sequential exit by two firms is unprofitable for the initial leaver; 4's external stability follows from $5 \notin \sigma$. 3 and 5 are not farsighted stable because entry (exit) by one firm would lead to $4 \in \sigma$ and be profitable. $\{2\}$ or $\{4\}$ alone form no set of farsighted stable cartels as $4 \notin \sigma$ or $2 \notin \sigma$ would contradict that all $s \notin \sigma$ violate internal or external stability. For $\delta \approx 1$, Theorem 3 of Diamantoudi (2005) guarantees a unique non-empty set of farsighted stable cartels.

regime imply that only this is farsighted stable. Therefore, Propositions 2 and 3 almost directly extend from stability à la d'Aspremont et al. to stability à la Diamantoudi.

The only qualification is that $\mathcal{I}_n(\delta) < 0$ does not rule out farsighted stability of the all-inclusive cartel in the default regime. Hence umbrella compensation may reduce a non-singleton set σ of farsighted stable cartels including n and some $\underline{s} \leq s < n-1$ to a singleton set $\sigma' = \{n\}$, which would allow for a constant price. An unambiguous increase of cartel size can however be concluded if we complement $\mathcal{I}_n(\delta) < 0$ and $\mathcal{I}^u_n(\delta) > 0$ in Proposition 2 by the requirement $n-1 \in \sigma$: then $n \notin \sigma$ because exit from the all-inclusive cartel is profitable and leads to $n-1 \in \sigma$. For this, in turn, it is sufficient that two firms together wield enough capacity to serve the market at cost because a partial cartel of size $n-2$ would then be unable to affect the market price and thus $\mathcal{I}_{n-1}(\delta) > 0$. So we could tighten the range of admissible capacities from $\mu a < k < q^m$ to $\max\{\mu a, \frac{a}{2}\} < k < q^m$ and then obtain an unqualified analogue of Proposition 3 for stability à la Diamantoudi.

Another limitation of the stability definition in Section 2.3 is that it disregards coordinated entry or exit. A partial cartel of $s \leq n-2$ members is deemed externally stable if outsider profits $\tilde{\pi}(p^{\circledast}_s)$ exceed cartel profits $\pi(p^{\circledast}_{s+1})$, even though two or more firms might find joint entry into a new cartel with $s' \geq s+2$ members profitable. This applies, for instance, to non-members of the stable cartel with $s = 2$ in Table 1: $\pi(p^{\circledast}_3) = 0.93 < \tilde{\pi}(p^{\circledast}_2) = 1.5 < \pi(p^{\circledast}_4) = 2.14$. Tightening our external and internal stability definitions to requiring $\tilde{\pi}(p^{\circledast}_s) \geq \pi(p^{\circledast}_{s'})$ for all $s' \in \{s+1, \ldots, n\}$ and $\pi(p^{\circledast}_s) \geq \tilde{\pi}(p^{\circledast}_{s'})$ for all $s' \in \{\underline{s}, \ldots, s-1\}$, respectively, would again modify details but not the message of our findings. In Table 1's example, we would start out in the default regime with a "jointly stable" cartel of $s = 4$ members and this would grow to $s' = 5$ with higher prices in the umbrella regime. Moreover, Propositions 2 and 3 would continue to hold because stability of cartels with $n-1$ or n members is not affected by coordination: joint entry is no issue and, by Proposition 1(iii), exit of several firms is always less profitable than leaving alone.

4.2. Asymmetric Capacities

The symmetry assumption in Section 2 greatly simplified the presentation and has been uncritical for our conclusions. As in Bos and Harrington's (2010) original setup, we could consider a total capacity $K_N = \sum_{i \in N} k_i$ in a market with firms $N = \{1, \ldots, n\}$

where individual firm capacities are $k_1 \geq k_2 \geq \ldots \geq k_n > 0$, and then keep track of the aggregate capacity of a cartel $C \subseteq N$ and the respective fringe $F = N \setminus C$ of non-members, denoted by $K_C = \sum_{i \in C} k_i$ and $K_F = \sum_{i \in F} k_i \equiv K_N - K_C$. One would then need to impose

$$a < K_N - k_1 \ \wedge \ k_1 < q^m, \tag{A1'}$$

instead of (A1) to ensure, first, that firms $2, \ldots, n$ have enough capacity to meet competitive demand (so firm 1 could not, on its own, earn profit by pricing above cost) and, second, any firm that undercuts its peers produces at capacity.

Assuming that the residual demand $D_{K_C}(p) = \max\left\{D(p) - K_F, 0\right\}$ of a cartel C, fines, and compensation payments are all split in proportion to capacity, the critical discount factors are the same for every cartel member.[16] They depend on the total cartel capacity K_C and evaluate to

$$\delta_{K_C}(p) = 1 - \frac{(1 - \alpha(\beta + \tau))\left[D(p) - K_F\right]}{K_C} \tag{28}$$

and

$$\delta^u_{K_C}(p) = 1 - \frac{(1 - \alpha(\beta + \tau))[D(p) - \mu^{-1} K_F]}{K_C} \tag{29}$$

in the default and umbrella regimes. Given (A1′), eqs. (28) and (29) define thresholds $\underline{\delta}_{K_C} < \bar{\delta}_{K_C} < 1$ and $\underline{\delta}^u_{K_C} < \bar{\delta}^u_{K_C} < 1$ as well as constrained profit maximizers $p^\circ_{K_C}(\delta)$ and $p^{\circ u}_{K_C}(\delta)$ in close analogy to the symmetric case. The set $\mathcal{W} \subset 2^N$ of cartels C that are profitable and dynamically stable for $\delta \approx 1$ in the default regime is characterized by

$$C \in \mathcal{W} \text{ if and only if } K_C = \sum_{i \in C} k_i > K_N - a \tag{30}$$

in contrast to the set $\mathcal{W}^u$ in the umbrella regime, which is characterized by

$$C \in \mathcal{W}^u \text{ if and only if } K_C = \sum_{i \in C} k_i > K_N - \mu a. \tag{31}$$

Since umbrella coefficient $\mu < 1$, greater cartel capacity is required for profitability in the umbrella regime, i.e. $\mathcal{W}^u \subseteq \mathcal{W}$. The corresponding sets $\mathcal{M} \subseteq \mathcal{W}$ and $\mathcal{M}^u \subseteq \mathcal{W}^u$ of *minimal profitable cartels* – defined by $C \in \mathcal{M}$ implying that $C' \notin \mathcal{W}$ for any $C' \subset C$,

[16] Bos and Harrington (2010) provide empirical and theoretical support for proportional sharing.

etc. – may however comprise cartels with very different numbers of firms and slightly different aggregate capacity, depending on the combinatoric properties of $k_1, \ldots, k_n$.[17]

Making the natural notational changes, Proposition 1 extends to asymmetric capacities one-to-one. The minimal discount factor needed to sustain a cartel with a capacity $K_C < K_N$ is greater in the umbrella regime and decreases in K_C; the overall cartel price is higher in the default regime and increases in K_C.

