

SUCCESSFUL VOTING IN WEIGHTED COMMITTEES AND SHAREHOLDER MEETINGS

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ABSTRACT

The applicable voting rule determines how closely collective decisions between three or more options reflect the preferences of a given individual. We construct measures of this and ask if a specific decision maker is more successful using plurality voting, plurality with a runoff vote, pairwise majority voting or the Borda scoring method. Our first benchmark finding is that if all voting weights are equal, then plurality rule maximizes the probability of obtaining one's individual top choice and Borda rule maximizes the voter-specific average ranking of the outcome. This result generalizes to asymmetric weights in aggregate terms but not from a single voter's perspective. We use computational methods to identify the individually most advantageous rule for any given weight distribution among three generic voters and also for the ten largest shareholders in S&P 100 corporations. Practical recommendations for the latter coincide in unexpectedly many cases with the analytical benchmark for equal weights. We also find that the Borda rule translates voting weights into voting success the most transparently and that traditional power indices for binary voting games approximate success in weighted committees well.

Keywords: collective decisions · shareholder voting · weighted committee games · voting procedures · voting power · voting success

JEL codes: D71 · C71 · C63 · G30

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1 Introduction

Many important decisions are taken by vote not just in politics but also in business. This includes elections of directors or chief executives, resolutions on important acquisitions, critical facility choices, the selection of an auditor, etc. The adopted voting procedure can make a big difference when the respective stakeholders disagree. This paper therefore addresses a natural but computationally non-trivial research question: Which default voting rule maximizes a given decision maker's expected voting success in the sense of reflecting his or her preferences the best? As far as we know, we are the first to provide answers for decisions on three or more options.

To see what is at stake, consider three shareholders who command 45%, 35% and 20% of corporate votes, respectively.¹ These might be exercised directly in an annual meeting or indirectly by controlling 5, 4 and 2 positions on the board of directors. Let the shareholders have different views of five CEO candidates, labeled a, b, c, d and e . For instance, the first (largest) shareholder ranks the candidates $a > d > e > c > b$ in strictly decreasing order. The second shareholder's preferences are $b > c > d > e > a$ and the third one's $c > e > d > b > a$.

One of the simplest methods to resolve the disagreement is a plurality vote: everybody indicates their favorite candidate, and the one with the most votes wins. Then a beats its competitors by 45% : 35% : 20% : 0% : 0% in the shareholder meeting or 5 : 4 : 2 : 0 : 0 in a board vote, assuming that preferences are expressed sincerely. However, a is ranked last by two of the shareholders and a plurality that is not a majority is legally insufficient in many settings (cf., e.g., §216(2) of Delaware General Corporation Law; or §44 of Robert's Rules of Order). So some board member may propose a runoff vote between the plurality leaders a and b . If the suggestion is taken up, b wins by 6 : 5 (or, analogously, 55% : 45% in a shareholder meeting). Pairwise comparisons might also be extended beyond a and b : in a round-robin tournament between all candidates, c would beat b by 7 : 4 and also win against a, d and e . This would make c the new CEO. The directors could alternatively translate their preference rankings into scores for the candidates – ascribing, say, 0 points to their lowest-ranked candidate, 1 to their respective second-lowest-ranked candidate and so forth – and hire the top scorer. This method is commonly associated with the French scientist Jean-Charles de Borda (1733–1799) and, in our example, candidate

¹This is an artificial example, but identical outcomes would result for some real share distributions – e.g., the Eurofighter Fighter Aircraft GmbH (Airbus 46%; BAE Systems 33%; Leonardo 21%) or, with suitable tie breaking, the early Apple Inc. (S. Jobs 45%; S. Wozniak 45%; R. Wayne 10%).

d would obtain a total ‘Borda score’ of $5 \cdot 3 + 4 \cdot 2 + 2 \cdot 2 = 27$ from the board. This exceeds scores of 20, 18, 25 and 20 for a , b , c and e – so d wins. Or directors could each approve as many candidates as they like and pick the one with the highest approval. The first shareholder might approve a , d and e from top down; the second only b and c ; the third c and e . This yields a top $5 + 2 = 7$ approval for e versus only 5, 4, 6 and 5 for a , b , c and d .

So *every* candidate is a winner: it all depends on the voting rule. Having a say on the latter is hence valuable for any forward-looking decision maker who wants their own favorite to win. In the example, the first shareholder would succeed with plurality rule, the second with plurality runoff rule and the third if enough pairwise comparisons are made. But situation-specific deductions of which rule to use demand considerable information and not even the chairperson of a board or committee can pick a voting rule as he or she pleases ad hoc. The applicable method for taking decisions is typically determined beforehand in statutes, charters, by-laws or laws; or there are institutional defaults with deviations requiring a justification.²

It is important, consequently, to evaluate the success implications of adopting one voting rule rather than another from a longer term or a priori perspective. Is there any general (dis)advantage of using, say, plurality rule vs. pairwise voting when an individual’s objective is to elect his or her personal favorite with maximal likelihood? What if he or she wants to induce choices with a high average subjective rank? Should the respective default rule depend on the distribution of voting rights? Answers to these questions matter not just for shareholders and corporate boards but many political institutions.

We consider a stylized collective decision-making body, generically referred to as a ‘committee’, and evaluate the agreement between collective choices and each member’s preferences when either plurality voting, plurality with a runoff vote, pairwise majority voting or the Borda scoring method is invoked.³ We do so for all possible preference configurations. The relevant players may wield asymmetric voting weights, which can reflect votes controlled by shareholders, parties or alliances in a parliament, representatives in a regional assembly, population shares in a polarized

²Implicit forms of voting are common and have defaults too: leaders may tacitly adopt the majority view in their team to secure their position (cf. Leeson 2007); scores that independent interviewers assign to applicants are totaled similar to Borda rule; or competing engineering proposals are dropped successively according to which one is favored by the fewest project members.

³The rules are prototypical instances of a *Condorcet method*, a *runoff rule* and *scoring rules* (see, e.g., Felsenthal and Nurmi 2018). Approval voting needs a different model of preferences and is left aside. We focus on sincere voters but also study success maximizers for strategic players (see Appendix B).

electorate, and so on. Player-specific weights will yield player-specific answers to the question of which voting rule, a priori, reflects individual preferences the best.

Our first contribution to the literature is the construction of indices which operationalize individual success under a given voting rule as the expected congruence between personal preference and a collective decision. The *top choice index* represents the probability that one’s most preferred option is selected; the *average rank index* captures the expected position of the selected option in the personal ranking. With these indices, we generalize the Rae index of success from binary to non-binary choices (Rae 1969; also see historical notes by Felsenthal and Machover 1998, p. 46). The new indices quantify differences across voting rules and between asymmetric voters by drawing preferences from probability distributions that are familiar from traditional measures of voting power for ‘yes’-or-‘no’ decisions – namely, the Penrose-Banzhaf (Penrose 1946; Banzhaf 1965) and the Shapley-Shubik power indices (Shapley and Shubik 1954).⁴ Neither matches the distribution of preferences in a given board, annual meeting or parliament exactly but they provide complementing benchmarks: one assumes independent idiosyncratic preferences and the other incorporates correlated attitudes that reflect a common interest. The respective success computations can improve corporate and political decision-making defaults. They can also provide decision support on whether acquiring additional voting rights is worthwhile or address concern that – for given voting weights – a particular method (dis)favors, for example, some large shareholder or a minority group.

As a second contribution, our extensive a priori computations provide a new reference point for anecdotal evidence on how sensitive collective decisions are to the adopted voting method for more than two options. We draw on structural equivalence results by Kurz, Mayer and Napel (2020) to investigate all possible distributions of voting weights among three players who decide on three or four alternatives. We show that success expectations can be locally very sensitive to rule or weight changes, that the individually most advantageous rule differs across players and that the respective success maximizer may vary non-monotonically in weight. The assumed a priori perspective complements illustrative a posteriori comparisons for single preference configurations (see, e.g., Riker 1982, Saari 2001 or Felsenthal and Nurmi 2018) and historical case studies: for instance, Leininger (1993) scrutinizes the ‘fatal’ voting procedure that moved the government of reunified

⁴Voting power and success differ conceptually. The former refers to the ability to influence the voting outcome, and the latter refers to the individual evaluation of outcomes. Your vote may make c , d or e the winner (power), but all may be evaluated low compared to your favorite option a (success).

Germany from Bonn to Berlin; Tabarrok and Spector (1999) suggest that Borda's method might have avoided the US civil war; Maskin and Sen (2016) reason that Donald Trump owes his 2016 election to the use of plurality rule. We also analytically derive success-maximizing rules if an arbitrary number of players have equal voting weights: choose plurality rule for the best chance to make your favorite win or Borda rule for the highest expected rank of the winner. These recommendations extend to arbitrary weight distributions if the objective – similar to Apesteguia, Ballester and Ferrer (2011) – is to maximize the weighted total success of all voters.

The third contribution of our study is to bring the obtained analytical insights to some real-world settings and to demonstrate that, despite high sensitivity of success maximizers to skewed voting weights in theory, a simple rule of thumb can be used in practice. We apply computational methods (complete enumeration and Monte Carlo simulation) to show that regardless of the pronounced asymmetry between the top investors in S&P 100 corporations – with mean holdings of $>10\%$ for the largest vs. $\approx 1\%$ for the tenth-largest – they have almost identical interests concerning which (straw) voting rule should be used to resolve disagreement. It turns out to matter more whether a high average rank or getting one's favorite better reflects individual objectives than whether one holds the first or tenth-most shares: for overwhelmingly many of the asymmetric shareholders, plurality rule maximizes the top choice probability and Borda rule produces the highest average outcome rank. This empirical finding echoes our analytical results for symmetric weights unexpectedly well. We also confirm the economic relevance of the research question: the voting method determines 1–3% of success a priori. This is in the same ballpark as the average success advantage of 2.2% for the seventh largest shareholder of an S&P 100 constituent compared to the respective tenth largest shareholder (with 1.78% vs. 1.14% of a firm's stock on average). We find that Borda rule links voting weights to success the most closely. This means that it provides particularly transparent incentives for investors who want an acquisition of additional ordinary shares to translate into additional voting success, not just nominal voting rights.

Fourth and finally, regression analysis shows that the advantage of having a higher relative voting weight is picked up surprisingly well for the S&P 100 data by standard indices of binary voting power, which have, e.g., featured in recent studies on common ownership (Azar, Schmalz and Tecu 2018; Backus, Conlon and Sinkinson 2021). The literature on a priori success has remained restricted to binary choices long after Rae's (1969) first investigation (see, e.g., Laruelle, Martínez and Valenciano 2006 and references therein). The present study generalizes success analysis to richer

decision-making problems but, at the same time, it contributes evidence in favor of using common power indices as viable approximations also for non-binary choices.

We acknowledge that voting in business or politics is usually a less formal affair than in our model. Controversial decisions are often predetermined in private by a few key players without fixed rules. Then a pre-selected favorite is proposed to the plenary meeting as a simple ‘yes’-or-‘no’ motion, potentially presenting even narrow winners from earlier stages as consensual choices.⁵ However, predetermination dynamics are likely to be influenced by who controls how many votes and procedural defaults for dealing with disagreement. To the extent that the defaults involve plurality-style endorsements (possibly with an eventual focus on two contenders), pairwise candidate comparisons or score assignments, the present investigation extends beyond settings with binding official rules. Weighted committees can thus be viewed as at least first approximations also of early straw voting stages in which, say, a CEO candidate or acquisition target is singled out from an unofficial shortlist.⁶

We next explain in more detail how this investigation connects to previous analysis of weighted voting and axiomatic assessments of social choice rules (Section 2). The framework of weighted committee games and our measures of a priori voting success are introduced in Sections 3 and 4. We then consider a wide range of small committees in Section 5 and actual voting share distributions in S&P 100 constituents in Section 6. We conclude in Section 7. Appendices A and B assess the robustness of our findings regarding the adopted tie breaking assumption and strategic voting.

2 Relation to the Literature on Weighted Voting

Individual a priori evaluations of voting systems date back to the Constitutional Convention in Philadelphia in 1787 (see Riker 1986). However, investigations have mostly been restricted to binary decisions. In particular, various measures of voting

⁵For instance, of the nearly 800 votes from 2017 to 2024 on issues that require only a qualified majority in the EU Council of Ministers, about 81% of the motions would have passed also under unanimity rule (own calculations, data retrieved from <https://www.consilium.europa.eu/en/documents-publications/public-register/votes/>). The IMF stipulates that “a shortlist of three candidates” is prepared for the position of IMF Managing Director and that choice from it is “by a majority of the votes cast” on the Executive Board (IMF Press Release 16/19). Quite magically, a single consensus candidate has always emerged before any competitors were officially shortlisted and rejected. See Mayer and Napel (2020).

⁶We point to McCahery, Sautner and Starks (2016) or Bowley, Hill and Kourabas (2024) for evidence on how shareholders exert power through behind-the-scenes interaction and Gantchev (2013) on the costs associated with different forms of shareholder influence.

power have been defined and applied to *weighted voting games* or more general *simple (voting) games* formalized by von Neumann and Morgenstern (1953, ch. 10). The Penrose-Banzhaf and Shapley-Shubik indices are the most prominent such measures (Penrose 1946; Banzhaf 1965; Shapley and Shubik 1954).⁷ They map a given distribution of voting weights for binary ‘yes’-or-‘no’ decisions (or sets of winning and losing coalitions) to individual decisiveness and a priori influence.

Analogous investigations of preference satisfaction and voting success have received less attention. A reason for this is that being successful is commonly seen as a trivial corollary to holding power (cf. Barry 1980). The probability of obtaining the preferred outcome (voting success) is, in fact, an affine transformation of the probability of being decisive for the outcome (voting power) if all ‘yes’-or-‘no’ configurations among players are equally likely (Dubey and Shapley 1979).⁸ The vast literature that has studied voting rules in the US Electoral College, the UN Security Council, the Council of the European Union, national parliaments, the European Central Bank or publicly traded corporations (see, e.g., Leech 1987, 1988, Azar et al. 2018 and the contributions in Holler and Nurmi 2013) hence focuses on decisiveness.

However, Laruelle et al. (2006, p. 197) note that “practitioners have often raised objections about the power indices approach . . . [and ask] why pay so much attention to decisiveness, when success seems a more important issue for the involved voters?” Moreover, the mathematical links between power and success are fragile. Their affine relation collapses already for mild interdependencies among voter preferences, such as the preference model underlying the Shapley-Shubik index (see Kirsch 2023).

The connection between power and success can even be non-monotonic if choices concern more than two options. For instance, every voting procedure designed to select the so-called Condorcet winner, i.e., the winner of a complete pairwise majority contest if such contest creates no cycle, is subject to the *no show paradox* (Moulin 1988). This implies that individuals or groups are sometimes strictly better off not casting all their eligible votes. Weighted voting analysis hence benefits from dedicated assessments of success especially for non-binary decisions.

Success analysis complements the traditional social choice literature by adding a neglected individual-focused perspective. Numerous scholars – with seminal con-

⁷See Felsenthal and Machover (1998), Laruelle and Valenciano (2008) or Napel (2019) for overviews. Extensions to weighted committee games are studied by Kurz, Mayer and Napel (2021).

⁸Rae (1969) conjectured and Taylor (1969) proved that deciding by simple majority then maximizes a priori success in the domain of symmetric rules. Dubey and Shapley (1979) extended the domain to simple games. For given weights, the only degree of freedom in voting on two options is the majority threshold. All rules we consider amount to using an optimal 50% quota in case of binary choices.

tributions by Arrow (1951), May (1952), Sen (1970), Gibbard (1973), Young (1974), Satterthwaite (1975) or Moulin (1988) – have investigated decision rules such as plurality or pairwise majority voting from an axiomatic viewpoint that highlights desirable aggregate properties. But every voting method with normatively appealing properties has turned out to have unappealing ones too. The corresponding investigations have produced detailed checklists on the (non-)fulfillment of various desirable criteria by common voting rules (cf., e.g., Felsenthal and Nurmi 2018), debate on how to prioritize them in specific contexts (Laslier 2012) and computations of the likelihood of a given rule violating a specific property (see, e.g., Gehrlein and Lepelley 2017). Voting experts still recommend entirely different methods for good normative reasons, while many practitioners know that a dictatorial rule would get them closest to what they want if only the by-laws permitted. We therefore take the position of a self-interested decision-maker and ask: How well does this vs. another eligible voting rule serve my personal goals a priori?

