Voting Rules and Voting Success in Weighted Committees and Shareholder Meetings

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Abstract

Collective decisions on more than two alternatives can vary widely in the adopted voting rule. This affects how closely collective choices reflect the preferences of a given individual. We ask if a specific voter is better off using plurality voting, plurality with a runoff vote, pairwise majority voting or the Borda scoring method. A partial answer is that if all voting weights are equal, then plurality rule maximizes the probability of obtaining one's individual top choice and Borda rule maximizes the voter-specific average ranking of the outcome. This result generalizes to asymmetric weights in aggregate terms but not from a single voter's point of view. We identify the individually most advantageous rule for any given weight distribution among three generic voters and also for the ten largest shareholders in S&P 100 corporations. Recommendations for the latter coincide in unexpectedly many cases with the analytical benchmark for equal weights. So although heterogeneity calls for caution in theory, the respective governance interests of investors with unequal holdings align well in practice. We also find that the Borda rule translates voting weights into voting success the most transparently and that traditional power indices for binary voting games approximate success in weighted committees well.

Keywords: collective decisions · shareholder voting · weighted committee games · voting procedures · voting power · voting success

JEL codes: D71 · C71 · C63 · G30

1 Introduction

Many important decisions are taken by vote not just in politics but also in business. This includes elections of directors or chief executives, resolutions on important acquisitions, the selection of an auditor or facility location, etc. When such decisions involve more than two options, the adopted voting rule can make a big difference. This paper therefore addresses the following research question: Which rule reflects the preferences of a given individual the best?

Even seasoned committee chairs can be surprised by how widely collective choices vary in the adopted voting method. For illustration, consider three shareholders who hold 45%, 35% and 20% of corporate votes, respectively.[1] These might be exercised directly in an annual meeting or indirectly by controlling 5, 4 and 2 positions on the board of directors. Let the shareholders hold different views of five CEO candidates, labeled a, b, c, d and e. For instance, the first (largest) shareholder ranks the candidates $a \succ d \succ e \succ c \succ b$ in strictly decreasing order. The second shareholder's preferences are $b \succ c \succ d \succ e \succ a$ and the third one's $c \succ e \succ d \succ b \succ a$.

A simple method to resolve their disagreement is a plurality vote: everybody indicates their favorite candidate, and the one with the most votes wins. Then a beats its competitors by 45% : 35% : 20% : 0% : 0% in the shareholder meeting or 5 : 4 : 2 : 0 : 0 in a board vote, assuming that preferences are expressed sincerely without strategic misrepresentation. However, a is ranked last by two of the shareholders. Furthermore, a plurality that is not a majority is legally insufficient in many settings (cf., e.g., §216(2) of Delaware General Corporation Law; or §44 of Robert's Rules of Order). So some board member may propose a runoff vote between the plurality leaders a and b. If the suggestion is taken up, b wins by 6 : 5 (or, analogously, 55% : 45% in a shareholder meeting). Pairwise comparisons might also be extended beyond a and b. In a round-robin tournament between all candidates, c would beat b by 7 : 4 and also win against a, d and e. This would make c the new CEO. The directors could alternatively translate their preference rankings into scores for the candidates – ascribing, say, 0 points to their lowest-ranked candidate, 1 to their respective second-lowest-ranked candidate and so forth – and hire the top scorer. This method is commonly associated with the French scientist Jean-Charles de Borda (1733–1799) and, in our example situation, candidate d would obtain a total 'Borda

[1] This is an artificial example, but identical outcomes would result for some real share distributions – e.g., the Eurofighter Fighter Aircraft GmbH (Airbus 46%; BAE Systems 33%; Leonardo 21%) or, with suitable tie breaking, the early Apple Inc. (S. Jobs 45%; S. Wozniak 45%; R. Wayne 10%).

score' of $5 \cdot 3 + 4 \cdot 2 + 2 \cdot 2 = 27$ from the board. This exceeds scores of 20, 18, 25 and 20 for a, b, c and e – so d wins. Or directors could each approve as many candidates as they like and pick the one with the highest approval. The first shareholder might then approve a, d and e from top down; the second only b and c; the third c and e. This yields a top $5 + 2 = 7$ approval for e versus only 5, 4, 6 and 5 for a, b, c and d.