As in Bos and Harrington (2010), a cartel turns out to be structurally stable if its smallest member has no incentive to leave and the largest non-member has no incentive to join. The respective formal conditions can be stated in terms of profit per unit of capacity. Namely, a cartel $C \subseteq N$ is structurally stable in the default regime if

$$\mathcal{I}(\delta; C) := (p^{\circledast}_{K_C}(\delta) - c)D^{*}_{K_C}\Big(p^{\circledast}_{K_C}(\delta)\Big)\Big/K_C - \Big(p^{\circledast}_{K_C - \min_{i \in C} k_i}(\delta) - c\Big) \geq 0 \tag{32}$$

and

$$\mathcal{E}(\delta; C) := \Big(p^{\circledast}_{K_C}(\delta) - c\Big) - (p^{\circledast}_{K_C + \max_{j \notin C} k_j}(\delta) - c)D^{*}_{K_C + k_j}\Big(p^{\circledast}_{K_C + \max_{j \notin C} k_j}(\delta)\Big)\Big/(K_C + \max_{j \notin C} k_j) \geq 0. \tag{33}$$

Analogous conditions apply to the umbrella regime – the difference being that the respective net cartel demand $D^{*u}_{K_C}(\cdot)$ and cartel price $p^{\circledast u}_{K_C}(\cdot)$ replace $D^{*}_{K_C}(\cdot)$ and $p^{\circledast}_{K_C}(\cdot)$.

For a cartel $C = \{1, 2, \ldots, s\}$ composed of the s largest firms and some cartel $C' = \{i_1, i_2, \ldots, i_{s'}\}$ with $s' > s$ firms that have identical aggregate capacities $K_C = K_{C'}$, stability of the latter implies stability of the former: C''s smallest member $i_{s'}$ wields (weakly) less capacity than C's smallest member s and hence $\mathcal{I}(\delta; C') \leq \mathcal{I}(\delta; C)$; similarly the largest non-member $s+1$ of C has smaller capacity than the largest non-member of C', $i' = \min N \setminus C'$, so $\mathcal{E}(\delta; C') \leq \mathcal{E}(\delta; C)$.[18] Otherwise, the number of infringers allows no conclusions about comparative stability, even conditional on capacity. For example, consider $n = 5$ firms with $(k_1, \ldots, k_5) = (60, 30, 20, 20, 10)$. $C = \{1, 5\}$ and $C' = \{2, 3, 4\}$ wield identical capacity, as do $\tilde{C} = \{1, 3\}$ and $\tilde{C}' = \{2, 3, 4, 5\}$. The 2-member cartel C has greater external and smaller internal stability than C' with more members; but it is exactly opposite for $\tilde{C}$ and $\tilde{C}'$. Moreover, there will typically not exist a unique

[17] $\mathcal{W}$ and $\mathcal{W}^u$ can be studied as *weighted simple games* à la von Neumann and Morgenstern (1953, Sec. 48-55). See, e.g., Taylor and Zwicker (1999) or Napel and Welter (2021).

[18] It follows, echoing Theorem 12 of Bos and Harrington (2010), that some stable cartel $C = \{1, 2, \ldots, s^{\circ}\}$ exists if $\delta \approx 1$. For linear demand, s° is generically unique but some cartels like our 'non-consecutive' C' above may be stable too (cf. Bos and Harrington 2010, Theorem 13). Our Proposition 5 hence has no direct analogue under asymmetry.

capacity K_C that is associated with stability.

If two cartels C and C' are stable, it is therefore hard to say if one or the other is more likely to form. An exception applies when $C' \subset C$. Then firms inside set $C \setminus C'$ have a strict incentive to jointly establish C (cf. fn. 10): stability of C generically entails $\pi_i(\delta; C) > \tilde{\pi}_i(\delta; C \setminus \{i\})$ for all $i \in C$, implying that $C \setminus C'$ contains two firms or more. Since prices increase in cartel capacity, any non-member profit $\tilde{\pi}_i(\delta; C \setminus \{i\})$ exceeds the corresponding non-member profit that would arise if yet more firms left C. Therefore $\pi_i(\delta; C) > \tilde{\pi}_i(\delta; C')$ for all $i \in C \setminus C'$ and we can presume C rather than $C' \subset C$ forms.

If we assume that, Proposition 2 extends straightforwardly: provided some (partial) cartel is dynamically stable in the umbrella (default) regime, legal standing of customers of non-infringing firms lowers welfare if $\mathcal{I}(\delta; N) < 0$ and $\mathcal{I}^u(\delta; N) > 0$.

Ensuring that these conditions are satisfied in analogy to Proposition 3 is more tedious than in the symmetric case. One can, however, fix a set $\mathcal{K}$ of capacities satisfying $\mu a < k < q^m$ for all $k \in \mathcal{K}$ and any assignment $\kappa\colon \mathbb{N} \to \mathcal{K}$ of capacities to firms. Then there must exist $\hat{n} > 2$ such that for all $N = \{1, \ldots, n\}$ with $n \geq \hat{n}$ we have $\mathcal{I}(\delta; N) < 0$ and $\mathcal{I}^u(\delta; N) > 0$ provided $\delta \in (\hat{\delta}_{K_N}, 1)$ where $\hat{\delta}_{K_N} := \max\{\underline{\delta}_{K_N - \min_{i \in N} k_i}, \underline{\delta}^u_{K_N}\} < 1$. The proof is analogous to the symmetric case but involves additional attention on verifying condition (A1′).

Since the price drop caused by exit of cartel member $i \in C$ increases in k_i – i.e., smaller members always have greater incentive to become an outsider – an equal distribution of a given cartel capacity K_C minimizes the maximal temptation to leave. So an industry with highly asymmetric capacities is more prone, ceteris paribus, to exhibit a partial cartel. Therefore, because only industries with partial cartels are affected by the legal standing of umbrella victims, asymmetries in real markets increase the relevance of our call for caution and justifications of umbrella compensation that provide other reasons than an alleged contribution to effective competition.

4.3. Endogenous Detection Probabilities

We have throughout assumed a fixed cartel detection probability $\alpha \in (0, 1)$. In reality, more enforcement effort may be exerted, the worse prevailing prices compare to 'yardstick' markets. Detection can also depend on cartel size, as more members make information leaks more likely, and possibly even the compensation regime: entitling more customers may prompt more complaints that authorities can follow up on.

Including a flexible detection probability changes the relative importance of beneficial deterrence vs. detrimental structural effects of umbrella compensation, but the qualitative conclusions from the baseline model are robust. Suppose, for illustration, that the umbrella regime per se renders detection of partial cartels more likely. The increase of critical discount factors from $\underline{\delta}_s$ and $\bar{\delta}_s$ in the default regime to $\underline{\delta}_s^u$ and $\bar{\delta}_s^u$ for $s < n$ in the umbrella regime would then be more pronounced (see eqs. (14) and (15)). Increased detection risk also strengthens the beneficial price effects captured by Proposition 1 and ranges of δ where $p^{\circledast u}_{s^{*u}(\delta)} > p^{\circledast}_{s^*(\delta)}$ in Figure 2 would shrink. However, the latter remain non-empty. Moreover, the standing of umbrella victims cannot plausibly alter detection odds, patience requirements, or prices for $s = n$. Unchanged $p_n^{\circledast u}$ and a reduced price $p_{n-1}^{\circledast u}$ imply a greater internal stability gain for the all-inclusive cartel. Therefore even additional cases of detrimental cartel growth may arise with regime-dependent α compared to the baseline.