3 Weighted Committees

We build on the generalization of binary weighted voting games (von Neumann and Morgenstern 1953) to multi-option weighted committee games as developed by Kurz et al. (2020). These games consider a set $N = \{1, \dots, n\}$ of voters or players such that each voter $i \in N$ has strict preferences P_i over a finite set $A = \{a_1, \dots, a_m\}$ of $m \geq 2$ alternatives. We may write $a > b > c$ or abc for P_i when the player’s identity is clear. The set of all $m!$ strict preference orderings on A is denoted by $\mathcal{P}(A)$. A *voting rule* $r: \mathcal{P}(A)^n \rightarrow A$ maps each preference profile $\mathbf{P} = (P_1, \dots, P_n)$ to a winning alternative $a^* = r(\mathbf{P})$.⁹ Rule r is *anonymous* if for any $\mathbf{P} \in \mathcal{P}(A)^n$ and any permutation $\rho: N \rightarrow N$ we have $r(\mathbf{P}) = r(\rho(\mathbf{P}))$ where $\rho(\mathbf{P}) := (P_{\rho(1)}, \dots, P_{\rho(n)})$. It is *neutral* if for any $\mathbf{P} \in \mathcal{P}(A)^n$ and any permutation $\rho: A \rightarrow A$ we have $r(\rho(\mathbf{P})) = \rho(r(\mathbf{P}))$ where, with slight abuse, $\rho(\mathbf{P})$ denotes the application of ρ to each alternative in the full preference profile.

We focus on truthful voting under one of the four anonymous rules that are summarized in Table 1, assuming lexicographic tie breaking.¹⁰ Under *plurality rule* r^P

⁹Shareholders’ rankings of options may differ if they maximize expected utility on incomplete markets (see, e.g., DeMarzo 1993). Investment horizons and business philosophies often vary between and even within founders and venture capitalists, activist hedge funds and pension funds, or retail and institutional investors (cf. Bolton et al. 2020 or Bubb and Catan 2022). One may also think of proxy battles, disputes in family-owned firms, shareholders who co-own different other firms and internalize spillovers, international investors subject to distinct standards for good governance, etc.

¹⁰Deterministic tie breaking simplifies the presentation. We show in Appendix A that all results

Table 1: Considered baseline voting rules

Rule	Winner $a^* \in \{a_1, \dots, a_m\}$ at preference profile $\mathbf{P} = (P_1, \dots, P_n)$
Borda	$r^B(\mathbf{P}) \in \arg \max_{a \in A} \sum_{i \in N} b_i(a, \mathbf{P})$
Copeland	$r^C(\mathbf{P}) \in \arg \max_{a \in A} \{a' \in A \mid a \succ_M^{\mathbf{P}} a'\} $
Plurality	$r^P(\mathbf{P}) \in \arg \max_{a \in A} \{i \in N \mid \forall a' \neq a \in A: a P_i a'\} $
Plurality runoff	$r^{PR}(\mathbf{P}) \begin{cases} = r^P(\mathbf{P}) & \text{if } \{i \in N \mid \forall a' \in A \setminus \{r^P(\mathbf{P})\}: r^P(\mathbf{P}) P_i a'\} > \frac{n}{2}, \text{ else} \\ \in \arg \max_{a \in \{a_{(1)}, a_{(2)}\}} \{i \in N \mid \forall a' \neq a \in \{a_{(1)}, a_{(2)}\}: a P_i a'\} \end{cases}$

each voter indicates his or her top-ranked alternative and the one ranked first by the most voters is chosen. This is the winner also under *plurality (with) runoff rule* r^{PR} if the obtained plurality constitutes a majority (i.e., more than 50% of votes); otherwise a runoff vote is conducted between the alternatives $a_{(1)}$ and $a_{(2)}$ that obtained the highest and second-highest plurality scores in the first stage.

Borda rule r^B has each player i assign a score of $m - 1, m - 2, \dots, 0$ to the alternative that he or she ranks first, second and so on. These scores $b_i(a, \mathbf{P}) := |\{a' \in A \mid a P_i a'\}|$ coincide with the number of alternatives that i positions below a . The alternative with the highest total score is selected. *Copeland rule* r^C considers pairwise majority comparisons between all alternatives. They define the majority relation $a \succ_M^{\mathbf{P}} a' :\Leftrightarrow |\{i \in N \mid a P_i a'\}| > |\{i \in N \mid a' P_i a\}|$ and the alternative that beats the most others according to $\succ_M^{\mathbf{P}}$ is selected. r^C is the only Condorcet-consistent method among the rules in Table 1: whenever some alternative a beats all others, then $r^C(\mathbf{P}) = a$.

A *weighted committee (game)* $(N, A, r|\mathbf{w})$ combines a set of players N , a set of alternatives A and an anonymous baseline rule r with a vector $\mathbf{w} = (w_1, \dots, w_n) \in \mathbb{N}_0^n$ of voting weights: each player i can cast w_i votes, e.g., by virtue of owning multiple voting shares or controlling as many seats on a board. Preferences P_i thus enter into the final decision w_i times, whereby anonymity is relinquished. The applicable mapping from preference profiles to collective choices then is

$$r|\mathbf{w}(\mathbf{P}) := r([P_1]^{w_1}, [P_2]^{w_2}, \dots, [P_n]^{w_n}) = r(\underbrace{P_1, \dots, P_1}_{w_1 \text{ times}}, \underbrace{P_2, \dots, P_2}_{w_2 \text{ times}}, \dots, \underbrace{P_n, \dots, P_n}_{w_n \text{ times}}) \quad (1)$$

hold also for anonymous random tie breaking. Possibly non-truthful strategic voting is addressed in Appendix B. It creates non-trivial equilibrium selection problems but the analysis can be adapted. Our findings for truthful voting turn out to be relatively robust to strategic play.

Table 2: Effect of the voting rule on the winning option

P_1	P_2	P_3	
a	b	c	$r^P \mathbf{w}(\mathbf{P}) = a$ (a has max. plurality tally of 5)
d	c	e	$r^{PR} \mathbf{w}(\mathbf{P}) = b$ (b beats a in runoff vote by 6 : 5)
e	d	d	$r^C \mathbf{w}(\mathbf{P}) = c$ (c wins all pairwise votes)
c	e	b	$r^B \mathbf{w}(\mathbf{P}) = d$ (d has max. Borda score of 27)
b	a	a	

Note: The table reproduces the shareholder example from the Introduction, illustrating how standard voting rules imply different choices for $\mathbf{P} = (P_1, P_2, P_3)$ when $\mathbf{w} = (5, 4, 2)$.

for all $\mathbf{P} \in \mathcal{P}(A)^n$. The mapping is homogeneous of degree zero in $\mathbf{w}$ and so we may equivalently consider relative voting weights $\mathbf{w} / \sum w_i$.

Two committees $(N, A, r|\mathbf{w})$ and $(N, A, r'|\mathbf{w}')$ are called *equivalent* if they produce the same outcomes no matter which preferences $\mathbf{P}$ are considered, despite $r \neq r'$ or $\mathbf{w} \neq \mathbf{w}'$. For example, both $r^P|(3, 1, 1)$ and $r^C|(5, 2, 1)$ select player 1's top choice for every $\mathbf{P}$, making player 1 a dictator. By contrast, Section 1's shareholder example – summarized again in Table 2 – shows that committees which use r^P , r^{PR} , r^C and r^B are non-equivalent if $\mathbf{w} = (5, 4, 2)$ and $m = 5$. We are interested in committees that are non-equivalent and compare a player's success in such committees for all conceivable preference configurations $\mathbf{P} \in \mathcal{P}(A)^n$ from an a priori perspective.

4 Measuring A Priori Success

The a priori assessment of player i 's success in a given committee is contingent on how the collective decisions $r|\mathbf{w}(\mathbf{P})$ are evaluated relative to i 's individual preference P_i and on the applicable distribution of preferences. For the latter, we consider preference cultures that are standard in quantitative social choice analysis and are underlying the two most prominent power indices for binary voting: the Penrose-Banzhaf and the Shapley-Shubik indices (see fn. 7 above).

We highlight two benchmarks for how well a decision $r|\mathbf{w}(\mathbf{P})$ reflects given preferences of voter i . Neither has a claim to be the 'right' or universally recommended. We construct one a priori success index for each: The *top choice (probability) index (TCI)* takes a player's success to mean having his or her most preferred option become the collective choice. The *average rank index (ARI)* counts every outcome that is better

than the player's bottom-ranked alternative as a partial success at least. The latter index can also be interpreted as reflecting a linear utility function over the available options. One can conduct general expected utility assessments by combining both indices in the case of $m = 3$ alternatives.

4.1 Top Choice Index and Average Rank Index

While much of social choice theory focuses on ordinal preferences, our aim is to express each player's prospect of obtaining preferred outcomes – as choices from A are taken according to $r|\mathbf{w}$ – by an interpersonally comparable number. Let us therefore consider a player-independent success function $\sigma: A \times \mathcal{P}(A) \rightarrow \mathbb{R}$ such that $\sigma(a, P_i) > \sigma(a', P_i) \Rightarrow aP_i a'$, and $\sigma(a^*, P_i) = 1$ (0) if a given committee choice a^* is i 's most (least) preferred option. Our first benchmark is $\sigma \equiv s$ with

$$s(a^*, P_i) = \begin{cases} 1 & \text{if } a^* \text{ is ranked top in } P_i, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

This equates success to getting one's top choice.

A complementing, more gradual evaluation is achieved by s 's linear interpolation

$$\tilde{s}(a^*, P_i) = \frac{|\{a' \in A : a^* P_i a'\}|}{m - 1}. \quad (3)$$

This also attributes success to outcomes between the best and worst case and evaluates i 's median-ranked alternative as exactly half a success for player i . Function $\tilde{s}$ or variations based, e.g., on a hyperbolic interpolation of s are compatible with interpreting (a priori) success as an (expected) utility. By contrast, piecewise constancy of s clashes with P_i being a strict ordering when $m > 2$.

Given some player-independent success function σ and probability measure $\Pr$ on $\mathcal{P}(A)^n$, we refer to the expected value of $\sigma(r|\mathbf{w}(\mathbf{P}), P_i)$ as player i 's *a priori success* in committee $(N, A, r|\mathbf{w})$. In particular, we define the *top choice index*

$$TCI_i(N, A, r|\mathbf{w}) := \mathbb{E}[s(r|\mathbf{w}(\mathbf{P}), P_i)] = \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot s(r|\mathbf{w}(\mathbf{P}), P_i), \quad (4)$$

and, analogously, the *average rank index* $ARI_i(N, A, r|\mathbf{w}) := \mathbb{E}[\tilde{s}(r|\mathbf{w}(\mathbf{P}), P_i)]$ as the two success indicators of our interest. TCI_i equals the probability that collective decisions match player i 's top-ranked alternative. ARI_i is an inverse measure of the average

rank of decisions according to i 's preferences: a value of $x \in [0, 1]$ means that collective decisions on average correspond to the $(m - (m-1) \cdot x)$ -th best alternative from player i 's perspective. $TCI_i = ARI_i = 1$ if and only if i is a dictator. TCI_i and ARI_i coincide with the success index named after Rae (1969) when $m = 2$ (see, e.g., Laruelle et al. 2006).

In case of three alternatives, which many of our later computations focus on, working with TCI_i and ARI_i is without loss of generality. This is because the probability of the collective choice matching i 's second-ranked alternative evaluates to $2 \cdot (ARI_i - TCI_i)$ and that for i 's bottom rank is $1 + TCI_i - 2 \cdot ARI_i$. These probabilities and TCI_i suffice to compute a priori success $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$ for any rank-based success function¹¹ or expected utility $\mathbb{E}[u_i(r|\mathbf{w}(\mathbf{P}))]$ for cardinal utility functions $u_i: A \rightarrow \mathbb{R}$ as in the welfare analysis of voting with equal weights by Apesteguia et al. (2011).

Concerning the probability distribution over preference profiles $\mathbf{P} \in \mathcal{P}(A)^n$ that defines expectations, a popular default is the *impartial culture* (IC) assumption: all players' preferences $P_1, \dots, P_n \in \mathcal{P}(A)$ are taken to be independent and drawn at random. Then

$$\Pr(\mathbf{P}) = (m!)^{-n}. \quad (5)$$

The IC distribution is underlying the Penrose-Banzhaf voting power index and has served as the starting point for many computations in the analysis of voting. See Klahr (1966), Fishburn (1971), Merrill (1984) or Nurmi and Uusi-Heikkilä (1985) for pioneering assessments of voting paradoxes, and Gehrlein and Lepelley (2017) for many more recent findings.

The most prominent alternative to IC is the *impartial anonymous culture* (IAC), which is underlying the Shapley-Shubik index. The IAC model is impartial regarding all rankings $\pi \in \mathcal{P}(A)$, just like IC, but assumes positive correlation across players. The respective probability distribution is given by¹²

$$\Pr(\mathbf{P}) = \left[\binom{m! + n - 1}{n} \cdot \left(n_{n_1!}^{\mathbf{P}}, \dots, n_{n_m!}^{\mathbf{P}} \right) \right]^{-1}. \quad (6)$$

Preferences in a real shareholder meeting or hiring committee will typically violate

¹¹For instance, success could also mean avoiding one's worst option, i.e., $\hat{s}(a^*, P_i) = 1 \Leftrightarrow a^*$ is not ranked bottom in P_i . Proposition 1 below would then call for a method known as *anti-plurality voting*. We remark that if A represents a shortlist generated from a bigger set $A' \supset A$ of proposals then success evaluations *relative to* A' would need to account for possible asymmetries in the shortlisting process. For instance, the largest shareholder's favorite in A' might always automatically be included in A . However, such privilege does not affect any success evaluations *relative to* A because each player's favorite in A is, by definition, an element of A .

¹²See, e.g., Berg (1985) or Kurz et al. (2021) for details and generalizations using Pólya urn schemes.

the IC or IAC assumptions. Working with the probabilities in equations (5)–(6) means doing thought experiments that assess institutions from behind a ‘veil of ignorance’. One disregards historical preference patterns, recent alliances, logrolling, etc. partly for lack of adequate data but also purposely in order to obtain a neutral constitutional evaluation of voting rules. The resulting assessments – e.g., that player 1 will be 40% more successful a priori than player 2 under rule r , whereas their success is identical under rule r' – reflect general procedural tendencies and the implied (un)levelness of the playing field for decision-making. Corresponding numbers typically differ from the players’ actual (a posteriori) voting success in a committee since just a few preference configurations determine the latter and real decision-making has social, political or financial dimensions that are orthogonal to voting rules.

Because IC presumes all players to have independent preferences, it typically yields an upper bound for success in adversarial scenarios where i ’s preferences are negatively correlated to those of others. By contrast, IAC provides a more consensus-oriented outlook. It assumes some similarity in how players rank options and bounds individual success for potentially even greater preference affiliation from below.

4.2 Illustration

For illustration, let us evaluate a priori success when our stylized shareholders with voting weights of $\mathbf{w} = (45\%, 35\%, 20\%)$ choose between candidates $A = \{a, b, c\}$. The six possible individual rankings in $\mathcal{P}(A) = \{abc, acb, bac, bca, cab, cba\}$ give rise to $6^3 = 216$ different preference profiles that may obtain for a particular decision. Table 3 shows a selection of them, the respective winners $r|\mathbf{w}(\mathbf{P})$ implied by voting rules $r \in \{r^P, r^{PR}, r^C, r^B\}$ and associated success values $s(r|\mathbf{w}(\mathbf{P}), P_i)$.

For instance, the highlighted profile $\mathbf{P} = (cab, bca, abc)$ implies that c is selected under plurality and Borda rule. In contrast, b is selected under plurality with runoff and a under Copeland rule. So, at that profile, shareholder 1 is (fully) successful under plurality and Borda rule. Additionally, half a success would be attributed to player 1 by success function $\tilde{s}$ under Copeland rule (indicated by $\underline{0}$ in Table 3).

The corresponding expected values are shown at the bottom of Table 3 for the IC and IAC assumptions. The preference similarity reflected by IAC raises a priori success for all players relative to the preference independence assumed by IC. By definition, success figures for $ARI(\cdot)$ are greater than those for $TCI(\cdot)$.