We can see that *every* candidate is a winner: it all depends on the voting rule. Having a say on the voting rule can hence be very valuable for a self-interested voter. However, neither a regular member nor the chairperson of a board or committee can pick a voting rule as he or she pleases, since methods for taking collective decisions are typically fixed a priori. They are either determined in laws, charters, by-laws or statutes; or there are institutional defaults with deviations requiring a justification.[2]

It is important, therefore, to study the implications of adopting one voting rule rather than another from an a priori perspective: Can we identify any general (dis)advantage of using, say, plurality rule vs. pairwise voting when an individual's objective is to elect his or her personal favorite with maximal likelihood? What if he or she wants to induce choices with a high subjective rank on average? Answering these questions can help identify preferable voting rules not just for corporate boards or shareholder meetings but also political committees, party conventions, electoral colleges, etc.

We consider a collective decision-making body, generically referred to as a 'committee', and evaluate the individual a priori success of all members when either plurality voting, plurality with a runoff vote, pairwise majority voting or the Borda scoring method is invoked to aggregate their preferences.[3] We are especially interested in cases where the relevant players wield asymmetric voting weights because these arise in many real-world settings: unequal weights can not only reflect unequal shareholdings but also votes controlled by parties, coalitions or alliances in parliaments or associations, regional assemblies, polarized electorates, etc.

We build our analysis on the framework of *weighted committee games* developed by Kurz, Mayer and Napel (2020) and define two measures of a priori voting success: the *top choice index* represents the probability that one's most preferred option is se-

[2] Implicit forms of voting are common and have defaults, too: leaders may tacitly adopt the majority view in their team to secure their position (cf. Leeson 2007); scores that independent interviewers assign to applicants are totaled similar to Borda rule; or competing engineering proposals are dropped successively according to which one is favored by the fewest project members.

[3] The rules are prototypical instances of a *Condorcet method*, a *runoff rule* and *scoring rules* (e.g., Felsenthal and Nurmi 2018). Approval voting needs a different model of preferences and is left aside. We focus on sincere voters but also report success maximizers for strategic players (see Appendix B).

lected; the *average rank index* captures the expected position of the selected option in one's personal ranking. We complement either index by probabilistic assumptions on preferences. There, we focus on distributions that are familiar from traditional measures of voting power for binary 'yes'-or-'no' decisions, namely the independence assumption of the Penrose-Banzhaf index (Penrose 1946; Banzhaf 1965) versus positive preference correlation as captured by the Shapley-Shubik index (Shapley and Shubik 1954).[4] The success indices can identify winners and losers of voting rule changes and quantify the respective gains or losses. They are useful for improving default governance rules and for addressing concern that – for given voting weights – a particular method (dis)favors, for example, some large shareholder or a minority group.

We study all possible distributions of voting weights among three players who decide on three or four alternatives. We show that *(i)* top choice success and average rank success can be locally very sensitive to rule or weight changes, *(ii)* the individually most advantageous rule differs across players and *(iii)* the respective success maximizer may vary non-monotonically in weight. Then we analytically derive success-maximizing rules if an arbitrary number of players have equal voting weights. The resulting recommendations extend to arbitrary weight distributions if the objective is to maximize the weighted aggregate success of all voters.

We finally explore data on actual corporate voting rights and demonstrate how success indices can support shareholders in selecting a voting rule. In particular, we study the distribution of voting shares in S&P 100 corporations. The high sensitivity of recommendations to the exact distribution of voting weights that is observed for three players suggests differently, but computations for the ten largest shareholders in the respective corporations reveal a robust pattern. Echoing the result for symmetric weights, we find that in most cases the plurality rule maximizes top choice success, while the Borda rule maximizes average rank success.

Regression analysis shows that the advantage of having a higher relative voting weight can be picked up surprisingly well for the S&P 100 data by standard indices of binary voting power, which have, e.g., featured in recent studies on common ownership (Azar, Schmalz and Tecu 2018; Backus, Conlon and Sinkinson 2021). The voting method determines additional 1–3% of success a priori. This is in the same ballpark

[4]Voting power and success differ conceptually. The former refers to the ability to influence the voting outcome, and the latter refers to the individual evaluation of outcomes. Your vote may make c, d or e the winner (influence), but all may be evaluated low compared to your favorite option a (success).

as the average success advantage of 2.2% for the seventh largest shareholder of an S&P 100 constituent compared to the respective tenth largest shareholder (holding 1.78% vs. 1.14% of a firm's stock on average).