Alternatively, let α be an increasing function of the price p selected by the cartel. The respective net demand functions $\hat{D}_s^*(p)$ and $\hat{D}_s^{*u}(p)$ then have slopes

$$\frac{\partial \hat{D}_s^*}{\partial p} = \frac{\partial D_s^*}{\partial p} - \underbrace{\frac{(\beta + \tau)}{s}\Big[D(p) - (n-s)k\Big]\frac{\partial \alpha}{\partial p}}_{>0} \tag{34}$$

and

$$\frac{\partial \hat{D}_s^{*u}}{\partial p} = \frac{\partial D_s^*}{\partial p} - \underbrace{\frac{1}{s}\Big[\beta D(p) + \tau\Big(D(p) - (n-s)k\Big)\Big]\frac{\partial \alpha}{\partial p}}_{>0}. \tag{35}$$

They are steeper than $D_s^*(p)$ and $D_s^{*u}(p)$ in Section 2 because a marginal price increase not just lowers net sales but additional units must be deducted to offset increased expected fines and compensation if $\partial \alpha / \partial p > 0$.

Assuming $(p-c)\cdot \hat{D}_s^*(p)$ and $(p-c)\cdot \hat{D}_s^{*u}(p)$ remain strictly concave, the corresponding unconstrained profit maximizers are characterized by somewhat unwieldy analogues of eqs. (6) and (7). They are lower than before but we still have $p_s^{\circledast u} < p_s^{\circledast}$ for all $s < n$ and all comparative statics remain as in Proposition 1. The conditions in Propositions 2 and 3 continue to imply a negative welfare assessment of umbrella compensation. So an increasing probability $\alpha(p)$ renders net demand more elastic, which lowers prices, but leaves Section 2's key findings unaffected.

If we let the probability of detection grow directly (rather than via higher prices) in a cartel's size s as in Bos and Harrington (2015), a stable cartel is ceteris paribus less likely to attract additional members in the umbrella regime. However, replacing constant $\alpha < 1$ by an increasing function $\alpha(s)$ with sufficient variation makes partial cartels (facing low risk of detection) more likely vis-à-vis all-encompassing cartels (facing high risk). Echoing the discussion in Section 4.2, this generates more situations with victims of umbrella pricing and ambiguous welfare consequences of extending legal standing. Even pronounced size dependence of detection allows for detrimental stability effects of adopting the umbrella regime. For instance, $\alpha(s)$ might increase from nearly zero for $s \leq n - 1$ to slightly below 1 for $s = n$ (with β and τ such that condition (A2) is satisfied). This would yield a high threshold $\hat{n}$ but Proposition 3 continues to hold.

The numerical example in Section 3 is very robust, too. For $\delta > \bar{\delta}_2$ and fixed probability $\alpha = 1/5$, the umbrella regime raises the stable cartel size from $s^* = 2$ to $s^{*u} = 3$ and the overall cartel price from $p_2^{\circledast} = 0.5$ to $p_3^{\circledast u} = 1.14$. If we rather assume a function $\alpha(s)$ with $\alpha(3) > \alpha(2) = 1/5$, sizes s^* and s^{*u} remain equal to the respective minimal profitable sizes $\underline{s} = 2$ and $\underline{s}^u = 3$, i.e., the cartel grows. Moreover, $p_2^{\circledast} < p_3^{\circledast u}$ for all $\alpha(3) < 2/7$. So an extra member would need to raise the detection probability by more than 42% for the umbrella regime to improve rather than lower welfare.

4.4. Revenue-based Public Fines

A revenue-based public penalty structure is more common in practice than the profit-based one that we have assumed above (e.g., cartel penalties in the EU are levied as a fraction of the relevant turnover, capped by 10% of overall annual turnover). Contributions by, e.g., Bageri et al. (2013) and Katsoulacos et al. (2015, 2020) have shown that revenue-based penalties result in higher prices than fines based on profit and that, importantly, their price-raising effect is amplified if public fines are complemented by private damage claims. Extending private antitrust enforcement by entitling more customers to compensation of their overcharges could, therefore, conceivably change the conclusion that the umbrella regime reduces prices conditional on an unchanged cartel size $s < n$ (Proposition 1(ii)).

This and the related findings in Proposition 1 are robust, however, to basing fines on revenue if we add a mild curvature restriction on demand. Namely, take public fines to be a fixed share $\tau > 0$ of a cartel member's revenue, while private compensation stays a multiple $\beta > 0$ of applicable overcharge damages. Per period profits in the default and umbrella regime then become

$$\hat{\pi}_s(p) = \Big[(p-c)(1-\alpha\beta) - \alpha\tau p\Big](D(p) - (n-s)k)/s \tag{36}$$

$$\hat{\pi}_s^u(p) = \Big[(p-c-\alpha\tau p)(D(p)-(n-s)k) - (p-c)\alpha\beta D(p)\Big]/s \tag{37}$$

and the corresponding maximizers $\hat{p}_s^*$ and $\hat{p}_s^{*u}$ are characterized in direct analogy to equations (6) and (7) by

$$\Big(1-\alpha(\beta+\tau)\Big)D(p) + \Big[(p-c)(1-\alpha\beta) - \alpha\tau p\Big]D'(p) = \Big(1-\alpha(\beta+\tau)\Big)(n-s)k \tag{38}$$

and

$$\Big(1-\alpha(\beta+\tau)\Big)D(p) + \Big[(p-c)(1-\alpha\beta) - \alpha\tau p\Big]D'(p) = \Big(1-\alpha(\beta+\tau)\Big)(n-s)k/\mu. \tag{39}$$

The identical left-hand sides of (38) and (39) are decreasing in p if

$$\underbrace{2\cdot\Big(1-\alpha(\beta+\tau)\Big)D'(p) + \Big[(p-c)\Big(1-\alpha(\beta+\tau)\Big)\Big]D''(p)}_{:=A<0} + \alpha\tau c D''(p) < 0 \tag{40}$$

with $A < 0$ guaranteed by the concavity of monopoly profits. We hence obtain $\hat{p}_s^* > \hat{p}_s^{*u}$ for $s < n$ if $D''(p) < |A|/(\alpha\tau c)$, i.e., $D(p)$ is "not too convex".

Constrained optimizers $\hat{p}_s^\circ(\delta)$ and $\hat{p}_s^{\circ u}(\delta)$ have the same properties as for profit-based fines. Also the key finding that obliging cartel members to compensate umbrella losses can do more harm than good is robust. In particular, the right-hand side of eq. (38) decreases in cartel size s exactly as that in eq. (6). Therefore, for concave or not too convex demand, prices increase in cartel size. In particular, the price set by the all-inclusive cartel with n members, which coincides in both regimes, exceeds the price set by any stable partial cartel in the default regime. Hence Proposition 2 continues to hold and the sufficient condition $\mathcal{I}_n^u > 0 > \mathcal{I}_n$ remains satisfied for a non-negligible set of parameters. The respective constraint $\mu a < k < q^m$ in Proposition 3 can even be relaxed because $\hat{\pi}_s^u(p) < \pi_s^u(p)$ for $c > 0$ tightens the minimal size requirement for a profitable cartel.