The benchmark success of an independent outsider or ‘dummy player’ who has

Table 3: Illustration of success computations

Pr(P) for			s(·, P _i) for player i =															
P = (P ₁ , P ₂ , P ₃)	IC	IAC	r ^P w(P)	1	2	3	r ^{PR} w(P)	1	2	3	r ^C w(P)	1	2	3	r ^B w(P)	1	2	3
abc, abc, abc	$\frac{1}{216}$	$\frac{1}{56}$	a	1	1	1	a	1	1	1	a	1	1	1	a	1	1	1
abc, abc, acb	$\frac{1}{216}$	$\frac{1}{168}$	a	1	1	1	a	1	1	1	a	1	1	1	a	1	1	1
abc, abc, cab	$\frac{1}{216}$	$\frac{1}{168}$	a	1	1	<u>0</u>	a	1	1	0	a	1	1	0	a	1	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
cab, bca, abc	$\frac{1}{216}$	$\frac{1}{336}$	c	1	<u>0</u>	0	b	0	1	<u>0</u>	a	<u>0</u>	0	1	c	1	<u>0</u>	0
cab, bca, acb	$\frac{1}{216}$	$\frac{1}{336}$	c	1	<u>0</u>	<u>0</u>	c	1	<u>0</u>	<u>0</u>	c	1	<u>0</u>	<u>0</u>	c	1	<u>0</u>	<u>0</u>
cab, bca, bac	$\frac{1}{216}$	$\frac{1}{336}$	b	0	1	1	b	0	1	1	b	0	1	1	c	1	<u>0</u>	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
cba, cba, cba	$\frac{1}{216}$	$\frac{1}{56}$	c	1	1	1	c	1	1	1	c	1	1	1	c	1	1	1
Sum total	1	1		168	120	120		144	144	120		136	136	136		147	129	111
TCI _i (·) for IC				$\frac{168}{216}$	$\frac{120}{216}$	$\frac{120}{216}$		$\frac{144}{216}$	$\frac{144}{216}$	$\frac{120}{216}$		$\frac{136}{216}$	$\frac{136}{216}$	$\frac{136}{216}$		$\frac{147}{216}$	$\frac{129}{216}$	$\frac{111}{216}$
				≈ .78	.56	.56		≈ 0.67	.67	.55		≈ .63	.63	.63		≈ .68	.60	.51
TCI _i (·) for IAC				$\frac{264}{336}$	$\frac{216}{336}$	$\frac{216}{336}$		$\frac{240}{336}$	$\frac{240}{336}$	$\frac{216}{336}$		$\frac{232}{336}$	$\frac{232}{336}$	$\frac{232}{336}$		$\frac{243}{336}$	$\frac{225}{336}$	$\frac{195}{336}$
				≈ .79	.64	.64		≈ .71	.71	.64		≈ .69	.69	.69		≈ .72	.67	.58
ARI _i (·) for IC				$\frac{180}{216}$	$\frac{144}{216}$	$\frac{144}{216}$		$\frac{162}{216}$	$\frac{162}{216}$	$\frac{156}{216}$		$\frac{162}{216}$	$\frac{162}{216}$	$\frac{162}{216}$		$\frac{177}{216}$	$\frac{162}{216}$	$\frac{142.5}{216}$
				≈ .83	.67	.67		≈ 0.75	.75	.72		≈ .75	.75	.75		≈ .82	.75	.66
ARI _i (·) for IAC				$\frac{282}{336}$	$\frac{246}{336}$	$\frac{246}{336}$		$\frac{264}{336}$	$\frac{264}{336}$	$\frac{258}{336}$		$\frac{264}{336}$	$\frac{264}{336}$	$\frac{264}{336}$		$\frac{282}{336}$	$\frac{267}{336}$	$\frac{238.5}{336}$
				≈ .84	.73	.73		≈ 0.79	.79	.77		≈ .79	.79	.79		≈ .84	.79	.71

Note: This table illustrates success computations when voters $N = \{1, 2, 3\}$ decide on options $A = \{a, b, c\}$ by rule $r|\mathbf{w}$ for $\mathbf{w} = (45\%, 35\%, 20\%)$ and $r \in \{r^P, r^{PR}, r^C, r^B\}$. Cases where $s(\cdot, P_i) \neq \tilde{s}(\cdot, P_i) = \frac{1}{2}$ are indicated by $\underline{0}$.

no say in the collective decision is $1/3$ and $1/2$ for the TCI and ARI , respectively.¹³ A figure of, e.g., $TCI_3(\cdot) = 111/216 \approx 0.51$ under Borda rule shows that shareholder 3's voting rights clearly improve the chances to get what he or she wants. Taking decisions by pairwise majority voting further raises these chances: Copeland rule is the best from shareholder 3's perspective (lilac highlights), no matter which preference culture or success function is considered. By contrast, the large shareholder 1 is, a priori, most successful if plurality rule is used. Shareholder 2's individual success maximizer differs for an all-or-nothing conception of success (plurality runoff) and an average rank perspective (Borda).

We see that player-specific voting weights lead to player-specific answers to the question of which voting rule reflects one's personal preferences the best a priori and that differences between the considered rules are non-negligible. The respective gap between, say, $TCI_2(\cdot) \approx 0.67$ vs. 0.56 under IC arises even though many highly consensual profiles enter the calculation (cf. first column of Table 3). If, in practice, voting is reserved for issues without an obvious consensus, the benefit to player 2 of requesting runoff votes will be greater than what a difference of ≈ 0.11 suggests.

5 General Success Evaluations

The above illustration concerned one of the many conceivable distributions of voting weights among three players. Suppose that shareholder 3 sells a 10% stake to shareholder 2, resulting in the initial ownership structure $\tilde{\mathbf{w}} = (45\%, 45\%, 10\%)$ of Apple. How would the switch from $\mathbf{w} = (45\%, 35\%, 20\%)$ change 2's and 3's prospects for implementing their respective preferences? How is shareholder 1 affected?

Such questions could be answered by redoing the computations illustrated in Table 3 case by case. However, seemingly different weight distributions often imply an identical mapping of preferences to outcomes. For instance, under Copeland, plurality or plurality runoff rule the collective decision will always match shareholder 1's top choice for $(51\%, 42\%, 7\%)$, $(70\%, 15\%, 15\%)$ and all other weight distributions in which shareholder 1 commands more than 50% of the votes. The latter are – from a voting perspective – equivalent. The previous identification of rule-specific weight equivalence classes by Kurz et al. (2020) allows the determination of a player's

¹³For general m , a dummy player d with independent preferences has a $TCI_d(\cdot) = 1/m$ chance to see its top choice win and must expect an outcome exactly in the middle, implying $ARI_d(\cdot) = 1/2$. Random share-based dictatorship constitutes another theoretical benchmark with $TCI_i(\cdot) = \frac{1+w_i \cdot (m-1)}{m}$ and $ARI_i(\cdot) = \frac{1+w_i}{2}$ if i has relative weight w_i ; e.g., $TCI_3(\cdot) \approx 0.47$ and $ARI_3(\cdot) = 0.60$ in the example.

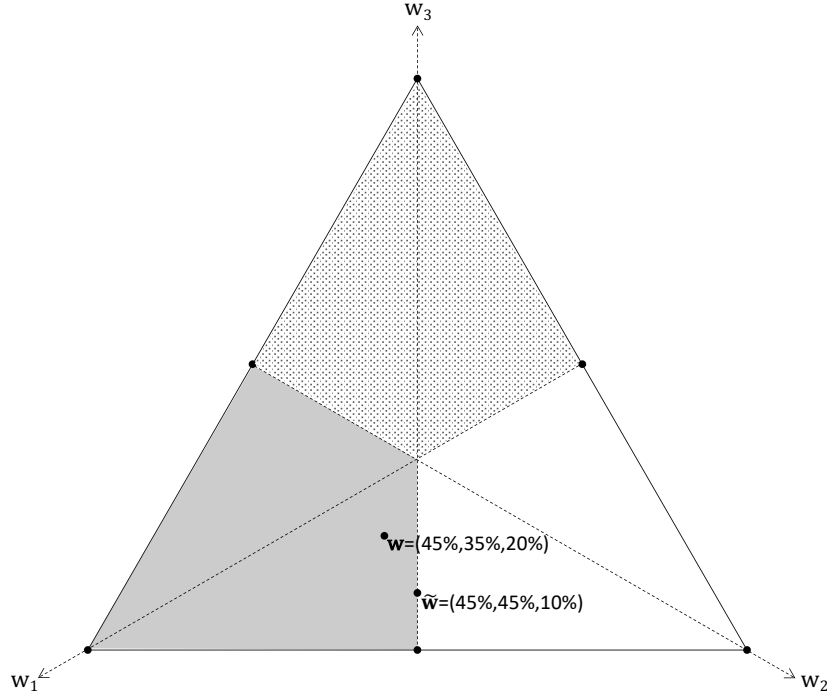


Figure 1: Simplex of all distributions of relative voting weights for $n = 3$

success for *all* possible distributions of voting weights: one needs to conduct the computations behind Table 3 only for one representative of each equivalence class. We will here cover all 6, 7, 4 and 51 classes that exist for plurality, plurality runoff, Copeland and Borda rule if $n = m = 3$.¹⁴

5.1 Success for Three Players with Arbitrary Voting Weights

To present our results, we use the standard projection of the three-dimensional simplex of relative voting weights into the plane. It is illustrated in Figure 1: vertices give 100% of voting weight to the indicated player, e.g., player 1 in the bottom left corner; the midpoint corresponds to symmetric weights of $(1/3, 1/3, 1/3)$. Player 1 (2; 3) wields a plurality of votes in the shaded (blank; dotted) quadrangle.

Figures 2–4 respectively show all achievable $ARI(\cdot)$ -vectors under IC for plurality with or without runoff and the Copeland method for $m = 3$ options, rounded to

¹⁴For $m = 4$ options there are 6 plurality, 7 plurality runoff, 4 Copeland and 505 Borda equivalence classes, corresponding to committees that differ structurally rather than just nominally. Having only $n \leq 3$ relevant players may be unrealistic for big corporations and parliaments but fits many private firms, startups, joint ventures or party alliances and government coalitions.

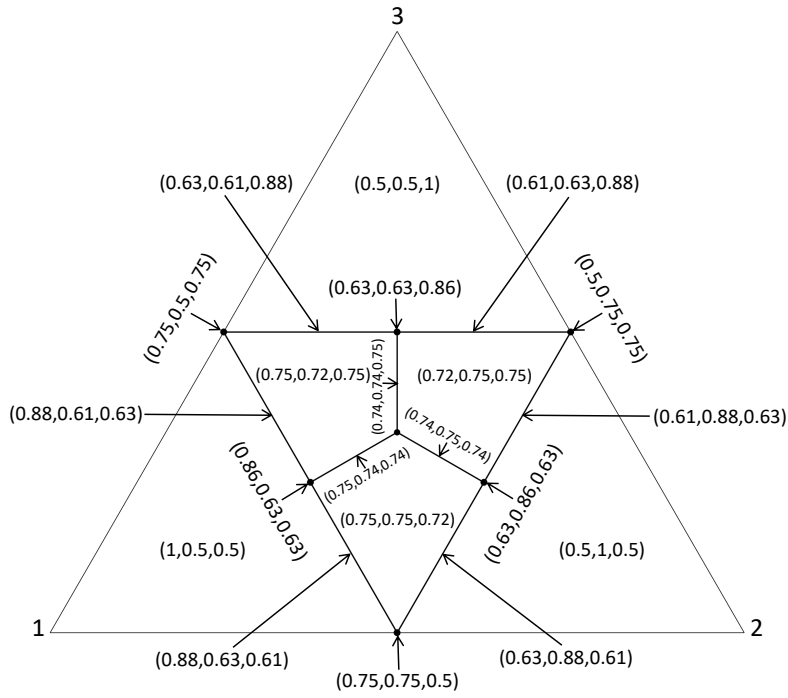


Figure 3: $ARI(\cdot)$ for plurality runoff rule r^{PR} and all weight distributions under IC when $n = m = 3$; $ARI_i = 0.75$ for all players if $\mathbf{w}^* = (1/3, 1/3, 1/3)$

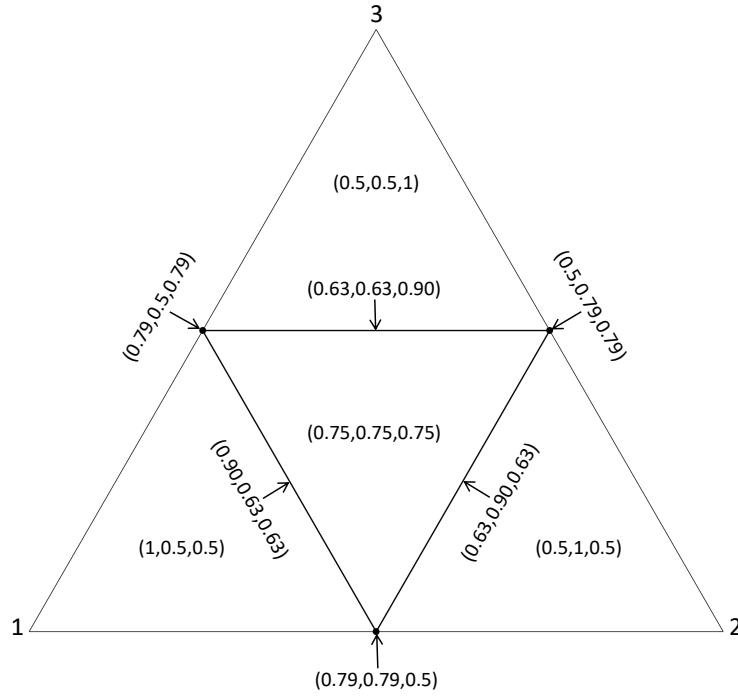


Figure 4: $ARI(\cdot)$ for Copeland rule r^C and all weight distributions under IC when $n = m = 3$

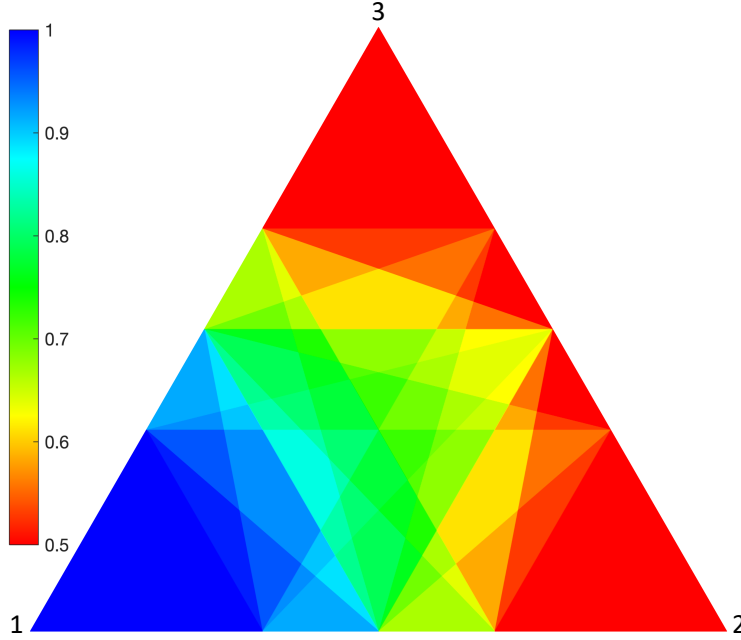


Figure 5: $ARI_1(\cdot)$ for Borda rule r^B and all weight distributions under IC when $n = m = 3$

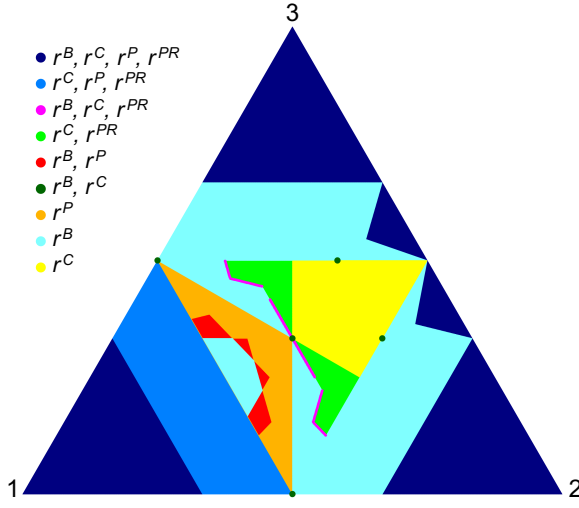
in cheek, the figures provide a map for any self-interested member of a committee with a say on its default voting rule, such as a shareholder in a corporation with few co-owners. At the same time, the figures can also help others prevent foul play.