While our analytical approach is descriptive of voting weights in real-world settings, actual voting often works differently. Controversial issues may be predetermined by a few key players without clear rules and their pre-selected favorite is proposed to a plenary or general meeting as a simple 'yes'-or-'no' motion – potentially framing even the narrow winner of earlier decision stages as consensual.[5] Our model is likely to be descriptive of the dynamics of these predeterminations. In that sense, the present investigation extends beyond settings with binding legal rules, such as elections between mayoral candidates or legislative choices in parliaments: weighted committees can be viewed as at least first approximations also of the early decision stages in which, say, a CEO candidate or acquisition strategy is selected from three or more options in unofficial straw votes before being approved at an annual meeting.[6]

Our study contributes to the literature on corporate governance and general collective choice analysis in several respects. First, we provide a systematic computational basis for the existing anecdotal evidence on how sensitive collective decisions are to the adopted voting method, given more than two options and asymmetric vote numbers. This complements insights from rule comparisons for selected examples (see, e.g., Riker 1982, Saari 2001 or Felsenthal and Nurmi 2018) and historical case studies (e.g., Leininger 1993 scrutinizes the 'fatal' voting procedure that moved the government of reunified Germany from Bonn to Berlin; Tabarrok and Spector 1999 suggest that Borda's method might have avoided the US civil war; Maskin and Sen 2016 reason that Donald Trump owes his 2016 election to the use of plurality rule in the Republican primaries). We show how the adopted voting method matters on average and for a wide range of voting weight distributions.

[5]For instance, of the more than 600 votes from 2017 to 2023 on issues that require only a qualified majority in the EU Council of Ministers, about 84% of the motions would have passed also under unanimity rule (own calculations, data retrieved from `https://www.consilium.europa.eu/en/documents-publications/public-register/votes/`). The IMF stipulates that "a shortlist of three candidates" is prepared for the position of IMF Managing Director and that choice from it is "by a majority of the votes cast" on the Executive Board (IMF Press Release 16/19). Quite magically, a single consensus candidate has always emerged before any competitors were officially shortlisted and rejected. See Mayer and Napel (2020) for further details.

[6]We point to McCahery, Sautner and Starks (2016) or Bowley, Hill and Kourabas (2023) for evidence on how shareholders exert power through behind-the-scenes interaction and Gantchev (2013) on the costs associated with different forms of shareholder influence.

Second, we construct success indices that quantify the differences between rules. These indices allow the assessment of charters, by-laws, statutes, etc. from an a priori perspective. Neither of the probabilistic preference cultures that we invoke in this assessment will match the distribution of preferences in a given board, annual meeting or parliament exactly, but they provide valuable benchmarks. One captures independent idiosyncratic preferences (impartial culture) and the other incorporates correlated attitudes that reflect a common interest (impartial anonymous culture). Success indices are similar in this respect to power indices for binary voting. We demonstrate that the latter do a good job at predicting success also for non-binary decision-making. Our investigation thus lends additional credence to many previous studies of weighted voting – in corporations, the US Electoral College, the EU Council, the IMF Board of Directors, etc. – that rely on traditional power indices (see Holler and Nurmi 2013).

Third, we bring our analytical insights to a real-world setting and find that, despite high sensitivity of success under a given voting rule to the applicable voting weights in theory, a simple rule of thumb captures empirical patterns well. We show that even with the significant asymmetries among big investors in S&P 100 corporations – involving mean holdings of >10% for the largest vs. ≈1% for the tenth-largest – the shareholders have almost identical interests concerning which (straw) voting rule should be used when, e.g., a location for a big facility, new CEO or auditor needs to be singled out from multiple options prior to an official 'yes'-or-'no' vote. It turns out to matter more whether a high average rank or getting one's favorite better reflects individual objectives than whether one holds the first or tenth-most shares: plurality rule maximizes the top choice probability and Borda rule produces the highest average outcome rank. We also find that Borda rule generally links voting weights to success the most closely. This means that it provides particularly transparent incentives for investors who want an acquisition of additional ordinary shares to translate into additional voting success, not just voting rights.

We next explain in more detail how this investigation connects to previous analysis of weighted voting and axiomatic assessments of social choice rules (Section 2). The framework of weighted committee games and our measures of a priori voting success are introduced in Sections 3 and 4. We then consider a wide range of small committees in Section 5 and actual voting share distributions in S&P 100 constituents in Section 6. We conclude in Section 7. Appendices A and B assess the robustness of our findings regarding the adopted tie breaking assumption and strategic voting.