4.5. Liability and Leniency Assumptions

When we equated the one-off deviation profit with approximately $(p - c)k$ above, we followed, e.g., Motta and Polo (2003) or Katsoulacos et al. (2015, 2020) in assuming that only active cartel members are fined and liable for redress. This can alternatively be interpreted as a regime in which it is optimal for a deviating cartel member to enter a leniency program (cf. Aubert et al. 2006) and where this fully exempts it from public and private payment obligations. The situation is generally less comfortable in reality. For instance, the US Antitrust Criminal Penalty Enhancement and Reform Act of 2004 (ACPERA) limits liability to single damages (instead of treble) but calls for compensation of an applicant's direct customers. Leniency rules in the EU may require compensation even of customers of other cartel members and of umbrella damages in special cases (e.g., bankruptcy of former cartel members).

If consequently we consider deviation profits below $(p - c)k$ then the dynamic cartel stability conditions are relaxed. This does not affect our findings, however. For instance, the critical discount factors $\delta_s(p)$ and $\delta_s^u(p^u)$ identified in eq. (5) would become

$$\hat{\delta}_s(p) = \frac{nk - D(p)}{sk} \quad \text{and} \quad \hat{\delta}_s^u(p^u) = \frac{(1 - \alpha\beta - \alpha\tau)(nk - D(p^u))}{k(s - \alpha\tau s - n\alpha\beta)} \tag{41}$$

if also a deviating firm is fined in case of detection, must compensate its direct clients and has to bear an equal share of possible umbrella compensation. The thresholds in eq. (41) are smaller than $\delta_s(p)$ and $\delta_s^u(p^u)$ in eq. (5). They go with smaller minimal discount factors $\underline{\hat{\delta}}_s := \hat{\delta}_s(c + 2\varepsilon)$ and $\underline{\hat{\delta}}_s^u := \hat{\delta}_s^u(c + 2\varepsilon)$, and imply higher cartel prices $\hat{p}_s^{\circ}(\delta)$ and $\hat{p}_s^{\circ u}(\delta)$ if the dynamic stability constraint is binding, while unconstrained cartel prices remain unaffected. The umbrella regime continues to have a beneficial deterrence effect, i.e., $\underline{\hat{\delta}}_s^u > \underline{\hat{\delta}}_s$ and $\hat{p}_s^{\circledast u}(\delta) < \hat{p}_s^{\circledast}(\delta)$ for any $s < n$. The comparative statics with respect to cartel size s also remain as in Proposition 1.

The profits of permanent cartel outsiders and non-deviating insiders determine the structural stability of a cartel. They vary in the applicable liability and leniency assumptions only if the dynamic stability constraint is binding. Even in that case, Propositions 2 and 3 remain valid. In particular, partial cartels always charge less than the all-inclusive one and profitability in the umbrella regime requires collusion by more firms than in the default regime ($\underline{s}^u \geq \underline{s}$). Hence leniency and liability assumptions that differ from the baseline model[19] modify the deterrence effect of the umbrella regime but neither change its sign, nor the possibility that it is overwhelmed by cartel growth.

4.6. Differentiated Products

The horizontal price spillovers that harm non-cartel customers are higher, the greater the substitutability between cartel and non-cartel products (see, e.g., Inderst et al. 2014). This has led Blair and Durrance (2018, p. 252) to conclude that "[t]he economic argument in favor of antitrust standing for customers of the nonconspirators is most compelling when the product is homogeneous".

The problems that we identified for the supposedly most compelling case in favor of the umbrella regime are, alas, not restricted to it. Adverse cartel size effects can arise just as well in markets for vertically or horizontally differentiated products.

Differentiation reduces the optimal adjustment of non-member prices in response to cartel formation. This diminishes umbrella damages and their internalization via expected compensation obligations, mitigating the reduction of cartel prices in the umbrella regime. Differentiation however does not change the logic behind the baseline findings: umbrella claims lower the profitability of smaller partial cartels but not of an all-inclusive one. This holds also if there exist other (non-differentiation) reasons for competitive prices in excess of marginal costs, such as decreasing returns to scale or imperfectly attentive consumers.

Unfortunately, the tractability of Bos and Harrington's (2010) model – with particularly simple price reactions of cartel outsiders – does not extend well to imperfect substitutes. For a concrete illustration of how the umbrella regime can cause cartel growth and reduce welfare for differentiated goods, we therefore adapt a model of endogenous cartel formation suggested by Merker (2019). It considers n firms that can produce at a unit cost $c \geq 0$ without restriction. Each firm i faces demand

$$D_i(p_1, \ldots, p_n) = a - bp_i + \sum_{j \neq i} dp_j \tag{42}$$

[19]We exclude an entirely different liability regime that would make cartel outsiders, who enjoyed supracompetitive profits by undercutting the cartel, compensate umbrella victims, i.e., their clients. This would not just create some intricate legal problems (cf. Inderst et al. 2014, p. 742), but entail detrimental structural stability effects (by reducing the attractiveness of being an outsider vs. joining a cartel) *without* beneficial dynamic stability effects.

Cartel size s	$p_s^{\circledast}$	$\tilde{p}_s^{\circledast}$	$\pi_s(p_s^{\circledast})$	$\tilde{\pi}(p_s^{\circledast})$	$p_s^{\circledast u}$	$\tilde{p}_s^{\circledast u}$	$\pi_s(p_s^{\circledast u})$	$\tilde{\pi}(p_s^{\circledast u})$
1	18.09	18.09	522.95	522.95	18.09	18.09	522.95	522.95
2	18.91	18.24	526.35	543.29	18.47	18.16	523.72	532.49
3	20.33	18.67	543.12	601.11	19.89	18.55	536.36	585.46
4	22.25	19.41	577.37	708.85	21.80	19.27	567.51	687.58
5	24.98	20.33	627.86	900.19	24.54	20.19	618.12	872.92
6	29.19	22.21	731.23	1281.72	28.81	22.08	721.78	1251.03
7	36.48	36.48	930.56	n.a.	36.48	36.48	930.56	n.a.

Table 2: Unconstrained (non-)member prices and profits for differentiated goods (rounded)

for $a > c$ and $b, d > 0$. The extensive computations by Merker (2019) demonstrate that the comparative statics for stable cartel sizes are already quite non-trivial without antitrust enforcement. Obtaining general results when we add fines and private litigation is therefore beyond the scope of the present investigation. However, it is not hard to verify the possibility of detrimental size effects of the umbrella regime.