5.2 Success for an Arbitrary Number of Symmetric Players

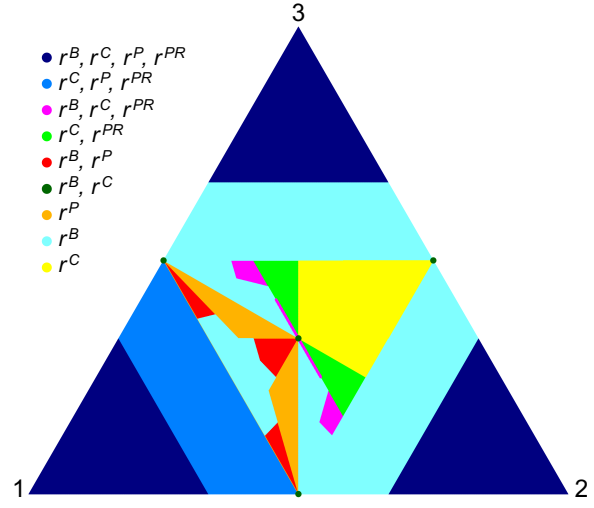
As can be seen in Figures 6 and 7, plurality rule and Borda rule respectively maximize the ‘all-or-nothing’ top choice success $TCI_1(\cdot)$ and average-rank success $ARI_1(\cdot)$ for an equal distribution of voting weight. This observation extends to arbitrary numbers of players or alternatives and comparisons with any anonymous voting rule. Analogous statements apply for other success functions. Namely, we have the following general recommendation for which voting rule r to use if voters are symmetric, i.e., all have voting weight $w_i = 1/n$ and their preferences are statistically exchangeable a priori, such as for IC or IAC:

Proposition 1. *Consider anonymous voting rules r , symmetric voters $N = \{1, \dots, n\}$ whose preferences over $A = \{a_1, \dots, a_m\}$ are statistically exchangeable random orderings a priori and a success function σ^s such that $\sigma^s(a^*, P_i) = s_k$ with $1 = s_1 \geq s_2 \geq \dots \geq s_m = 0$ whenever P_i ranks a^* in k -th place. Then any player i ’s a priori success $\mathbb{E}[\sigma^s(r(\mathbf{P}), P_i)]$ is maximized by*

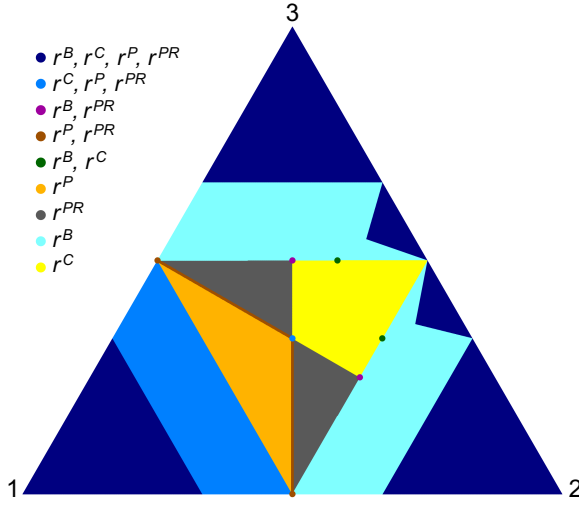
(a) *ARI* and *IC*



(b) *ARI* and *IAC*



(c) *TCI* and *IC*



(d) *TCI* and *IAC*

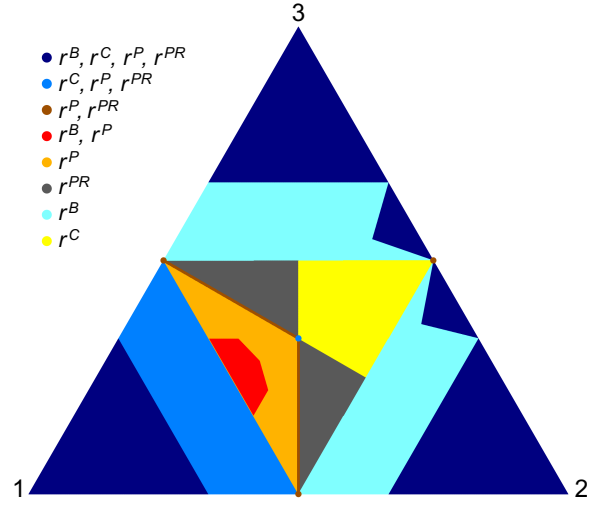
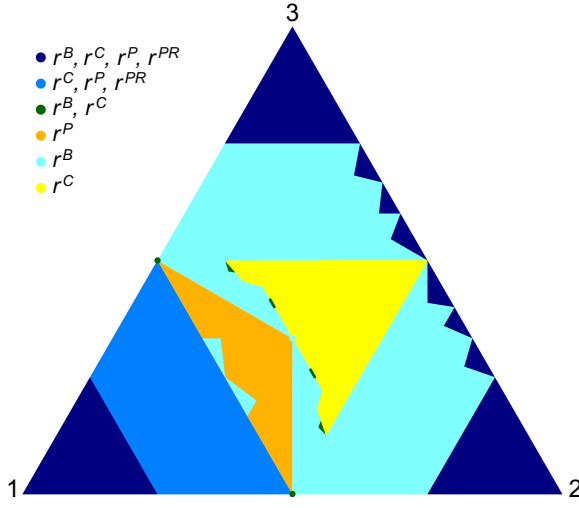
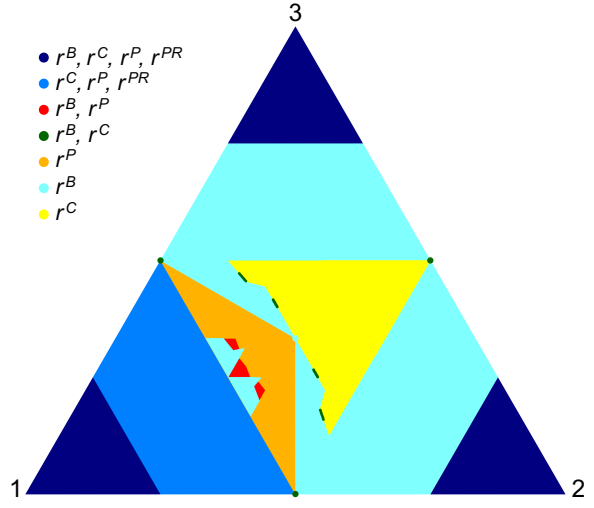


Figure 6: Maximizers of player 1's voting success for $n = m = 3$

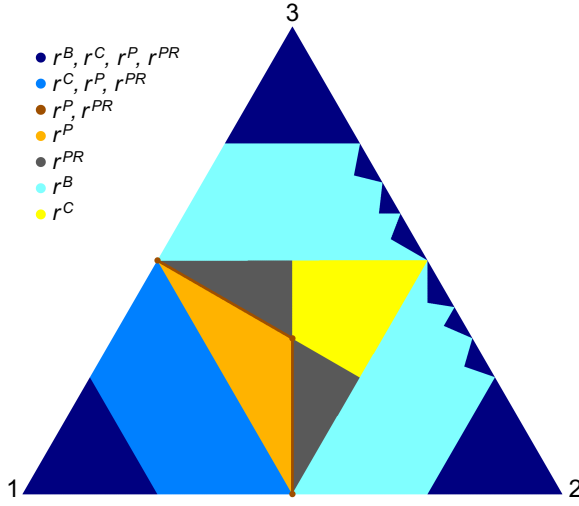
(a) *ARI* and *IC*



(b) *ARI* and *IAC*



(c) *TCI* and *IC*



(d) *TCI* and *IAC*

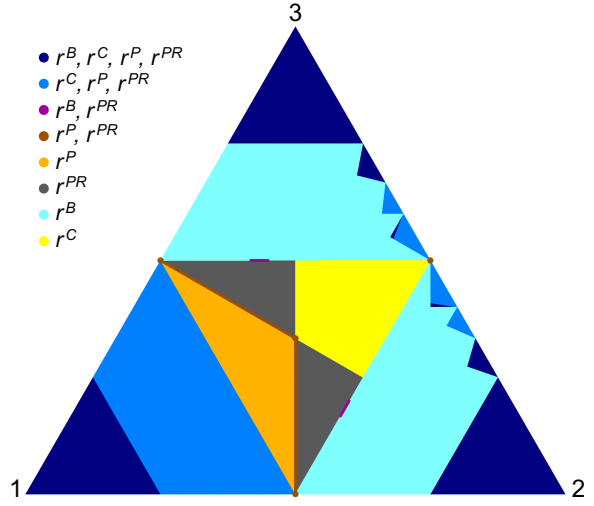


Figure 7: Maximizers of player 1's voting success for $n = 3$ and $m = 4$

the scoring rule r^s that selects

$$r^s(\mathbf{P}) \in \arg \max_{a \in A} \sum_{i=1}^n \sum_{k=1}^m s_k \cdot \chi_k^{P_i}(a)$$

where $\chi_k^{P_i}(a) = 1$ if P_i ranks a in k -th place and 0 otherwise. In particular, $\text{TCI}_i(\cdot)$ is maximized by plurality rule r^P and $\text{ARI}_i(\cdot)$ is maximized by Borda rule r^B .

Proof. Exchangeability and anonymity imply $\mathbb{E}[\sigma^s(r(\mathbf{P}), P_i)] = \mathbb{E}[\sigma^s(r(\mathbf{P}), P_j)]$ for any players $i, j \in N$. Maximization of $\mathbb{E}[\sigma^s(r(\mathbf{P}), P_i)]$ with respect to r is therefore equivalent to the maximization of

$$\sum_{j=1}^n \mathbb{E}[\sigma^s(r(\mathbf{P}), P_j)] = \sum_{j=1}^n \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \sigma^s(r(\mathbf{P}), P_j) = \sum_{j=1}^n \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \sum_{k=1}^m s_k \cdot \chi_k^{P_j}(r(\mathbf{P})) \quad (7)$$

$$= \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \left[\sum_{j=1}^n \sum_{k=1}^m s_k \cdot \chi_k^{P_j}(r(\mathbf{P})) \right]. \quad (8)$$

By definition, $r^s(\mathbf{P})$ maximizes the bracketed term in equation (8) for every $\mathbf{P} \in \mathcal{P}(A)^n$. Hence r^s maximizes $\mathbb{E}[\sigma^s(r(\mathbf{P}), P_i)]$. It remains to note that $1 = s_1 > s_2 = \dots = s_m = 0$ for all-or-nothing success function $s(\cdot)$ and that then $r^s = r^P$. Similarly, $s_k = (m-k)/(m-1)$ holds for $k = 1, \dots, m$ for the more gradual function $\tilde{s}(\cdot)$. Then, $r^s = r^B$ because re-scaling Borda scores $b_i(a, P)$ by $1/(m-1) > 0$ leaves the score maximizers and thus the selected outcomes unchanged. ■

The intuition behind Proposition 1 is straightforward: plurality rule is defined as maximizing the number of voters who see their top choice win. In a perfectly symmetric world, this is equivalent to maximizing the probability that a fixed voter sees its top choice win. Similarly, Borda rule picks the option with the highest average rank among all voters, which entails maximizing the expected rank assigned to the outcome by any fixed voter under symmetry. Changing the order of summation in equations (7) and (8) formalizes just this. We are unaware of previous statements of this generalization of the Rae-Taylor theorem for binary decisions (Rae 1969; Taylor 1969). In the special case where voter preferences are independent and identically distributed, Proposition 1 echoes Theorem 3.1 of Apesteguia et al. (2011) on the utilitarian social optimality of r^s if all s_k equal the expected utility of choosing a voter's k -th ranked alternative.

Importantly, even the slightest deviation from perfect symmetry can destroy the optimality of r^P and r^B identified in Proposition 1. The combinatorics behind achieving a plurality, majority or score maximum make equal voting weights a knife-edge case. To see this, move slightly to the northeast or northwest of the midpoint of the simplex, e.g., in Figure 6(d): r^P immediately stops being TCI_1 -optimal. Also the other panels of Figures 6 and 7 attest that it is tempting but wrong to extrapolate Proposition 1 and the intuitive advantage of plurality rule (Borda rule) in yielding high top choice success (average rank success) to ‘nearly symmetric’ weights.

There is not even a monotonic pattern of which voting rules are optimal for player 1 once perfect symmetry is broken. Consider Figure 6(d) and $\mathbf{w} = (w_1, (1 - w_1)/2, (1 - w_1)/2)$ for illustration: all rules make 1 a dummy player for $w_1 = 0$ but r^P and r^{PR} maximize 1’s success given the correlation with players 2 and 3’s preferences;¹⁷ r^C is the unique TCI_1 -maximizer for $w_1 \in (0, 1/3)$; then r^C , r^P and r^{PR} are success maximizing for $w_1 = 1/3$; only r^P maximizes TCI_1 for $w_1 \in (1/3, 3/7]$ and $w_1 = 1/2$, while r^P is tied with r^B for $w_1 \in (3/7, 1/2)$; again r^C , r^P and r^{PR} are optimal for $w_1 \in (1/2, 2/3]$; and finally all four rules induce $TCI_1 = 1$ for $w_1 \in (2/3, 1]$. A similar back and forth can be seen in the other panels of Figures 6 and 7.

Based on these observations we should expect Proposition 1’s theoretical results to provide little to no practical guidance for shareholder voting in publicly traded companies, parliaments, etc. unless all of the relevant stakeholders have equal voting weights. Quite surprisingly, the findings obtained in Section 6 for prominent US share distributions will offer evidence against this conjecture.

5.3 Other Evaluation Criteria

The premise motivating our investigation is that a given player i cares only about its own success, not some greater good. Let us nonetheless comment on two additional, normatively appealing aspects of voting rules: the weighted total of individual success values and the extent to which voting success correlates with voting weight.

5.3.1 Aggregate Success of All Players

By-laws or rules of procedure may need to appeal also to future investors, not just the founders or current owners of a company, and it can be unclear who will be a large or a small shareholder in a few years’ time. Instituting a rule that implies a high

¹⁷ r^P and r^{PR} are equivalent if only two players have positive weight. The rules pick the top option for at least one of them, while r^C and r^B can yield ties involving options that are mid-ranked by both.

average or sum total of individual success values then becomes an attractive default. High aggregate a priori success is also desirable from the welfare perspective of a regulator. So it is good to know that plurality (Borda) rule maximizes the sum of all voters' top choice (average rank) success. This is a direct corollary of Proposition 1 for symmetric voters but extends to any asymmetric weights if we treat each share equally, i.e., consider the weighted sum of individual success values:

Proposition 2. *Under the conditions of Proposition 1, the scoring rule r^s maximizes the $\mathbf{w}$ -weighted total success $\sum_{j=1}^n w_j \mathbb{E}[\sigma^s(r|\mathbf{w}(\mathbf{P}), P_j)]$ for any given $\mathbf{w}$. In particular, $\sum_{j=1}^n w_j TCI_j(\cdot)$ is maximized by plurality rule r^P , and $\sum_{j=1}^n w_j ARI_j(\cdot)$ is maximized by Borda rule r^B for any distribution of voting weights.*

The proof follows directly from replacing $\sum_{j=1}^n \mathbb{E}[\sigma^s(r(\mathbf{P}), P_j)]$ by $\sum_{j=1}^n w_j \mathbb{E}[\sigma^s(r|\mathbf{w}(\mathbf{P}), P_j)]$ in equation (7) in the proof of Proposition 1. For Proposition 2, one may even drop the exchangeability condition that is needed for Proposition 1.

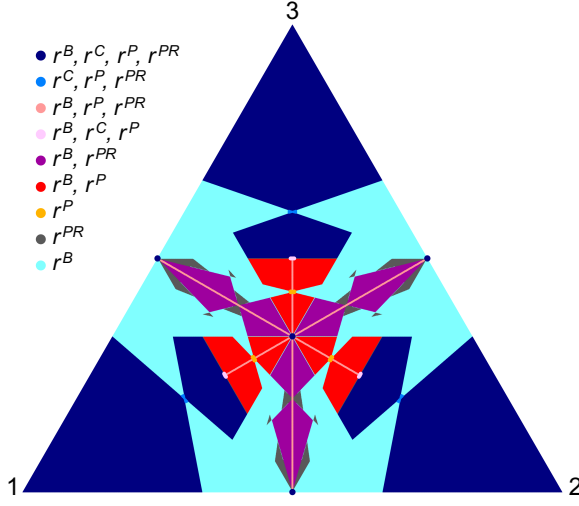
5.3.2 Transparency

The extent to which larger shareholdings imply greater success is another aspect that investors and authorities may care about. Voting rights are the defining feature of common shares. Thus, differences in ownership should go with differences in how well the respective shareholder preferences are reflected in corporate decisions.