2 Relation to the Literature on Weighted Voting

Individual-based a priori evaluations of voting systems date back to the Constitutional Convention in Philadelphia in 1787 (see Riker 1986). However, they have mostly been restricted to binary decisions. In particular, various measures of voting power have been applied to *weighted voting games* and more general *simple (voting) games* formalized by von Neumann and Morgenstern (1953, ch. 10). The Penrose-Banzhaf and Shapley-Shubik indices are the most prominent such measures (Penrose 1946; Banzhaf 1965; Shapley and Shubik 1954).[7] They map a given distribution of voting weights for 'yes'-or-'no' decisions (or sets of winning and losing coalitions) to individual decisiveness and a priori influence.

Analogous investigations of preference satisfaction and voting success have received less attention. A reason for this is that being successful is commonly seen as a corollary to holding power (cf. Barry 1980). The probability of obtaining the preferred outcome (voting success) is, in fact, an affine transformation of the probability of being decisive for the outcome (voting power) if all 'yes'-or-'no' configurations among players are equally likely. The vast literature that has studied voting rules in the US Electoral College, the UN Security Council, the Council of the European Union, national parliaments, the European Central Bank (see the contributions in Holler and Nurmi 2013) or publicly traded corporations (e.g., Leech 1987, 1988 or Azar et al. 2018) hence focuses on decisiveness.

However, voting power need not be what decision-makers care about. Laruelle, Martínez and Valenciano (2006, p. 197) remark that "practitioners have often raised objections about the power indices approach . . . [and ask] why pay so much attention to decisiveness when success seems a more important issue for the involved voters?" Moreover, the mathematical links between power and success are fragile. Their affine relation collapses already for mild interdependencies among voter preferences, such as the preference model underlying the Shapley-Shubik index (see Kirsch 2023).

The connection between power and success can even be non-monotonic if choices concern more than two options. For instance, every voting procedure designed to select the so-called Condorcet winner, i.e., the winner of a complete pairwise majority contest if such contest creates no cycle, is subject to the *no show paradox* (Moulin 1988). This implies that individuals or groups are sometimes strictly better off not casting all their eligible votes. Weighted voting analysis hence benefits from dedicated

[7] See Felsenthal and Machover (1998), Laruelle and Valenciano (2008) or Napel (2019) for overviews. Extensions to weighted committee games are studied by Kurz, Mayer and Napel (2021).

assessments of success especially for non-binary decisions.

Analysis of a priori success also complements the traditional social choice literature by adding a neglected individual-based perspective. Numerous scholars – with seminal contributions by Arrow (1951), May (1952), Sen (1970), Gibbard (1973), Young (1974), Satterthwaite (1975) or Moulin (1988) – have investigated decision rules such as plurality or pairwise majority voting from an axiomatic viewpoint that highlights desirable aggregate properties. But every voting method with normatively appealing properties also has unappealing ones. The corresponding investigations have produced detailed checklists on the (non-)fulfillment of various desirable criteria by common voting rules (cf., e.g., Felsenthal and Nurmi 2018), debate on how to prioritize them in specific contexts (Laslier 2012) and computations of the likelihood of a given rule violating a specific property (see, e.g., Gehrlein and Lepelley 2017). Voting experts may still recommend several different methods for good normative reasons, while many practitioners know: if the by-laws permitted, the optimal rule for getting what you want would be dictatorial. We therefore take the position of a self-interested decision-maker and ask: How well does this vs. another standard voting rule serve my personal goals a priori? This is a profane but natural question. To the best of our knowledge, it has been addressed only for binary decisions so far (cf. Rae 1969 and Taylor 1969).

3 Weighted Committee Games

We build on the generalization of binary weighted voting games (von Neumann and Morgenstern 1953) to multi-option weighted committee games as developed by Kurz et al. (2020). These games consider a set $N = \{1, \ldots, n\}$ of voters or players such that each voter $i \in N$ has strict preferences P_i over a finite set $A = \{a_1, \ldots, a_m\}$ of $m \geq 2$ alternatives. We may write $a \succ b \succ c$ or abc for P_i when the player's identity is clear. The set of all $m!$ strict preference orderings on A is denoted by $\mathcal{P}(A)$. A *voting rule* $r\colon \mathcal{P}(A)^n \to A$ maps each preference profile $\mathbf{P} = (P_1, \ldots, P_n)$ to a winning alternative $a^* = r(\mathbf{P})$.[8] Rule r is *anonymous* if for any $\mathbf{P} \in \mathcal{P}(A)^n$ and any permutation $\rho\colon N \to N$