In analogy to Table 1, we collect the price and profit implications for an example parameter configuration – namely, $n = 7$, $a = 90$, $b = 8$, $d = 1.1$, $\alpha = 0.14$, $\beta = 1$, $\tau = 0$ and $c = 10$ – in Table 2. $p_s^{\circledast}$ and $p_s^{\circledast u}$ again denote regime-specific equilibrium prices of cartel members; $\tilde{p}_s^{\circledast}$ and $\tilde{p}_s^{\circledast u}$ indicate those of non-members.[20]

The stable cartel size in the default regime can be seen to be $s^* = 2$ in the table. The corresponding cartel-induced overcharge is $p_2^{\circledast} - p_1^{\circledast} = 18.91 - 18.09 = 0.82$ for all products sold by cartel members and $\tilde{p}_2^{\circledast} - \tilde{p}_1^{\circledast} = 18.24 - 18.09 = 0.15$ for those of non-members. In the umbrella regime, only a cartel of size $s^{*u} = 3$ is stable. Corresponding overcharges increase to $p_3^{\circledast u} - p_1^{\circledast u} = 1.80$ and $\tilde{p}_3^{\circledast u} - \tilde{p}_1^{\circledast u} = 0.45$. So all customers face higher prices. And though non-cartel customers are newly compensated in the umbrella regime, unatoned overpayments increase: given a probability $1 - \alpha = 0.86$ of non-detection, expected uncompensated overcharges to cartel and non-cartel customers rise from a total of 135.89 in the default regime to 372.88 in the umbrella regime.

5. Concluding Remarks

The key message of above analysis is that the size of cartels should be expected to vary with the adopted compensation regime, just as cartel profitability and the time

[20] Merker's setup requires higher n for a stable partial cartel to exist than our setup in Section 3. The critical discount factor increases in cartel size s, i.e., optimal prices are unconstrained if $\delta \geq \bar{\delta}_7$.

preference required to sustain collusion do. The effect of extending legal standing to umbrella victims on the latter is beneficial, as one would expect. However, there is an effect on cartel size that goes into the opposite direction: large cartels that were hitherto unstable can become stable. They could not form when freeriding on a cartel with comparatively small market coverage was sufficiently lucrative, but can when these small cartels are deterred by the obligation to compensate umbrella victims.

As illustrated in Section 3, it is possible that the unique stable cartel size remains the same in both legal regimes, or that prices fall even though cartel size increases. There are many configurations, however, in which this is not the case. Then cartels grow because of the umbrella regime. They charge higher prices and may induce greater uncompensated damages in expectation – despite a greater set of victims having standing in court.[21]

In light of these observations it would seem quite optimistic to uphold the presumption of extended standing "making a significant contribution to the maintenance of effective competition" (CJEU C-557/12 2014, 23) or "hav[ing] greater deterrent effect than recovery limited to direct purchasers" (Blair and Maurer 1982). Of course, we do not doubt that good arguments for entitling *all* victims of an antitrust infringement to redress, no matter whether they were harmed directly or indirectly, can be put forward. The point is that, in contrast with first intuition, judges and policymakers should emphasize reasons of distributional justice or legal principle – not deterrence and effective competition.

[21]Let us point to Argenton et al. (2020, p. 269) for the related wider question: "[W]here to stop the causal chain set in motion by the initial liability-generating behaviour?" Upstream firms that supplied (non-) cartel members, producers of complement goods and their suppliers, etc. may all have suffered indirect harm and qualify as victims, too. Going beyond a partial equilibrium framework one might even pinpoint ripple effects on wages or bond rates.

Appendix: Proof of Proposition 4

First consider part (i). That $\mathcal{I}^u_s(\delta) > 0$ does not imply $\mathcal{I}_s(\delta) > 0$ is obvious from our example. It is also obvious that implication (26) is true for $\delta \in (\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$ because $\mathcal{I}^u_s(\underline{\delta}^u_s) = 0$ and $\mathcal{I}^u_s(\delta)$ strictly increases on $(\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$. (Recall that a cartel of s firms is dynamically stable in both regimes if $\delta \in (\underline{\delta}^u_s, 1)$.)

It remains to show $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on the interval $(\underline{\delta}^u_{s-1}, 1)$ where sub-cartels of size $s - 1 \geq \underline{s}^u$ would be profitable and dynamically stable. Depending on market parameters and considered cartel size s, this interval can be split into quadratic, linear and constant parts of $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ differently from Figure 1. Namely, several pairwise comparisons of the critical discount factors that determine if $p^{\circledast}_s(\delta) = p^{\circ}_s(\delta)$ or p^*_s and if $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ or p^*_{s-1} in eq. (18), and its analogue, can go either way. Figure 3 shows the Hasse diagram of the corresponding partially ordered set. This gives rise to seven possible cases. Before turning to each, we establish some properties that hold whenever one or two pairwise comparisons, e.g., between $\underline{\delta}_{s-1}$ and $\bar{\delta}_s$, go in a particular way.

Claim 1. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} < \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is strictly concave and decreasing on $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left((\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ and linearly decreasing on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ $\left((\bar{\delta}^u_s, \bar{\delta}^u_{s-1})\right)$. $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ falls faster in the latter intervals than in the former.

Proof. For $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ dynamic stability constraints are binding for cartel sizes s and $s-1$. Substituting $p^{\circledast}_s(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast u}_s(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ and $p^{\circledast u}_{s-1}(\delta) = p^{\circ u}_{s-1}(\delta)$ into $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ and abbreviating $e := 1 - \alpha(\beta + \tau) \in (0, 1)$, one obtains

$$\mathcal{I}_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta + k\delta(n+1-s\delta) - k\alpha(1+n\delta-s\delta)(\beta+\tau)\Big] \tag{43}$$

and

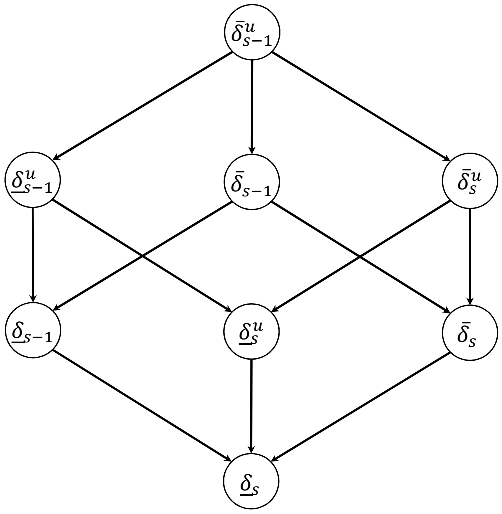
$$\mathcal{I}^u_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta - k\alpha\tau + k\delta\Big(n+1-s\delta-(n-s)\alpha\tau\Big)\Big] \tag{44}$$

after some algebra. Corresponding derivatives with respect to δ are

$$\frac{\partial\mathcal{I}_s(\delta)}{\partial\delta} = \frac{k}{b}\Big[-a + k(n-s) - k\Big(s(2\delta-1)-1\Big)e^{-1}\Big] \tag{45}$$

and

$$\frac{\partial\mathcal{I}^u_s(\delta)}{\partial\delta} = \frac{k}{b}\Big[-a + k\Big(1+n-2s\delta-(n-s)\alpha\tau\Big)e^{-1}\Big] \tag{46}$$



Case	Ordering
1	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s-1}$
2	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
3	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$
4	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
5	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$
6	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
7	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$

Figure 3: Hasse diagram for critical discount factors ($x \to y$ indicating $x > y$) and compatible orderings that partition $(\underline{\delta}^u_{s-1}, 1)$

with

$$\frac{\partial^2 \mathcal{I}_s(\delta)}{\partial \delta^2} = \frac{\partial^2 \mathcal{I}^u_s(\delta)}{\partial \delta^2} = \frac{-2k^2 s}{be} < 0. \tag{47}$$

So $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ are strictly concave in δ.