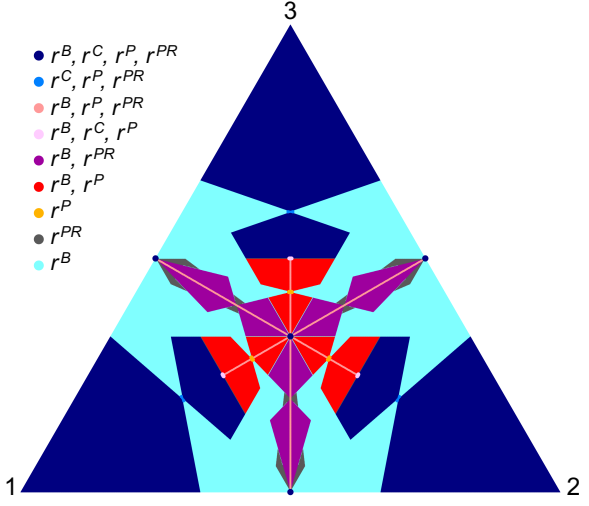
A simple measure of how transparently a priori voting success is aligned to voting rights is their correlation. For instance, under Borda rule, IC and $m = 3$, the correlation coefficient for the weight distribution $\mathbf{w} = (45\%, 35\%, 20\%)$ and the success distribution $ARI(\cdot) = (177/216, 162/216, 142.5/216)$ in Table 3 is 0.9992. This number is considerably higher than the respective figures for plurality and plurality runoff (0.8030 and 0.9177); the uniform success values for Copeland are entirely uncorrelated with the given weights.

Figure 8 shows which method maximizes the respective correlation coefficient for all conceivable voting weight configurations among three players who decide on three alternatives. The winner is mostly Borda rule – no matter if one considers ARI or TCI success vectors for the IC or IAC preference distribution. An analogous evaluation using the Kendall rank correlation coefficient gives similar results (with Borda rule being even more dominant). The underlying reason is that the Borda method comes with many more weight equivalence classes than our other voting rules (cf. Kurz et al. 2020). So, weight variations are more likely to make a difference.

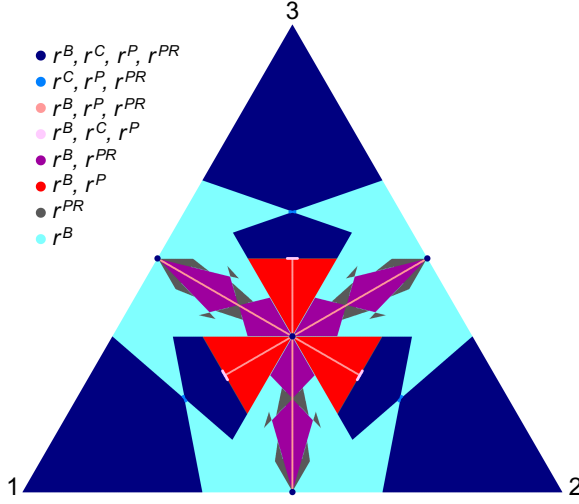
(a) *ARI* and *IC*



(b) *ARI* and *IAC*



(c) *TCI* and *IC*



(d) *TCI* and *IAC*

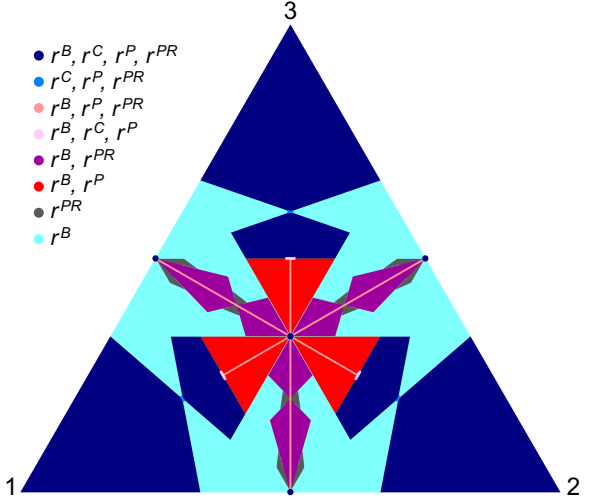


Figure 8: Maximizers of correlation between voting weights and success when $n = m = 3$

We also confirmed that Borda rule translates the large shareholdings in S&P 100 corporations, which we study next, into success the most transparently.

6 Application to S&P 100 Corporations

As we saw above, the individual optimality of the plurality and Borda voting rules can break down even for marginal deviations from symmetric voting weights. We therefore apply our success measures to a range of actual distributions of voting weights in order to assess the practical relevance of our findings (cf. Leech 1988). We consider the ownership structure of the companies in the S&P 100 stock index as composed at the beginning of 2022. To achieve reasonable computation times of ≈ 6 hours per firm and success index on average, we focus on $m = 3$ alternatives and the ten largest shareholdings of each index constituent. The remaining shares are treated as a homogeneous, perfectly divisible ‘ocean’ of free float.¹⁸

Under the IC assumption, the law of large numbers induces a uniform distribution of the float’s cumulative weight across the $m! = 6$ possible rankings. We evaluate success in each of the resulting $6^{10} \approx 60$ mio. distinguishable preference configurations. The situation is more complicated under the IAC assumption: positive correlation between shareholders – both large and small – generates infinitely many distinct configurations. So, we approximate a priori success values under IAC in an extensive Monte Carlo simulation.¹⁹

6.1 Data

The considered shareholder data comes from the Thomson Reuters Global Ownership database as of January 19, 2022, accessed via Refinitiv Eikon. The data combines

¹⁸As a robustness check, we alternatively ignore all but the ten largest shareholders and evaluate their 6^{10} possible preference profiles. Ignoring smaller shareholders is in line with Azar et al. (2018), who eliminated all holdings below 0.5%. Disregarding the float does not change any results for IC, while corresponding findings for IAC become somewhat more similar to the IC findings.

¹⁹The exchangeable preferences under IAC can be simulated by first drawing common preference inclinations $\mathbf{p} = (p_1, \dots, p_{m!})$ uniformly from the $(m! - 1)$ -dimensional unit simplex and then determining individual preferences via (conditionally) independent single draws from the $\mathbf{p}$ -multinomial distribution (cf., e.g., Berg 1985). We approximate TCL_i (ARI_i) by averaging 350 000 iterations of the following steps: (1) draw $\mathbf{p}$; (2) draw P_i for $i = 1, \dots, 10$; (3) shortcut draws for float shareholders by dividing the float’s total weight in proportion to $(p_1, \dots, p_{m!})$; (4) determine winner a^* ; (5) evaluate $s(a^*, P_i)$ ($\tilde{s}(a^*, P_i)$) for $i = 1, \dots, 10$. This takes about 4 hours/firm on a desktop computer with a 2.3 GHz i7 CPU; complete enumeration for IC takes about 2 hours/firm (compared to 0.1 seconds for $m = 2$).

Table 4: Descriptive statistics for shareholdings in the 92 included S&P 100 corporations

Shareholdings	Percentage values for the i^{th} largest shareholder						
	$i =$	1	2	3	...	10	11-...
Mean		10.76	7.23	5.38	...	1.14	62.33
Mean (cumulative)		10.76	18.00	23.38	...	37.67	100.00
Standard deviation		7.19	1.24	1.26	...	0.34	8.50
Standard deviation (cumulative)		7.19	7.15	7.13	...	8.50	0.00
Maximum		48.87	15.14	9.11	...	2.29	72.68
Maximum (cumulative)		48.87	53.38	56.65	...	68.92	100.00
Minimum		6.96	4.52	3.27	...	0.44	31.08
Minimum (cumulative)		6.96	13.24	17.90	...	27.32	100.00

information from various sources, such as mandatory disclosures. It has previously been used, e.g., by Bushee and Noe (2000), Azar et al. (2018) or Backus et al. (2021). We consider only ordinary shares with equal voting rights and exclude corporations with dual or multi-share classes that entail different voting privileges (cf. Backus et al. 2021). This leaves 92 constituents of the S&P 100 index in our sample.²⁰ We consolidated the shareholdings of all BlackRock entities in analogy to Backus et al. (2021) and Ben-David, Franzoni, Moussawi and Sedunov (2021).

The descriptive statistics in Table 4 provide an overview of the consolidated share distributions. The largest shareholders of the S&P 100 constituents hold $\approx 7\%$ to almost 50% of the corporate stock with a standard deviation of 7.19% and a mean of 10.76%. The ten largest shareholders hold a cumulative stake of 37.67% on average. The remaining holdings define the respective free float mentioned above.

For the independent preferences assumed by IC, a sixth of the float's voting weight can be associated with each of the six alternative rankings of $m = 3$ options. Then, despite not wielding the required majority formally, the largest shareholder of three S&P 100 firms (Oracle, T-Mobile US, Walmart) can effectively dictate collective choices under any of our voting rules with a top choice and average rank index value of $TCI_1 = ARI_1 = 1$ (and $TCI_i = 1/3$ or $ARI_i = 1/2$ for shareholders $i \geq 2$). The three corporations will be ignored in IC-based rule comparisons, and so we report results for altogether 89 (92) out of 100 corporations included in the S&P 100 for IC (IAC).

²⁰The excluded corporations are: Alphabet, Berkshire Hathaway, Charter Communications, Comcast, Ford Motor, Meta Platforms, Nike and United Parcel Service.

Table 5: Success maximizers for S&P 100 shareholders

Success index and Prob. distr.	Success maximizers for the i^{th} largest shareholder										
	r	1	2	3	4	5	6	7	8	9	10
ARI IC $n = 89$	Borda	76	85	56	84	71	76	78	70	73	70
	Copeland	2	4	33	3	11	7	7	9	4	13
	Plurality	11	0	0	0	0	0	0	0	0	0
	Plurality runoff	0	0	0	2	7	6	4	10	12	6
ARI IAC $n = 92$	Borda	87	92	92	92	92	92	92	92	92	92
	Copeland	0	0	0	0	0	0	0	0	0	0
	Plurality	5	0	0	0	0	0	0	0	0	0
	Plurality runoff	0	0	0	0	0	0	0	0	0	0
TCI IC $n = 89$	Borda	0	10	7	6	8	9	8	9	9	9
	Copeland	0	0	19	0	1	0	0	0	0	2
	Plurality	89	50	60	78	61	70	63	62	66	58
	Plurality runoff	0	29	3	5	19	10	18	18	14	20
TCI IAC $n = 92$	Borda	0	4	3	3	3	3	3	3	3	3
	Copeland	0	0	0	0	0	0	0	0	0	0
	Plurality	92	88	89	89	89	89	89	89	89	89
	Plurality runoff	0	0	0	0	0	0	0	0	0	0

6.2 Results

Table 5 summarizes the success maximizers for the included S&P 100 companies. The striking message is that the theoretical conclusions of Proposition 1 for perfectly symmetric voting weights extend to the considered voting share distributions. These distributions are highly asymmetric even among the largest investors. And yet, assuming preference correlation à la IAC, Borda is the best rule for all but five of the 920 included shareholders (gross count) if success is evaluated according to the individual average ranking of collective choices (ARI_i).²¹ Similarly, plurality rule maximizes success in overwhelmingly many cases if a shareholder i cares only about the probability with which it obtains its top choice (TCI_i).

Under IC, plurality rule maximizes the TCI_1 value of the largest shareholder in all 89 companies under consideration, the respective TCI_2 number of the second-largest shareholder in 50 companies, the TCI_3 level of the third-largest in 60 and so on. We obtain similar findings for Borda rule and the average rank index: Borda is the ARI_1 (ARI_2 , ARI_3 , $\dots$, ARI_{10}) maximizer for the respective largest (second-largest, third-largest, $\dots$, tenth-largest) shareholder in 76 (85, 56, $\dots$, 70) of the included 89

²¹The five exceptions arise when the largest shareholder owns a share of $w_1 \geq 20\%$.

companies. These results are very unexpected given the distribution sensitivity and the non-monotonicities observed in Section 5 for three voters.

A robust rule of thumb emerges: make a case for deciding by Borda rule if your objective is to maximize the ARI_i value, and for plurality rule if you want to maximize TCI_i . This is essentially independent of whether you have the largest, second-largest, etc. holding in a corporation and whether the IC or IAC preference distribution is deemed more relevant for a personal a priori assessment. The recommendation also holds for small or infinitesimal shareholders in the float²² and, as shown in Proposition 2, Borda and plurality rule maximize the weighted total of all shareholders' success values.

Copeland and plurality runoff rule maximize a player i 's ARI_i and TCI_i levels only in a few cases. The key exceptions are ARI_3 numbers for the third-largest shareholders (Copeland is optimal in 33 cases) and TCI_2 values for the second-largest shareholders (plurality runoff is optimal for 29 of them) if preferences are independent.

Having such exceptions may, of course, make it worthwhile not to rely on the above rule of thumb but to assess voting methods individually and case by case. One can, for instance, zoom in on the success maximizers for the largest institutional investors: Vanguard and BlackRock. Both are among the ten largest shareholders of all examined S&P 100 companies and usually in the top three. It turns out that the advantage of Borda (plurality) in maximizing their ARI_i (TCI_i) success level is pronounced for both of them.²³

Figure 9 illustrates the mean values of ARI_i and TCI_i for the ten largest shareholders under the applicable voting rule. On the one hand, the figure corroborates the above rule of thumb: the interpolated lines for Borda (plurality) lie consistently above those of the other three voting rules when considering ARI_i (TCI_i).²⁴ On the other hand, comparing the spaces between the lines to their slopes highlights that, non-surprisingly, the voting advantage that an investor derives from having more shares than another tends to matter more than whether this or that rule applies.

²²Positive correlation under IAC aligns the interests of any dummy players with the average interests of the shareholders in Table 5, while all rules yield identical dummy success $TCI_d = 1/3$ and $ARI_d = 1/2$ under IC. We also cross-checked the findings reported in Table 5 by repeating the analysis with an additional shareholder 11 owning 0.1% of shares: all conclusions continued to hold.

²³Exceptions arise under IC, with results for $TCI_{\text{BlackRock}}$ similar to those for the second largest shareholders in Table 5. We remark that broad agreement among investors about the success implications of different voting rules makes it unlikely that a rule switch would prompt portfolio readjustments. This supports treating voting weights as exogenous givens.

²⁴Using medians instead of means leads to the same inferences.