[8] Aggregating heterogeneous preferences presents a challenge different from aggregating information among voters with identical preferences. On the latter see, e.g., Nitzan and Procaccia (1986), Young (1995) or Pivato (2013). Preferences for specific options can be heterogeneous among shareholders because these have distinct incentives, time preferences or attitudes towards risk and uncertainty. Investment horizons and business philosophies typically differ, e.g., between founders and venture capitalists, retail and institutional investors, or activist hedge funds and pension funds. Or consider

Table 1: Considered baseline voting rules

Rule	*Winner* $a^* \in \{a_1, \dots, a_m\}$ *at preference profile* $\mathbf{P} = (P_1, \dots, P_n)$
Borda	$r^B(\mathbf{P}) \in \arg\max_{a\in A} \sum_{i\in N} b_i(a, \mathbf{P})$
Copeland	$r^C(\mathbf{P}) \in \arg\max_{a\in A} \left\|\{a' \in A \mid a \succ_M^{\mathbf{P}} a'\}\right\|$
Plurality	$r^P(\mathbf{P}) \in \arg\max_{a\in A} \left\|\{i \in N \mid \forall a' \neq a \in A\colon aP_i a'\}\right\|$
Plurality runoff	$r^{PR}(\mathbf{P}) \begin{cases} = r^P(\mathbf{P}) \text{ if } \left\|\{i \in N \mid \forall a' \in A \setminus \{r^P(\mathbf{P})\}\colon r^P(\mathbf{P})P_i a'\}\right\| > \frac{n}{2}, \text{ else} \\ \in \arg\max\limits_{a\in\{a_{(1)},a_{(2)}\}} \left\|\{i \in N \mid \forall a' \neq a \in \{a_{(1)}, a_{(2)}\}\colon aP_i a'\}\right\| \end{cases}$

we have $r(\mathbf{P}) = r(\rho(\mathbf{P}))$ where $\rho(\mathbf{P}) := (P_{\rho(1)}, \dots, P_{\rho(n)})$. It is *neutral* if for any $\mathbf{P} \in \mathcal{P}(A)^n$ and any permutation $\rho\colon A \to A$ we have $r(\rho(\mathbf{P})) = \rho(r(\mathbf{P}))$ where, with slight abuse, $\rho(\mathbf{P})$ denotes the application of ρ to each alternative in the full preference profile.

We focus on truthful voting under one of the four anonymous rules that are summarized in Table 1, assuming lexicographic tie breaking.[9] Under *plurality rule* r^P each voter indicates his or her top-ranked alternative and the one ranked first by the most voters is chosen. This is the winner also under *plurality (with) runoff rule* r^{PR} if the obtained plurality constitutes a majority (i.e., more than 50% of votes); otherwise a runoff vote is conducted between the alternatives $a_{(1)}$ and $a_{(2)}$ that obtained the highest and second-highest plurality scores in the first stage.

Borda rule r^B has each player i assign a score of $m-1, m-2, \dots, 0$ to the alternative that he or she ranks first, second and so on. These scores $b_i(a, \mathbf{P}) := \left|\{a' \in A \mid aP_i a'\}\right|$ coincide with the number of alternatives that i positions below a. The alternative with the highest total score is selected. *Copeland rule* r^C considers pairwise majority comparisons between all alternatives. They define the majority relation $a \succ_M^{\mathbf{P}} a' \;:\Leftrightarrow \left|\{i \in N \mid aP_i a'\}\right| > \left|\{i \in N \mid a'P_i a\}\right|$ and the alternative that beats the most others according to $\succ_M^{\mathbf{P}}$ is selected. r^C is the only Condorcet-consistent method among the rules in Table 1: whenever some alternative a beats all others, then $r^C(\mathbf{P}) = a$.

A *weighted committee (game)* $(N, A, r|\mathbf{w})$ combines a set of players N, a set of alternatives A and an anonymous baseline rule r with a vector $\mathbf{w} = (w_1, \dots, w_n) \in \mathbb{N}_0^n$ of

family fighting over the succession to a patriarch CEO; a (non-)diversified investor with (no) stakes in competing firms who does (not) internalize profit spillovers; international investors subject to distinct standards for good governance; etc.

[9] Deterministic tie breaking simplifies the presentation. We show in Appendix A that all results are robust to anonymous random tie breaking. Strategic voting is addressed in Appendix B.