Substituting $\delta = \underline{\delta}_{s-1}$ from eq. (22) into (45) yields

$$\frac{\partial \mathcal{I}_s(\underline{\delta}_{s-1})}{\partial \delta} = \frac{k}{be}\left(\frac{e(s+1)(a-kn)}{s-1} - k(s\alpha(\beta+\tau)-1)\right) < 0. \tag{48}$$

Inequality (48) is satisfied iff

$$\frac{(s+1)(a-kn)}{s-1} < \frac{k(s\alpha(\beta+\tau)-1)}{1-\alpha(\beta+\tau)}. \tag{49}$$

Making the LHS as large as possible, that is, substituting $a = k(n-1)$ yields

$$\begin{aligned}
&-\frac{s+1}{s-1} < \frac{s\alpha(\beta+\tau)-1}{1-\alpha(\beta+\tau)} \\
\Leftrightarrow\ & -(s+1)(1-\alpha(\beta+\tau)) < (s-1)(s\alpha(\beta+\tau)-1) \\
\Leftrightarrow\ & 0 < 2 - \alpha(\beta+\tau) + s^2\alpha(\beta+\tau) - 2s\alpha(\beta+\tau) \\
\Leftrightarrow\ & 0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha(\beta+\tau)(s-2)}_{>0}.
\end{aligned} \tag{50}$$

Similarly substituting $\delta = \underline{\delta}^u_{s-1}$ from eq. (25) into (46) gives

$$\frac{\partial \mathcal{I}^u_s(\underline{\delta}^u_{s-1})}{\partial \delta} = \frac{-k}{b(s-1)e}(-a(1+s)e - k(n(1+s)(-1+\alpha\tau) + (s-1)(1-s\alpha\tau))) < 0 \tag{51}$$

which is satisfied iff

$$0 < -a(1+s)e - k(n(1+s)(-1+\alpha\tau) + (s-1)(1-s\alpha\tau)). \tag{52}$$

Making the RHS as small as possible by substituting $a = k(n-1)$ yields

$$\begin{aligned}
& 0 < -(n-1)(1+s)e - n(1+s)(-1+\alpha\tau) - (s-1)(1-s\alpha\tau) \\
\Leftrightarrow\ & 0 < -n(1-\alpha\beta-\alpha\tau) - ns(1-\alpha\beta-\alpha\tau) + (1-\alpha\beta-\alpha\tau) + s(1-\alpha\beta-\alpha\tau) + n - n\alpha\tau + ns \\
& - ns\alpha\tau - s + s^2\alpha\tau + 1 - s\alpha\tau \\
\Leftrightarrow\ & 0 < 2 - \alpha\beta - \alpha\tau + s^2\alpha\tau - 2s\alpha\tau + n\alpha\beta + ns\alpha\beta - s\alpha\beta \\
\Leftrightarrow\ & 0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha\tau(s-2)}_{\geq 0} + \underbrace{s\alpha\beta(n-1)}_{>0}.
\end{aligned} \tag{53}$$

So both derivatives $\frac{\partial \mathcal{I}_s(\delta)}{\partial \delta}$ and $\frac{\partial \mathcal{I}^u_s(\delta)}{\partial \delta}$ are negative at the respective left endpoints of $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$. By (47) they are falling. Hence the slopes of $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ must be negative for all $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$, respectively.

For $\delta \in (\bar{\delta}_s, \bar{\delta}_{s-1})$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Profits of s cartel members consequently become constant, too, and $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have slope

$$-k\frac{\partial p^{\circ}_{s-1}(\delta)}{\partial \delta} = -k\frac{\partial p^{\circ u}_{s-1}(\delta)}{\partial \delta} = -\frac{(s-1)k^2}{eb}. \tag{54}$$

This is less than the slopes identified for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ in eq. (45) $\left(\text{eq. (46)}\right)$ where cartel profits still increase in δ. This proves Claim 1. ■

Claim 2. If $\underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is linearly decreasing for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_{s-1})\right)$.

Proof. For $\delta > \underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\delta > \underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Collusive profits of s members then are constant in δ whereas cartel prices and profits of an outsider are linearly increasing in δ for a cartel of size $s-1$. Hence $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have the slope already identified in eq. (54). ■

Claim 3. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$ then $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$.

Proof. $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ are then given by eqs. (43) and (44) above. Hence

$$\begin{aligned}\mathcal{I}^u_s(\delta) - \mathcal{I}_s(\delta) &= \frac{k}{be}\Big[-k\alpha\tau - k\delta(n-s)\alpha\tau + k\alpha(1+n\delta - s\delta)(\beta+\tau)\Big] \\ &= \frac{k^2\alpha\beta}{be}\Big(1+(n-s)\delta\Big) > 0.\end{aligned} \tag{55}$$

■

Claim 4. $\Delta\bar{\delta}^u_s := \bar{\delta}^u_{s-1} - \bar{\delta}^u_s > \bar{\delta}_{s-1} - \bar{\delta}_s := \Delta\bar{\delta}_s$.

Proof. Applying eq. (8) and simplifying yields

$$\Delta\bar{\delta}_s = \frac{-e(a-(n-s)k)2(s-1)k + 2ske(a-(n-(s-1))k)}{4sk^2(s-1)} = -\frac{(a-kn)e}{2k(s-1)s} \tag{56}$$

and similarly eq. (24) gives

$$\Delta\bar{\delta}^u_s = -\frac{ae - kn(1-\alpha\tau)}{2k(s-1)s}. \tag{57}$$

So

$$\Delta\bar{\delta}^u_s - \Delta\bar{\delta}_s = \frac{-ae + kn(1-\alpha\tau) + ae - kn(1-\alpha\beta-\alpha\tau)}{2k(s-1)s} = \frac{n\alpha\beta}{2(s-1)s} > 0. \tag{58}$$

■

Claim 5. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}^u_{s-1}, 1)$.