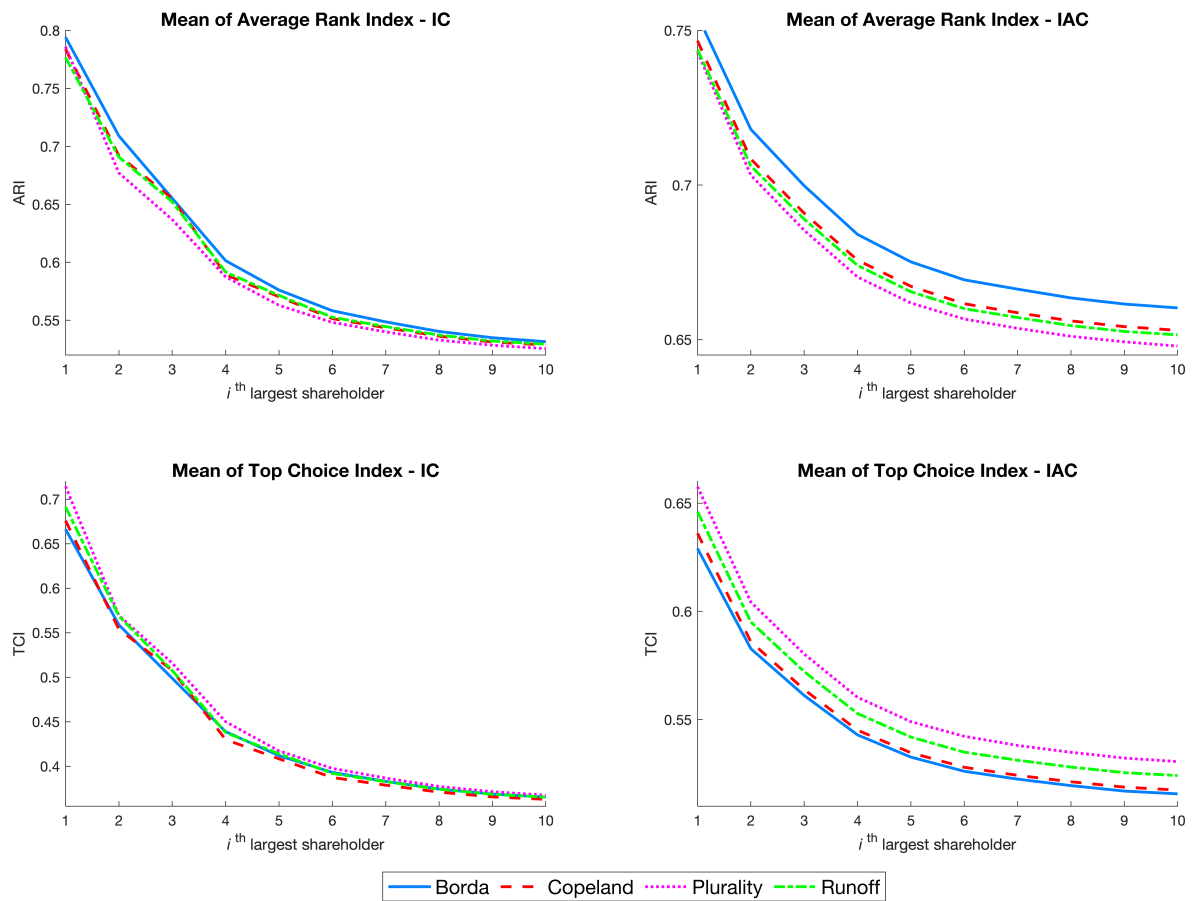


Figure 9: Success indices averaged per shareholding rank across S&P 100 companies

We use a linear regression model in order to quantify expected voting rule differences and to determine their statistical significance. Since it is well-known from the analysis of binary votes that it is not higher voting weight as such that gives an advantage, but how this weight facilitates the formation of winning majorities (or blocking minorities) with other voters, we opted to include proxies of shareholders' voting power instead of the holdings themselves. We draw on values of the Penrose-Banzhaf index (*PBI*) for IC and the Shapley-Shubik index (*SSI*) for IAC. Because these indices were developed for binary decisions under the IC and IAC assumptions, they are attuned to the respective correlation pattern.²⁵ We then estimate the following regression model using an ordinary least squares (OLS) regression with heteroscedasticity robust standard errors (White 1980):²⁶

$$VotingSuccess_{ir} = \alpha_0 + \beta_1 PowerIndex_i + \sum_{j=2}^4 \beta_j VotingRules_r + \varepsilon_{ir}, \quad (9)$$

where *VotingSuccess* is the shareholder's success index ARI_i (TCI_i) and *VotingRules* are *Copeland*, *Plurality* and *PluralityRunoff* (*Borda*, *Copeland* and *PluralityRunoff*) indicator variables, respectively. *PowerIndex* represents either shareholder *i*'s *PBI* or *SSI* value.

We estimate the regressions for 3 560 (3 680) voting rule-shareholder observations consisting of the ten largest shareholders of the 89 (92) S&P 100 companies for our four voting rules considering the IC (IAC) distribution. We present the corresponding results in Table 6. One can see that the adjusted R^2 values are very high. This is expected as a shareholder's power and success are driven by the underlying voting rights and so both – even if power is approximated by indices that presume binary voting – covary. The correlation is particularly high under the IC assumption but the fit is very good for IAC too. These observations retrospectively support the many previous investigations that employed the Penrose-Banzhaf or Shapley-Shubik indices of binary voting power in contexts where collective choices are not from the start simple 'yes'-or-'no' affairs, but at some stage require a selection from multiple candidates or motions.

²⁵The respective power index computations assume binary decisions by shareholders $i = 1, \dots, 10$ with a 50% majority quota. Dedicated power measures for non-binary decisions (Kurz et al. 2021) are more challenging to compute and already capture some rule effects.

²⁶As a robustness check, we alternatively control for the holding percentage of shareholder *i* or the ownership concentration among shareholders $j \neq i$ measured by the Herfindahl index (cf. Demsetz and Lehn 1985 and Ajinkya, Bhojraj and Sengupta 2005). The model then loses some explanatory power but results are qualitatively unchanged. Including shareholder rank fixed effects does not alter our conclusions either, nor does using tobit regressions for censored dependent variables.

Table 6: Multivariate regression results

	Dependent variable: Success index for given preference distribution			
	<i>ARI</i>		<i>TCI</i>	
	IC (1)	IAC (2)	IC (3)	IAC (4)
Variable	Coefficient	Coefficient	Coefficient	Coefficient
<i>PBI</i>	0.4950***		0.6166***	
<i>SSI</i>		0.3249***		0.4195***
<i>Borda</i>			-0.0108***	-0.0180***
<i>Copeland</i>	-0.0068***	-0.0080***	-0.0126***	-0.0154***
<i>Plurality</i>	-0.0124***	-0.0130***		
<i>PluralityRunoff</i>	-0.0071***	-0.0098***	-0.0065***	-0.0078***
<i>Intercept</i>	0.5078***	0.6528***	0.3356***	0.5210***
<i>Adjusted R²</i>	0.9924	0.8644	0.9885	0.8797
<i>n</i>	3 560	3 680	3 560	3 680

Note: This table presents the coefficients and significance levels of our four OLS regressions with heteroscedasticity robust standard errors (White 1980). The superscripts *, ** and *** represent significance levels of 10%, 5% and 1% (two-tailed *t*-tests), respectively. We estimate the following regression models: $VotingSuccess_{ir} = \alpha_0 + \beta_1 PowerIndex_i + \sum_{j=2}^4 \beta_j VotingRules_r + \varepsilon_{ir}$ where *VotingRules* are *Copeland*, *Plurality* or *PluralityRunoff* indicators in columns (1) and (2) and *Borda*, *Copeland* or *PluralityRunoff* indicators in columns (3) and (4). The dependent variable *VotingSuccess* is the shareholder's success index *ARI* in columns (1) and (2) and *TCI* in columns (3) and (4), respectively. The success indices are calculated assuming IC in columns (1) and (3) and IAC in columns (2) and (4) for the probability distribution over preference profiles. Consequently, the Penrose-Banzhaf index (*PBI*) represents the *PowerIndex* in columns (1) and (3) the Shapley-Shubik index (*SSI*) is used in columns (2) and (4).

The results in columns (1) and (2) of Table 6 confirm the inferences we drew from Table 5 and Figure 9: individual voting success as measured by *ARI* is significantly lower when the voting rules *Copeland*, *Plurality* or *PluralityRunoff* are applied instead of Borda rule, which is the baseline in these regressions. Similarly, columns (3) and (4) show significantly lower success levels for *Borda*, *Copeland* and *PluralityRunoff* in comparison to the benchmark plurality rule considering *TCI*. All differences are statistically significant at the 1% level.

Using the optimal voting rules on average leads to 1 to 3% higher success rates. This may not seem a big difference but in a majority of cases the tenth largest shareholders (with mean holdings of 1.14%) would benefit more from switching from the worst to the best voting rule than from switching their holdings with the respective seventh largest shareholders (with mean holdings of 1.78%). Considering only IAC, the effects are yet more pronounced: the tenth largest shareholders in more than 80%

(50%) of the firms would benefit more from a rule change than from swapping their holdings with those of the sixth (fifth) largest shareholders.²⁷ Recall also that a priori figures include many preference configurations with broad agreement among voters. If, a posteriori, one conditions on situations where disagreement has motivated an otherwise unnecessary call for a vote, the success gap between the individually best vs. worst rule grows. The simulated rule differences then serve as lower bounds.

7 Concluding Remarks

The key takeaway from this investigation is that the adopted voting procedure matters from an a priori and day-to-day perspective – not only in selected textbook examples or under rare historical circumstances (cf. Leininger 1993, Tabarrok and Spector 1999, Maskin and Sen 2016, but also Darmann and Klamler 2023 or Lachat and Laslier 2024). Voting rules entail different prospects of seeing one’s most preferred option win and they make a difference to where the expected collective outcome lies in one’s ranking of all options. This can be quantified and predicted for the a priori distribution of preferences that one considers relevant. Corresponding evaluations may inform the design of by-laws, statutes and other governance instruments.

Of course, our findings have practical and theoretical limitations. First, a wide range of voting rules exist, and we have focused on just four. For instance, after a plurality vote without a majority winner, one may delete only the alternative with the lowest support and vote again; a chairperson may put just specific pairwise comparisons on the agenda; and there are many different ways to translate candidates’ positions in individual preference rankings into scores. The rules investigated here include particularly prominent representatives from the three main classes of single-winner methods (Condorcet methods, scoring rules and runoff rules), but there are ample opportunities for follow-up work. Borda rule is probably the least frequently used of the voting procedures that we have looked at. In case of symmetric voters, it operationalizes a straightforward ideal of justice (see Apesteguia et al. 2011) and has particularly compelling axiomatic properties.²⁸ Ambuehl and Bernheim (2024) doc-

²⁷Considering Apple Inc., for instance, the ARI_{10} -gain of $0.6619 - 0.6497 = 0.0121$ from replacing plurality rule by Borda rule exceeds the difference between $ARI_{10} = 0.6497$ and $ARI_5 = 0.6616$ under plurality rule, which reflect holdings of 0.66% vs. 2.07%. An acquisition of $2.07\% - 0.66\% = 1.41\%$ of Apple stock would have cost around USD 38 billion in Jan. 2022.

²⁸Borda rule is closely connected to both May’s and Arrow’s classical axioms of rational collective choice. As shown by Maskin (2025), the rule is unique in satisfying anonymity, neutrality, responsiveness, unrestricted domain, a Pareto ranking condition and modified independence of irrelevant

ument that Borda winners receive better ex post support than plurality or pairwise winners in decision experiments. Our findings that Borda rule yields higher individual ranks also for practically relevant asymmetry and induces higher correlation between voting rights and success than the competing rules strengthens the case for using it more.

Second, we are aware that very many votes by boards or general assemblies are ‘yes’-or-‘no’ decisions with little dissent. The existing sets of options are often reduced to singletons behind the scenes without formal votes long before an official meeting. However, decision making can involve disagreement. Then it is advantageous to know how different procedures to resolve it translate into different success expectations. In addition, the non-public processes that lead to a consensus proposal may implicitly involve multiple pairwise comparisons, a runoff-like focus on the two options with the most initial support or the accumulation of scores. Understanding the comparative merits of these approaches can be as beneficial in their informal application as in explicit votes.

Third, we have focused on sincere voting. This is a restrictive theoretical assumption despite rather scant empirical support for its most compelling alternative: strategic voting. The latter comes with a pervasive non-uniqueness of the resulting voting outcomes and entails limitations of its own. We present a detailed investigation of strategic voting equilibria in Appendix B. It shows that many of the weight-specific maximizers identified in Section 5 continue to maximize success if one selects from non-singleton sets of equilibria according to their Kemeny distance to truthful voting.

Despite these caveats, the reported investigation highlights that taking decisions by vote is a lot less trivial than it may appear. The combinatorial complexity of weighted voting is enormous. Analysis of collective decision rules is therefore laborious already for two alternatives.²⁹ The computational burden increases steeply when three or more options compete but corresponding a priori assessments are still worthwhile: they can make voting more successful.

alternatives. This comes on top of the desirable consistency properties that distinguish Borda rule from, e.g., pairwise voting (cf. Young 1974).

²⁹See, e.g., Kurz and Napel (2016) and Kober and Weltge (2021) on an open computational problem created by the qualified majority rule that is used in the Council of the European Union.

Appendices

Appendix A: Robustness to Random Tie Breaking

Lexicographic tie breaking assumes some fixed ordering $a_1 <_L a_2 <_L \dots <_L a_m$ of the alternatives such that if options $a_{i_1}, \dots, a_{i_k}$ receive the same plurality score (Borda score, etc.) at a given preference profile $\mathbf{P}$ then a_{i^*} with $i^* = \min\{i_1, \dots, i_k\}$ is selected as the unique winner $r|\mathbf{w}(\mathbf{P})$.

To demonstrate that the tie-breaking assumption is innocuous for our analysis, consider the *set-valued version of a given voting rule* r like r^P (r^B , etc.). This maps each preference profile $\mathbf{P}$ to the non-empty set $\hat{r}|\mathbf{w}(\mathbf{P}) = A^* \subseteq A$ of all alternatives that have the highest plurality score (Borda score, etc.). In contrast to our point-valued baseline, the respective set-valued version of r^P (r^B , etc.) is neutral, i.e., $\hat{r} = \hat{r}^P$ ($\hat{r}^B$, etc.) satisfies $\hat{r}(\rho(\mathbf{P})) = \rho(\hat{r}(\mathbf{P}))$ for any permutation $\rho: A \rightarrow A$ and $\mathbf{P} \in \mathcal{P}(A)^n$.

Now take an arbitrary alternative-based tie breaking method. It can be described by a family $\{\beta_B\}_{B \in 2^A \setminus \emptyset}$ of probability distributions that assign winning probabilities $\beta_B(a)$ to all $a \in B$ with $\sum_{a \in B} \beta_B(a) = 1$ for any set of tied alternatives B . We will write $\{\beta_B\}$ for short. Lexicographic tie breaking $\{\beta_B^{\text{lex.}}\}$ amounts to $\beta_B^{\text{lex.}}(a) = 1$ iff a is the lexicographically minimal element of B . A popular alternative is uniform random tie breaking $\{\beta_B^{\text{uni.}}\}$ where $\beta_B^{\text{uni.}}(a) = 1/|B|$ for any $a \in B$. A given method $\{\beta_B\}$ might also apply uniform tie breaking if $|B| = 2$, lexicographic tie breaking if $|B| = 3$, prescribe particular B -specific probabilities if $|B| = 4$, etc. All we require is that preferences affect the outcome via the baseline voting rule while tie break probabilities are independent of $\mathbf{P}$.³⁰

For a given success function $\sigma: A \times \mathcal{P}(A) \rightarrow \mathbb{R}$ let σ 's extension to $\{\beta_B\}$ -tie breaking $\hat{\sigma}: 2^A \times \mathcal{P}(A) \rightarrow \mathbb{R}$ be defined by

$$\hat{\sigma}(B, P_i) := \sum_{a^* \in B} \beta_B(a^*) \sigma(a^*, P_i). \quad (10)$$

This equals the expectation of $\sigma(a^*, P_i)$ under the pertinent tie break probabilities. With these definitions we have

Proposition 3. Consider voters $N = \{1, \dots, n\}$ with voting weights $\mathbf{w} = (w_1, \dots, w_n)$ whose preferences over $A = \{a_1, \dots, a_m\}$ are drawn from a probability distribution on $\mathcal{P}(A)^n$ that satisfies $\Pr(\mathbf{P}) = \Pr(\rho(\mathbf{P}))$ for any permutation $\rho: A \rightarrow A$. Let $\hat{r}|\mathbf{w}$ be the neutral set-valued

³⁰Making tie break probabilities a function of the preferences of, e.g., the committee's chairperson or the largest shareholder would shift the distribution of a priori success in the expected direction.

version of $r|\mathbf{w}$ and $\hat{\sigma}$ be the extension of success function σ to $\{\beta_B\}$ -tie breaking. Then

$$\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)] = \mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)].$$

In particular, player i 's top choice or average rank success under lexicographic tie breaking, $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$, and the respective success under any other alternative-based tie breaking method, $\mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)]$, are identical.

Proof. For any non-empty subset $B \subseteq A$ of alternatives, denote the set of preference profiles that yield a tie between the alternatives in B by

$$\mathcal{P}_B := \{\mathbf{P} \in \mathcal{P}(A)^n : \hat{r}|\mathbf{w}(\mathbf{P}) = B\}. \quad (11)$$

Also denote the permutations of A that only switch elements of $B \subseteq A$ by

$$\mathcal{S}^B := \{\rho: A \rightarrow A : [a \notin B \Rightarrow \rho(a) = a]\}. \quad (12)$$

If $\mathbf{P} \in \mathcal{P}_B$, then neutrality of $\hat{r}|\mathbf{w}$ implies that also $\mathbf{P}' = \rho(\mathbf{P}) \in \mathcal{P}_B$ for any $\rho \in \mathcal{S}^B$. Hence $\mathcal{P}_B$ can be partitioned into $k(B) = |\mathcal{P}_B|/|B|!$ subsets $\mathcal{P}_{B,1}, \dots, \mathcal{P}_{B,k(B)}$ that each contain $|B|!$ profiles which differ only by permutations of B 's elements. For any such partition element $\mathcal{P}_{B,j}$ let us fix a 'representative' profile $\mathbf{P}^{B,j} \in \mathcal{P}_{B,j}$ that ranks B 's elements $B = \{a_{r_1}, \dots, a_{r_{|B|}}\}$ from player i 's perspective by $a_{r_1} P_i^{B,j} a_{r_2} P_i^{B,j} \dots P_i^{B,j} a_{r_{|B|}}$.