Proof. $\delta > \bar{\delta}^u_{s-1}$ implies $\delta > \bar{\delta}_{s-1}$ and $\delta > \bar{\delta}^u_s \geq \bar{\delta}_s$. Hence, all relevant dynamically stable cartels can choose the respective unconstrained profit maximizers $p^{\circledast}_{s-1}(\delta) = p^*_{s-1} = \frac{1}{2b}\Big(a-(n-(s-1))k\Big)$, $p^{\circledast u}_{s-1}(\delta) = p^{*u}_{s-1} = \frac{1}{2b}\Big(a-(n-(s-1))k\cdot\mu^{-1}\Big)$, $p^{\circledast u}_s(\delta) = p^{*u}_s$ and $p^{\circledast}_s(\delta) = p^*_s$. Again abbreviating $e := 1-\alpha(\beta+\tau) \in (0,1)$ and using $\mu^{-1} = 1 + \frac{\alpha\beta}{e}$, $\mathcal{I}_s(\delta) < \mathcal{I}^u_s(\delta)$ holds iff

$$
\begin{aligned}
& \frac{e\Big(a-(n-s)k\Big)^2}{4bs} - \frac{a-(n-s+1)k}{2b}k < \frac{e\Big(a-(n-s)k\mu^{-1}\Big)^2}{4bs} - \frac{a-(n-s+1)k\mu^{-1}}{2b}k \\
\Leftrightarrow\quad & e\Big(a-(n-s)k\Big)^2 + 2ks\Big((n-s+1)k-a\Big) < \\
& \qquad e\Big(a-(n-s)k-\frac{\alpha\beta}{e}(n-s)k\Big)^2 + 2ks\Big((n-s+1)k\mu^{-1}-a\Big) \\
\Leftrightarrow\quad & 2k^2s(n-s+1) < -2e\Big(a-k(n-s)\Big)\frac{\alpha\beta}{e}k(n-s) + e\Big(\frac{\alpha\beta}{e}k(n-s)\Big)^2 + 2k^2s(n-s+1)\mu^{-1} \\
\Leftrightarrow\quad & 0 < 2\alpha\beta\Big(k(n-s)-a\Big)k(n-s) + \frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2 - 2k^2s(n-s+1)(1-\mu^{-1}) \\
\Leftrightarrow\quad & 0 < 2\alpha\beta\Big(k(n-s)-a\Big)k(n-s) + \frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2 + 2k^2s(n-s+1)\frac{\alpha\beta}{e} \\
\Leftrightarrow\quad & 0 < 2\Big(k(n-s)-a\Big)k(n-s)e + \alpha\beta\Big(k(n-s)\Big)^2 + 2k^2s(n-s+1). \qquad (59)
\end{aligned}
$$

The RHS of (59) is decreasing in a. So it suffices to observe it is positive for the maximal value $a = k(n-1) - \epsilon \approx k(n-1)$ that satisfies (A1′). In particular,

$$
\begin{aligned}
& 0 < 2k\Big(k(n-s)-k(n-1)\Big)(n-s)e + \alpha\beta\Big(k(n-s)\Big)^2 + 2k^2s(n-s+1) \\
\Leftrightarrow\quad & 0 < k^2\Big[2(-s+1)(n-s)e + \alpha\beta(n-s)^2 + 2s(n-s+1)\Big] \\
\Leftrightarrow\quad & 0 < 2(1-s)(n-s)e + \alpha\beta(n-s)^2 + 2s(n-s+1) \qquad (60) \\
\Leftrightarrow\quad & 0 < 2(n-s)e - 2s(n-s)\Big(1-\alpha(\beta+\tau)\Big) + \alpha\beta(n-s)^2 + 2s + 2s(n-s) \\
\Leftrightarrow\quad & 0 < \underbrace{2(n-s)e + 2s(n-s)\alpha(\beta+\tau) + \alpha\beta(n-s)^2}_{\geq 0} + \underbrace{2s}_{>0}.
\end{aligned}
$$

■

We are now ready to verify $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, 1)$ in the seven cases identified in Figure 3:

Case 1 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s-1}$. Then $\delta \in (\underline{\delta}^u_{s-1}, 1)$ implies $\delta > \bar{\delta}_{s-1}, \bar{\delta}_s$. So $p^{\circledast}_{s-1}(\delta) = p^*_{s-1}$ and $p^{\circledast}_s(\delta) = p^*_s$ in the default regime, which renders $\mathcal{I}_s(\delta)$ constant for $\delta \in (\underline{\delta}^u_{s-1}, 1)$. $\mathcal{I}^u_s(\delta)$ linearly decreases from $\mathcal{I}^u_s(\underline{\delta}^u_{s-1})$ to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1})$ according to claim 2 and then stays constant. By claim 5, $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$. Hence $\mathcal{I}^u_s(\underline{\delta}^u_{s-1}) > \mathcal{I}_s(\underline{\delta}^u_{s-1})$ to avoid a contradiction. So $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ holds for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 2 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. For $\delta \in (\bar{\delta}^u_{s-1}, 1)$ we have $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ according to claim 5. Claim 2 ensures that $\mathcal{I}^u_s(\delta)$ falls linearly on $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_{s-1})$ and in particular on sub-interval $(\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$ where $\mathcal{I}_s(\delta)$ is constant. Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$

in order not to contradict $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$. For $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$, both $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ decrease with identical slope (invoking claim 1 or 2 depending on $\bar{\delta}_s \lesseqgtr \underline{\delta}_{s-1}$). Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ must also hold for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$.

Case 3 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 1.

Case 4 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. That $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$ can be concluded from claims 2 and 5 just as for case 1. $\mathcal{I}_s(\delta)$ decreases linearly with slope $-\frac{(s-1)k^2}{eb}$ on $(\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ for $\bar{\delta}_s < \underline{\delta}_{s-1}$ and on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ for $\bar{\delta}_s > \underline{\delta}_{s-1}$, and so does $\mathcal{I}^u_s(\delta)$ on $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ (claim 2). Hence, considering $\delta \geq \underline{\delta}^u_{s-1}$, $\mathcal{I}_s(\delta)$ assumes a maximum of $\mathcal{I}_s(\underline{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1}) + \frac{(s-1)k^2}{eb} \cdot (\bar{\delta}_{s-1} - \underline{\delta}^u_{s-1})$ at $\delta = \underline{\delta}^u_{s-1}$. $\mathcal{I}^u_s(\delta)$ exceeds that value at $\bar{\delta}^u_s > \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\}$ and assumes even higher values on $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$ because it is decreasing on this interval to $\mathcal{I}_s(\underline{\delta}^u_{s-1})$.[22] Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 5 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 4.

Case 6 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. Given $\underline{\delta}_{s-1} < \underline{\delta}^u_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$, claim 3 yields $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$. Then $\mathcal{I}_s(\delta)$ falls linearly on $(\bar{\delta}_s, \bar{\delta}^u_s)$ while $\mathcal{I}^u_s(\delta)$ decreases in a slower strictly concave fashion for $\delta \in (\bar{\delta}_s, \bar{\delta}^u_s)$ (claim 1). So $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on $(\bar{\delta}_s, \bar{\delta}^u_s)$. For $\delta \in (\bar{\delta}^u_s, \bar{\delta}_{s-1})$, both functions fall linearly with same slope (claim 1), extending $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ at $\delta = \bar{\delta}^u_s$ to interval $(\bar{\delta}^u_s, \bar{\delta}_{s-1})$. $\mathcal{I}_s(\delta)$ turns constant for $\delta \in (\bar{\delta}_{s-1}, 1)$ while $\mathcal{I}^u_s(\delta)$ continues its decrease – but only to a value of $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1})$ (claim 5). Then $\mathcal{I}^u_s(\delta)$ turns constant too, implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 7 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$. For $\delta \in (\bar{\delta}_{s-1}, 1)$, $\mathcal{I}_s(\delta)$ is constant. By contrast, $\mathcal{I}^u_s(\delta)$ is constant to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$ on $(\bar{\delta}^u_{s-1}, 1)$ (claim 5) and, focusing on $(\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$, decreasing to this value from $\mathcal{I}^u_s(\bar{\delta}_{s-1})$ – implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ can be concluded in direct analogy to case 6.

This proves part (i) of the proposition. Part (ii) then follows from recalling $\mathcal{E}_n(\delta) = \mathcal{E}^u_n(\delta)$ and that $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}^u_s(\delta) = -\mathcal{I}^u_{s+1}(\delta)$ for $s < n$. □

[22] That $\mathcal{I}^u_s(\bar{\delta}^u_s) > \mathcal{I}_s(\underline{\delta}^u_{s-1})$ is most easily seen by looking at $\mathcal{I}^u_s$'s behavior from the right, i.e., moving down from $\delta = 1$ to $\delta = \underline{\delta}^u_{s-1}$: it switches from constant to increasing with slope $\left|-\frac{(s-1)k^2}{eb}\right|$ already at $\bar{\delta}^u_{s-1} > \bar{\delta}_{s-1}$ and continues this increase over an interval $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ that is wider than the corresponding interval $(\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ of $\mathcal{I}_s$'s increase for $\delta \geq \underline{\delta}^u_{s-1}$ because $\bar{\delta}_{s-1} - \bar{\delta}_s < \bar{\delta}^u_{s-1} - \bar{\delta}^u_s$ (claim 4) and $\bar{\delta}_s \leq \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\} < \underline{\delta}^u_{s-1}$ imply $\bar{\delta}_{s-1} - \bar{\delta}^u_{s-1} < \underline{\delta}^u_{s-1} - \bar{\delta}^u_s$.

References

Andersson, O. and E. Wengström (2007). Do antitrust laws facilitate collusion? Experimental evidence on costly communication in duopolies. *Scandinavian Journal of Economics 109*(2), 321–339.

Argenton, C., D. Geradin, and A. Stephan (2020). *EU Cartel Law and Economics*. Oxford: Oxford University Press.

Aubert, C., P. Rey, and W. E. Kovacic (2006). The impact of leniency and whistle-blowing programs on cartels. *International Journal of Industrial Organization 24*(6), 1241–1266.

Bageri, V., Y. Katsoulacos, and G. Spagnolo (2013). The distortive effects of antitrust fines based on revenue. *Economic Journal 123*(572), F545–F557.

Basso, L. J. and T. W. Ross (2010). Measuring the true harm from price-fixing to both direct and indirect purchasers. *Journal of Industrial Economics 58*(4), 895–927.

Blair, R. D. and C. P. Durrance (2018). Umbrella damages: Towards a coherent antitrust policy. *Contemporary Economic Policy 36*(2), 241–254.

Blair, R. D. and V. G. Maurer (1982). Umbrella pricing and antitrust standing: An economic analysis. *Utah Law Review 1982*(4), 761–796.

Bos, I. and J. E. Harrington, Jr. (2010). Endogenous cartel formation with heterogeneous firms. *RAND Journal of Economics 41*(1), 92–117.

Bos, I. and J. E. Harrington, Jr. (2015). Competition policy and cartel size. *International Economic Review 56*(1), 133–153.

Bos, I., W. Letterie, and N. Scherl (2019). Industry impact of cartels: Evidence from the stock market. *Journal of Competition Law and Economics 15*(2-3), 358–379.

Bos, I., W. Letterie, and D. Vermeulen (2015). Antitrust as facilitating factor for collusion. *B. E. Journal of Economic Analysis and Policy 15*(2), 797–814.

Bryant, P. G. and E. W. Eckard (1991). Price fixing: The probability of getting caught. *Review of Economics and Statistics 73*(3), 531–536.

Combe, E., C. Monnier, and R. Legal (2008). Cartels: The probability of getting caught in the European Union. Bruges European Economic Research Papers no. 12.

Connor, J. M. (2020). *Private International Cartels Full Data 2019 Edition.* `https://purr.purdue.edu/publications/2732/2`.

D'Aspremont, C., A. Jacquemin, J. J. Gabszewicz, and J. A. Weymark (1983). On the stability of collusive price leadership. *Canadian Journal of Economics 16*(1), 17–25.

De Roos, N. and V. Smirnov (2021). Collusion, price dispersion, and fringe competition. *European Economic Review 132*, #103640.

Diamantoudi, E. (2005). Stable cartels revisited. *Economic Theory 26*(4), 907–921.

Donsimoni, M.-P., N. Economides, and H. Polemarchakis (1986). Stable cartels. *International Economic Review 27*(2), 317–327.

Escrihuela-Villar, M. (2008). On endogenous cartel size under tacit collusion. *Investigaciones Económicas 32*(3), 325–338.

Escrihuela-Villar, M. (2009). A note on cartel stability and endogenous sequencing with tacit collusion. *Journal of Economics 96*(2), 137–147.

Gabszewicz, J., M. Marini, and O. Tarola (2019). Endogenous mergers in markets with vertically differentiated products. *B.E. Journal of Theoretical Economics 19*(1), 1–22.

Holler, E. and M. P. Schinkel (2017). Umbrella effects: Correction and extension. *Journal of Competition Law and Economics 13*(1), 185–189.

Inderst, R., F. P. Maier-Rigaud, and U. Schwalbe (2014). Umbrella effects. *Journal of Competition Law and Economics 10*(3), 739–763.

Katsoulacos, Y., E. Motchenkova, and D. Ulph (2015). Penalizing cartels: The case for basing penalties on price overcharge. *International Journal of Industrial Organization 42*, 70–80.

Katsoulacos, Y., E. Motchenkova, and D. Ulph (2020). Combining cartel penalties and private damage actions: The impact on cartel prices. *International Journal of Industrial Organization 73*, #102604.

Laborde, J.-F. (2021). Cartel damages actions: How courts have assessed cartel overcharges. *Concurrences N°3-2021*, 232–242.

Laitenberger, U. and F. Smuda (2015). Estimating consumer damages in cartel cases. *Journal of Competition Law and Economics 11*(4), 955–973.

McCutcheon, B. (1997). Do meetings in smoke-filled rooms facilitate collusion? *Journal of Political Economy 105*(2), 330–350.

Merker, T. (2019). The effect of product differentiation on cartel stability. Discussion Paper, University of Oslo.

Motta, M. and M. Polo (2003). Leniency programs and cartel prosecution. *International Journal of Industrial Oganization 21*(3), 347–379.

Napel, S. and D. Welter (2021). Simple voting games and cartel damage proportioning. *Games 12*(4), 74.

Ormosi, P. L. (2014). A tip of the iceberg? The probability of catching cartels. *Journal of Applied Econometrics 29*(4), 549–566.

Shaffer, S. (1995). Stable cartels with a Cournot fringe. *Southern Economic Journal 61*(3), 744–754.

Taylor, A. D. and W. S. Zwicker (1999). *Simple Games*. Princeton, NJ: Princeton University Press.

Von Neumann, J. and O. Morgenstern (1953). *Theory of Games and Economic Behavior* (3rd ed.). Princeton, NJ: Princeton University Press. (1st edition, 1944).

Weber, F. (2021). The volume effect in cartel cases: A special challenge for damage quantification? *Journal of Antitrust Enforcement 9*, 436–458.