Player i 's success $\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)$ for $\mathbf{P} \in \mathcal{P}_{B,j}$ equals

$$\hat{\sigma}(B, P_i) = \hat{\sigma}(B, P_i^{B,j}) = \beta_B(a_{r_1})\sigma(a_{r_1}, P_i^{B,j}) + \beta_B(a_{r_2})\sigma(a_{r_2}, P_i^{B,j}) + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P_i^{B,j}) \quad (13)$$

if $\mathbf{P} = \mathbf{P}^{B,j}$. For the related profile $\mathbf{P}' \in \mathcal{P}_{B,j}$ where, e.g., a_{r_1} and a_{r_2} are permuted, i 's success evaluates to

$$\begin{aligned} \hat{\sigma}(B, P'_i) &= \beta_B(a_{r_1})\sigma(a_{r_1}, P'_i) + \beta_B(a_{r_2})\sigma(a_{r_2}, P'_i) + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P'_i) \\ &= \beta_B(a_{r_1})\sigma(a_{r_2}, P_i^{B,j}) + \beta_B(a_{r_2})\sigma(a_{r_1}, P_i^{B,j}) + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P_i^{B,j}) \end{aligned} \quad (14)$$

and, more generally, we have $\hat{\sigma}(B, P'_i) = \sum_{a \in B} \beta_B(a)\sigma(\rho^{-1}(a), P_i^{B,j})$ if $\mathbf{P}' = \rho(\mathbf{P}^{B,j})$.

This implies

$$\begin{aligned} \sum_{\mathbf{P} \in \mathcal{P}_{B,j}} \Pr(\mathbf{P}) \hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) &= \sum_{\rho \in \mathcal{S}^B} \Pr(\rho(\mathbf{P}^{B,j})) \sum_{a \in B} \beta_B(a)\sigma(\rho^{-1}(a), P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \beta_B(a) \sum_{\rho \in \mathcal{S}^B} \sigma(\rho^{-1}(a), P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \beta_B(a) \sum_{\tilde{a} \in B} \frac{|B|!}{|B|} \sigma(\tilde{a}, P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \frac{|B|!}{|B|} \sigma(a, P_i^{B,j}). \end{aligned} \quad (15)$$

The second equality exploits that $\Pr(\mathbf{P}) = \Pr(\rho(\mathbf{P}))$ for any permutation $\rho: A \rightarrow A$ and changes the order of summation. The third equality uses that, as we go over all permutations of B 's elements supposing a given a prevails in the tie break, a is ranked in the location that $\tilde{a} = \rho^{-1}(a)$ has in reference ranking $P_i^{B,j}$ exactly $|B|!/|B|$ times for each $\tilde{a} \in B$. The final equality just replaces the expectation of a constant by this constant and renames $\tilde{a}$ to a .

We therefore have

$$\begin{aligned}
\mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)] &= \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) \\
&= \sum_{B \subseteq A} \sum_{j=1}^{k(B)} \sum_{\mathbf{P} \in \mathcal{P}_{B,j}} \Pr(\mathbf{P}) \hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) \\
&= \sum_{B \subseteq A} \sum_{j=1}^{k(B)} \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \frac{|B|!}{|B|} \sigma(a, P_i^{B,j})
\end{aligned} \tag{16}$$

independently of the specific family $\{\beta_B\}$ of tie break probabilities. A priori success under tie breaking method $\{\beta_B\}$ hence equals a priori success under lexicographic tie breaking $\{\beta_B^{\text{lex.}}\}$, which is $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$. $\blacksquare$

Appendix B: Robustness to Strategic Voting

The main analysis has assumed voters to express their preferences without strategic misrepresentation. From a theoretical point of view, this is restrictive: unless some voter is a dictator, the preference profiles for which sincere voting is a Nash equilibrium are a strict subset of preference domain $\mathcal{P}(A)^n$ (cf. Gibbard 1973 and Satterthwaite 1975).³¹

Strategic voting is arguably less problematic in practice than in theory. It requires information about other voters' preferences that is often unavailable or difficult to obtain. Manipulation attempts can have negative reputation effects; they may fail or even backfire. It can also be computationally expensive for a voter to evaluate which outcomes are achievable through which preference misrepresentation. This holds already if everyone else votes sincerely³² and further complexity is added if, potentially, other voters misrepresent their preferences too. The subjects in experiments by Van der Straeten et al. (2010) voted strategically only if the required computations were elementary. Other authors made similar observations (see, e.g., Kube and Puppe 2009, Pons and Tricaud 2018, Abeler, Nosenzo and Raymond 2019 or Baujard and Lebon 2022). Even from a theory perspective it is not clear if the assumptions for a particular strategic voting equilibrium are less restrictive than for sincere voting: players must be aware of the possibility to manipulate; their costs of exercising this option must be small; and they must somehow come to correctly anticipate their adversaries' strategies even when there are many alternative equilibria.

It is nonetheless worthwhile to assess the robustness of our success comparisons with respect to strategic voting. The key difficulty in doing this is non-uniqueness of equilibrium. If, for instance, our stylized shareholders with weights $\mathbf{w} = (45\%, 35\%, 20\%)$ choose between candidates $A = \{a, b, c\}$ and have sincere preferences $\mathbf{P} = (acb, bca, cba)$, there exist 40 Nash equilibria in pure strategies under r^P and r^{PR} , 39 under r^C and 14 under r^B .³³ If we eliminate weakly dominated strategies

³¹For instance, in Table 3's example with $\mathbf{w} = (45\%, 35\%, 20\%)$, at least one voter has an incentive to misreport their preference for 36 (24, 24, 72) out of the 216 preference profiles for r^P (r^{PR} , r^C , r^B).

³²Weighted votes using r^B and r^{PR} are NP-hard to manipulate for three or more alternatives, r^C for at least four alternatives. The manipulation problem has polynomial complexity for any number of alternatives only for r^P . See the survey by Conitzer and Walsh (2016).

³³The numbers refer to a normal-form game with strategy sets $S_i = \{abc, \dots, cba\}$ in which each player $i \in N$ has complete information about r and $\mathbf{P}$. For instance, strategy $s_1 = cab$ by voter 1 in a game where $P_1 = acb$ means that 1 acts like a sincere voter with preferences cab : under r^{PR} , 1 first votes for c and then for c (a) if there is a runoff (not) involving option c . This would, e.g., be better for

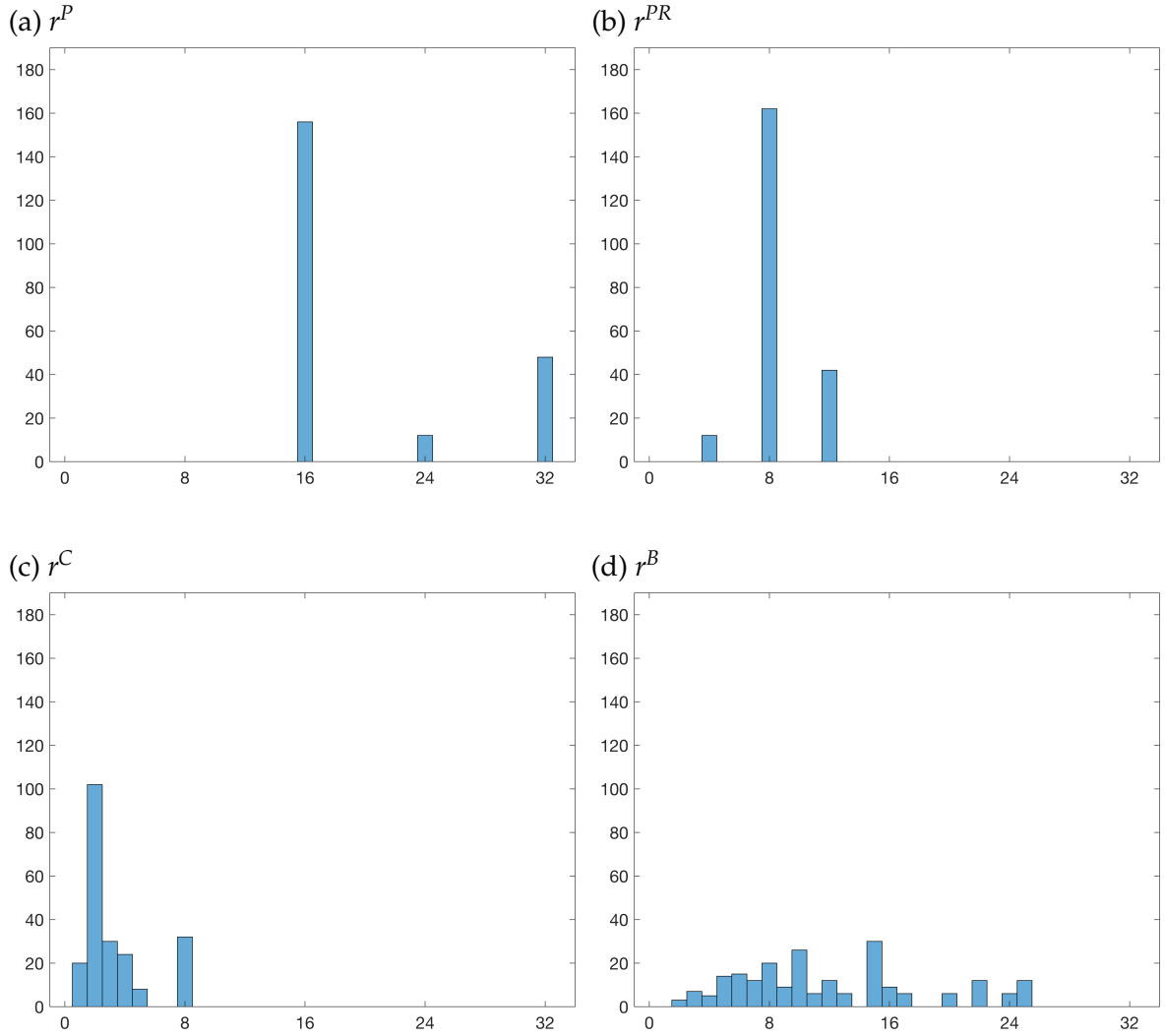


Figure A-1: Distribution of the number of Nash equilibria in undominated pure strategies for $\mathbf{w} = (45\%, 35\%, 20\%)$, $m = 3$ and $\mathbf{P} \in \mathcal{P}(A)^n$

(see Farquharson 1969), there are still 16, 8, 3 and 6 equilibria to choose from. Figure A-1 reports the distribution of the numbers of corresponding equilibria across the $(3!)^3 = 216$ preference profiles $\mathbf{P}$. For most $\mathbf{P}$, there are many Nash equilibria to select from as a theorist – and to coordinate between as actual voters. Success under strategic voting is highly contingent on everyone forming correct beliefs about the behavior of everybody else.

This holds for weight distributions other than $\mathbf{w} = (45\%, 35\%, 20\%)$ too. We have computed the sets of pure-strategy Nash equilibria for all profiles $\mathbf{P}$ for $n = 3$

1 than the sincere strategy $s_1^o = acb$ if 2 and 3 play $s_2 = bca$ and $s_3 = cba$. We disregard mixed-strategy equilibria because they have weak foundations and require an extension of $\mathbf{P}$ to lotteries.

players, $m = 3$ options and all non-dictatorial weight equivalence classes of r^P , r^{PR} , r^C and r^B . We checked in each case if (i) sincere voting is a Nash equilibrium or if (ii) the winning alternative is the same as under sincere voting in some equilibrium involving undominated strategies. The results are summarized at the end of this Appendix in Tables A-1 to A-4.³⁴ The share of profiles $\mathbf{P}$ in which the sincere voting outcome $r|\mathbf{w}(\mathbf{P})$, and hence success $\sigma(r|\mathbf{w}(\mathbf{P}), P_i)$, are in sense (i) or (ii) consistent with strategic voters often exceeds 90%. It falls below 80% only for what essentially are 2-player tie-breaking games. Hence, the conclusions from the analysis of sincere voting are unlikely to be far off.

This can be made more precise by, for instance, adopting a cost of lying-based selection criterion and then evaluating individual success in the corresponding equilibrium outcomes. Specifically, among all strategic voting equilibria, let us select those that require the lowest number of pairwise preference misrepresentations, i.e., we identify the equilibrium strategy profile $\mathbf{s}^* \in \mathcal{P}(A)^n$ with minimal Kemeny distance to the sincere profile $\mathbf{s}^\circ = \mathbf{P}$. Then we compute player i 's success $\sigma(r|\mathbf{w}(\mathbf{s}^*), P_i)$ at $\mathbf{P}$ and obtain the respective a priori success $TCI_i^*(\cdot)$ and $ARI_i^*(\cdot)$ under strategic voting by taking expectations with respect to $\mathbf{P}$.³⁵

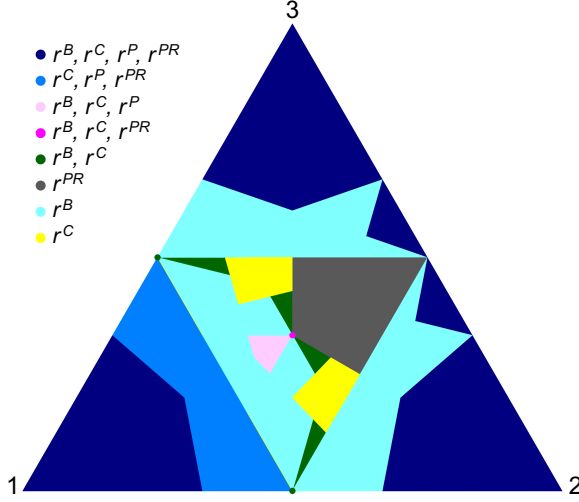
Figure A-2 shows the corresponding maximizers of $TCI_1^*(\cdot)$ and $ARI_1^*(\cdot)$ for strategic voters under the IC and IAC benchmark distributions for $n = m = 3$ in direct analogy to the results for sincere voters in Figure 6. There are visible differences. However, in most cases, at least one of the previously success-maximizing rules is still a success maximizer. The results for strategic voting thus help to select between rules that promise equal a priori success under sincere voting. Exceptions to this are (i) a few areas where r^C is the maximizer under sincere voting but now r^{PR} maximizes player 1's success, or vice versa, and (ii) some cases where r^P is the maximizer under sincere voting but r^B under strategic voting if $w_1 \in (\max\{w_2, w_3\}, 50\%)$. It is possible that, e.g., $TCI_1^*(\cdot) = ARI_1^*(\cdot) = 1$ for *all* considered rules although $w_1 \in (50\%, 66.67\%)$; cf. the extension of the dark blue area in the bottom left corners of the simplex.

As a complementing robustness check, we considered voting under uncertainty (see, e.g., Majumdar and Sen 2004) and verified if a sincere vote would maximize the expected utility of a voter with $u_i(a) = \tilde{s}(a, P_i)$ who – lacking better information –

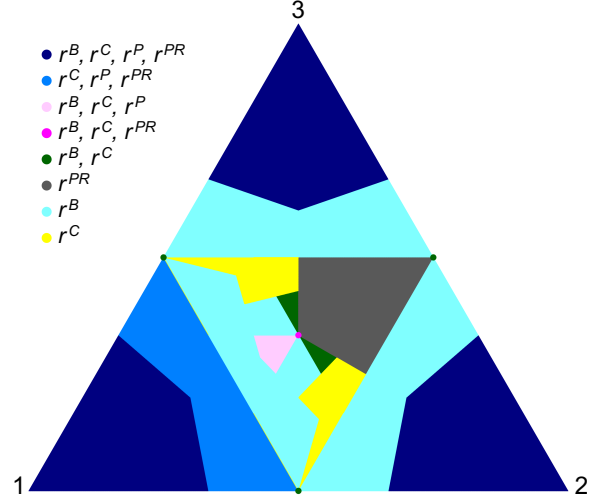
³⁴We indicate each equivalence class by an integer weight distribution with minimum sum. Our example $\mathbf{w} = (45\%, 35\%, 20\%)$ is equivalent to $\mathbf{w}' = [3, 2, 2]$ under r^P , to $\mathbf{w}' = [2, 2, 1]$ under r^{PR} , to $\mathbf{w}' = [1, 1, 1]$ under r^C and to $\mathbf{w}' = [5, 4, 2]$ under r^B (cf. Kurz et al. 2020).

³⁵If several equilibrium strategy profiles $\mathbf{s}^*$ minimize the Kemeny distance to $\mathbf{P}$, we pick at random and evaluate the expectation of $\sigma(r|\mathbf{w}(\mathbf{s}^*), P_i)$. For combinations of $r|\mathbf{w}$ and $\mathbf{P}$ where no pure strategy equilibrium exists, we take every player's success to be the same as under sincere voting.

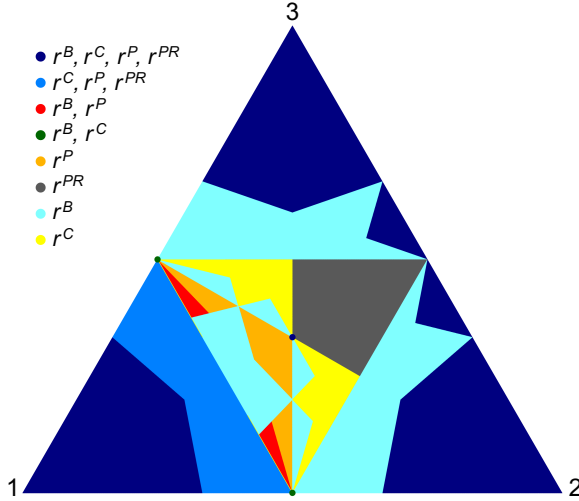
(a) ARI^* and IC



(b) ARI^* and IAC



(c) TCI^* and IC



(d) TCI^* and IAC

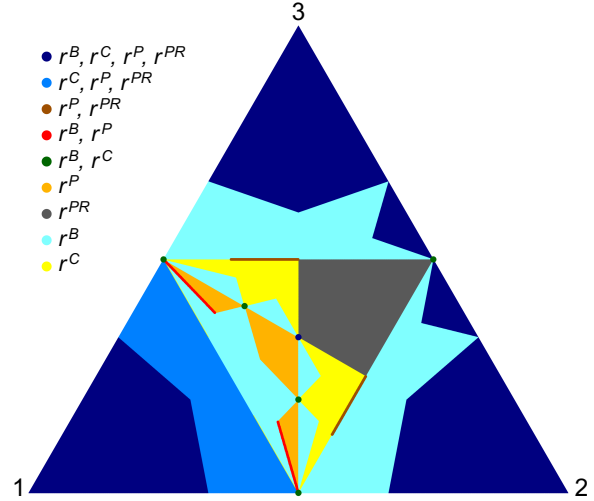


Figure A-2: Maximizers of ARI_1^* and TCI_1^* in strategic voting equilibria with minimal Kemeny distance to $\mathbf{P}$ when $n = m = 3$

assumes that preferences of the others are independent, distributed uniformly and expressed truthfully.³⁶ We checked for each voter i with any fixed preferences P_i if, writing $\mathbf{P}_{-i} = (P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_n)$,

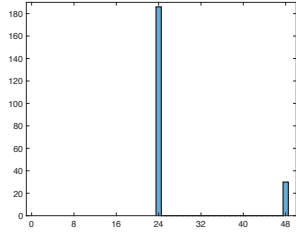
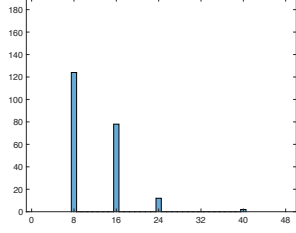
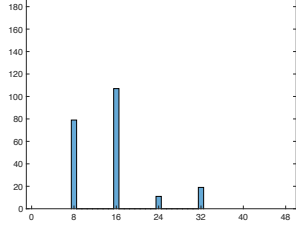
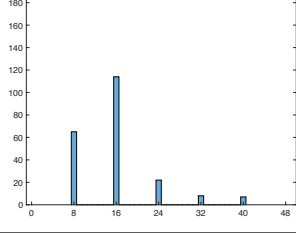
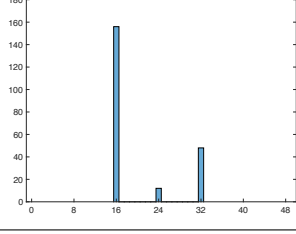
$$\sum_{\mathbf{P}_{-i} \in \mathcal{P}(A)^{n-1}} \Pr(\mathbf{P}_{-i}) \cdot \tilde{s}(r|\mathbf{w}(P_i, \mathbf{P}_{-i}), P_i) \geq \sum_{\mathbf{P}_{-i} \in \mathcal{P}(A)^{n-1}} \Pr(\mathbf{P}_{-i}) \cdot \tilde{s}(r|\mathbf{w}(P'_i, \mathbf{P}_{-i}), P_i) \quad \forall P'_i \neq P_i. \quad (17)$$

This inequality happens to hold for all voters i and preferences P_i in all plurality and plurality runoff equivalence classes when $n = m = 3$. It is also satisfied for 42 out of the 51 Borda equivalence classes and for 3 out of the 4 Copeland classes, where the remaining 9 and 1 represent non-generic distributions (i.e., locally isolated points or lines in the simplex).³⁷ In other words: sincere voting is typically the best strategy for a voter who applies the principle of insufficient reason to the unknown actions of other players. If we take a virtual walk through the simplex in Figure 1 and enumerate all $(1500 + 2)! / (1500! \cdot 2!) = 1127251$ games with non-negative integer voting weights $\mathbf{w} = (w_1, w_2, w_3)$ such that $w_1 + w_2 + w_3 = 1500$, the proportion of games where sincere voting is optimal in this sense – i.e., strategy $s_i = P_i$ satisfies inequality (17) for all i and P_i – evaluates to either $\approx 99\%$ or 100% under r^P , r^{PR} , r^C and r^B for both $m = 3$ or 4 alternatives.

³⁶This is a special case of the model by Majumdar and Sen (2004). Knightian preference uncertainty was earlier considered by Moulin (1981).

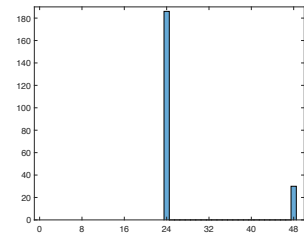
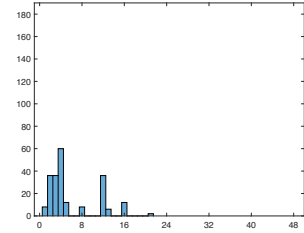
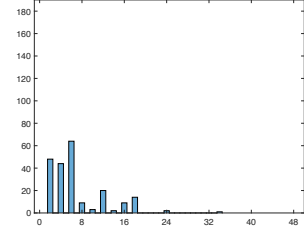
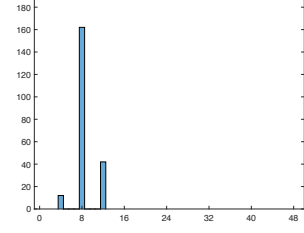
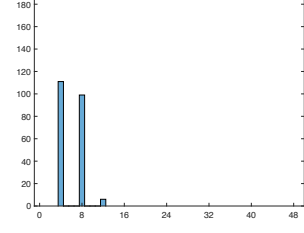
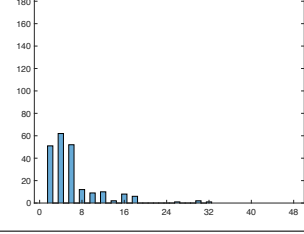
³⁷For $m = 4$ options, deviating from truthful voting does not pay in 3 of the 6 plurality classes, 4 of the 7 plurality runoff classes, 3 of the 4 Copeland classes and 354 of the 505 Borda classes.

Table A-1: Number and properties of undominated Nash equilibria under plurality rule

$r^P w$	Distribution of # NE for $\mathbf{P} \in \mathcal{P}(A)^n$	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		210	6	0	$192/216$ ≈ 0.89	$192/216$ ≈ 0.89
[1,1,1]		178	38	0	$180/216$ ≈ 0.83	$180/216$ ≈ 0.83
[2,1,1]		197	19	0	$192/216$ ≈ 0.89	$196/216$ ≈ 0.91
[2,2,1]		148	68	0	$180/216$ ≈ 0.83	$188/216$ ≈ 0.87
[3,2,2]		132	84	0	$180/216$ ≈ 0.83	$204/216$ ≈ 0.94

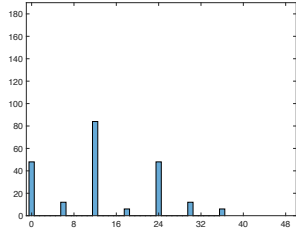
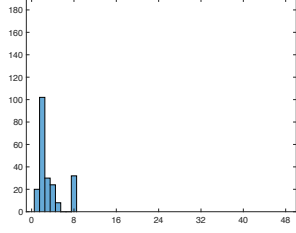
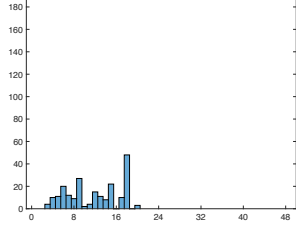
Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all five non-dictatorial r^P equivalence classes for $n = m = 3$ and all $\mathbf{P} \in \mathcal{P}(A)^3$.

Table A-2: Number and properties of undom. Nash equilibria under plurality runoff rule

$r^{PR} \mathbf{w}$	Distribution of # NE for $\mathbf{P} \in \mathcal{P}(A)^n$	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		210	6	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[1,1,1]		196	20	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[2,1,1]		194	22	0	198/216 ≈ 0.92	202/216 ≈ 0.94
[2,2,1]		186	30	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[3,2,1]		208	8	0	196/216 ≈ 0.91	196/216 ≈ 0.91
[3,2,2]		185	31	0	192/216 ≈ 0.89	196/216 ≈ 0.91

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all six non-dictatorial r^{PR} equivalence classes for $n = m = 3$ and all $\mathbf{P} \in \mathcal{P}(A)^3$.

Table A-3: Number and properties of undominated Nash equilibria under Copeland rule

$r^C w$	Distribution of # NE for $P \in \mathcal{P}(A)^n$	# different NE outcomes for $P \in \mathcal{P}(A)^n$			Share of P s.t. sincere voting is a NE	Share of P s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		168	0	0	132/216 ≈ 0.61	156/216 ≈ 0.72
[1,1,1]		178	38	0	192/216 ≈ 0.89	204/216 ≈ 0.94
[2,1,1]		149	67	0	167/216 ≈ 0.77	210/216 ≈ 0.97

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all three non-dictatorial r^C equivalence classes for $n = m = 3$ and all possible preference configurations $P \in \mathcal{P}(A)^3$. The outcome distribution does not sum up to 216 for $w = [1, 1, 0]$ because there is no pure NE for 48 profiles of sincere preferences.

Table A-4: Number and properties of undom. Nash equilibria under Borda rule

$r^B w$	Average # NE	# different NE outcomes for $P \in \mathcal{P}(A)^n$			Share of P s.t. sincere voting is a NE	Share of P s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]	13.50	168	0	0	132/216 $\approx$ 0.61	156/216 $\approx$ 0.72
[1,1,1]	11.96	69	141	6	165/216 $\approx$ 0.76	216/216 = 1
[2,1,0]	19.83	192	0	0	162/216 $\approx$ 0.75	186/216 $\approx$ 0.86
[2,1,1]	7.75	171	45	0	176/216 $\approx$ 0.81	208/216 $\approx$ 0.96
[2,2,1]	8.60	131	71	6	144/216 $\approx$ 0.67	198/216 $\approx$ 0.92
[3,1,1]	13.72	204	12	0	178/216 $\approx$ 0.82	208/216 $\approx$ 0.96
[3,2,0]	18.00	144	0	0	108/216 $\approx$ 0.50	144/216 $\approx$ 0.67
[3,2,1]	9.09	169	47	0	161/216 $\approx$ 0.75	208/216 $\approx$ 0.96
[4,1,1]	9.22	216	0	0	201/216 $\approx$ 0.93	213/216 $\approx$ 0.99
[3,2,2]	9.96	139	75	2	169/216 $\approx$ 0.78	204/216 $\approx$ 0.94
[3,3,1]	4.06	156	12	0	132/216 $\approx$ 0.61	168/216 $\approx$ 0.78
[4,2,1]	8.54	194	18	0	168/216 $\approx$ 0.78	203/216 $\approx$ 0.94
[3,3,2]	12.53	66	138	12	156/216 $\approx$ 0.72	216/216 = 1
[4,3,1]	10.06	179	35	0	138/216 $\approx$ 0.64	196/216 $\approx$ 0.91
[5,2,1]	9.72	212	0	0	174/216 $\approx$ 0.81	202/216 $\approx$ 0.94
[4,3,2]	7.38	147	65	0	155/216 $\approx$ 0.72	201/216 $\approx$ 0.93
[5,2,2]	12.06	180	36	0	174/216 $\approx$ 0.81	216/216 = 1
[5,3,1]	11.50	200	16	0	150/216 $\approx$ 0.69	198/216 $\approx$ 0.92
[4,3,3]	13.56	66	138	12	174/216 $\approx$ 0.81	216/216 = 1
[5,4,1]	7.41	174	22	0	130/216 $\approx$ 0.60	180/216 $\approx$ 0.83
[6,3,1]	4.50	192	0	0	162/216 $\approx$ 0.75	186/216 $\approx$ 0.86
[5,3,3]	6.56	132	84	0	168/216 $\approx$ 0.78	210/216 $\approx$ 0.97
[5,4,2]	12.06	126	90	0	144/216 $\approx$ 0.67	207/216 $\approx$ 0.96
[6,4,1]	7.56	186	10	0	123/216 $\approx$ 0.57	176/216 $\approx$ 0.81
[7,2,2]	15.50	216	0	0	186/216 $\approx$ 0.86	210/216 $\approx$ 0.97
[5,4,3]	12.24	100	110	6	159/216 $\approx$ 0.74	210/216 $\approx$ 0.97
[7,4,1]	7.78	196	0	0	144/216 $\approx$ 0.67	182/216 $\approx$ 0.84
[6,5,2]	10.07	156	51	0	138/216 $\approx$ 0.64	195/216 $\approx$ 0.90
[7,5,1]	6.82	163	9	0	114/216 $\approx$ 0.53	160/216 $\approx$ 0.74
[6,5,3]	12.06	126	90	0	144/216 $\approx$ 0.67	204/216 $\approx$ 0.94
[7,5,2]	10.60	170	46	0	145/216 $\approx$ 0.67	203/216 $\approx$ 0.94
[8,5,1]	7.75	172	0	0	117/216 $\approx$ 0.54	161/216 $\approx$ 0.75
[6,5,4]	12.58	76	130	10	165/216 $\approx$ 0.76	214/216 $\approx$ 0.99
[7,5,3]	9.21	132	84	0	156/216 $\approx$ 0.72	208/216 $\approx$ 0.96
[7,6,2]	6.00	163	17	0	132/216 $\approx$ 0.61	174/216 $\approx$ 0.81
[8,5,2]	10.94	186	30	0	156/216 $\approx$ 0.72	207/216 $\approx$ 0.96
[7,5,4]	6.08	162	42	0	165/216 $\approx$ 0.76	197/216 $\approx$ 0.91
[7,6,4]	12.29	93	116	7	150/216 $\approx$ 0.69	212/216 $\approx$ 0.98
[8,6,3]	12.06	126	90	0	144/216 $\approx$ 0.67	210/216 $\approx$ 0.97
[9,6,2]	9.83	192	24	0	138/216 $\approx$ 0.64	198/216 $\approx$ 0.92
[8,7,3]	8.18	151	48	0	138/216 $\approx$ 0.64	189/216 $\approx$ 0.88
[8,6,5]	9.35	133	74	5	168/216 $\approx$ 0.78	204/216 $\approx$ 0.94
[10,7,2]	9.13	187	21	0	129/216 $\approx$ 0.60	188/216 $\approx$ 0.87
[11,7,2]	10.17	196	12	0	132/216 $\approx$ 0.61	187/216 $\approx$ 0.87
[9,7,5]	8.10	149	55	0	153/216 $\approx$ 0.71	193/216 $\approx$ 0.89
[10,8,3]	10.06	158	54	0	138/216 $\approx$ 0.64	198/216 $\approx$ 0.92
[11,8,2]	8.42	174	18	0	120/216 $\approx$ 0.56	174/216 $\approx$ 0.81
[11,9,3]	7.94	168	24	0	132/216 $\approx$ 0.61	180/216 $\approx$ 0.83
[13,8,2]	10.50	192	0	0	126/216 $\approx$ 0.58	174/216 $\approx$ 0.81
[12,9,7]	4.83	180	12	0	162/216 $\approx$ 0.75	186/216 $\approx$ 0.86

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all 50 non-dictatorial r^B equivalence classes for $n = m = 3$ and all $P \in \mathcal{P}(A)^3$. If the outcome distribution does not sum up to 216, there are some P for which no pure NE exists.

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