Table 2: Effect of the voting rule on the winning option

P_1	P_2	P_3
a	b	c
d	c	e
e	d	d
c	e	b
b	a	a

$\Rightarrow$

$r^P|\mathbf{w}(\mathbf{P}) = a$ (a has max. plurality tally of 5)
$r^{PR}|\mathbf{w}(\mathbf{P}) = b$ (b beats a in runoff vote by 6 : 5)
$r^C|\mathbf{w}(\mathbf{P}) = c$ (c wins all pairwise votes)
$r^B|\mathbf{w}(\mathbf{P}) = d$ (d has max. Borda score of 27)

Note: This table illustrates an example of how standard voting rules imply different choices for $\mathbf{P} = (P_1, P_2, P_3)$ when $\mathbf{w} = (5, 4, 2)$.

voting weights: each player i can cast w_i votes, e.g., by virtue of owning multiple voting shares or controlling as many seats on a board. Preferences P_i thus enter into the final decision w_i times. The applicable mapping from preference profiles to collective choices then is

$$r|\mathbf{w}(\mathbf{P}) := r([P_1]^{w_1}, [P_2]^{w_2}, \ldots, [P_n]^{w_n}) = r(\underbrace{P_1, \ldots, P_1}_{w_1 \text{ times}}, \underbrace{P_2, \ldots, P_2}_{w_2 \text{ times}}, \ldots, \underbrace{P_n, \ldots, P_n}_{w_n \text{ times}}) \tag{1}$$

for all $\mathbf{P} \in \mathcal{P}(A)^n$. The mapping is homogeneous of degree zero in $\mathbf{w}$ and so we may equivalently consider relative voting weights $\mathbf{w}/\sum w_i$.

Two committees $(N, A, r|\mathbf{w})$ and $(N, A, r'|\mathbf{w}')$ are called *equivalent* if they produce the same outcomes no matter which preferences $\mathbf{P}$ are considered, despite $r \neq r'$ or $\mathbf{w} \neq \mathbf{w}'$ (cf. Kurz et al. 2020). For example, both $r^P|(3, 1, 1)$ and $r^C|(5, 2, 1)$ select player 1's top choice for every $\mathbf{P}$, making player 1 a dictator. By contrast, Section 1's shareholder example – summarized again in Table 2 – proves that committees which use r^P, r^{PR}, r^C and r^B are non-equivalent if $\mathbf{w} = (5, 4, 2)$ and $m = 5$. We are interested in committees that are non-equivalent and compare a player's success in such committees for all conceivable preference configurations $\mathbf{P} \in \mathcal{P}(A)^n$ from an a priori perspective.

4 Measuring A Priori Success

The a priori assessment of player i's success in a given committee is contingent on how the collective decisions $r|\mathbf{w}(\mathbf{P})$ are evaluated relative to i's individual preference P_i and on the applicable distribution of preferences. We will make several complementing assumptions that we consider informative. They do not come with a claim to be the 'right' or universally recommended ones.

We define two indices: The *top choice (probability) index (TCI)* takes a player's success to mean having his or her most preferred option become the collective choice. The *average rank index (ARI)* counts every outcome that is better than the player's bottom-ranked alternative as a partial success at least. The latter index can also be interpreted as reflecting a risk-neutral utility function over the available options. One can conduct more general expected utility assessments by combining both indices in the case of $m = 3$ alternatives.

4.1 Top Choice Index and Average Rank Index

Much of social choice theory focuses on ordinal preferences alone but we here want to condense a player's prospect of taking collective choices from A according to $r|\mathbf{w}$ into an interpersonally comparable number. Let us therefore consider a player-independent success function $\sigma\colon A \times \mathcal{P}(A) \to \mathbb{R}$ such that $\sigma(a, P_i) > \sigma(a', P_i) \Rightarrow aP_ia'$, and $\sigma(a^*, P_i) = 1$ (0) if a given committee choice a^* is i's most (least) preferred option. Our first benchmark is $\sigma \equiv s$ with

$$s(a^*, P_i) = \begin{cases} 1 & \text{if } a^* \text{ is ranked top in } P_i, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

This equates success to getting one's top choice.

A complementing, more gradual evaluation is achieved by s's linear interpolation

$$\tilde{s}(a^*, P_i) = \frac{\left|\{a' \in A : a^*P_ia'\}\right|}{m-1}. \tag{3}$$

This also attributes success to outcomes between the best and worst case and evaluates i's median-ranked alternative as exactly half a success for player i. Function $\tilde{s}$ or variations based, e.g., on a hyperbolic interpolation of s are compatible with interpreting (a priori) success as an (expected) utility. By contrast, piecewise constancy of s clashes with P_i being a strict ordering when $m > 2$.

Given some player-independent success function σ and probability measure Pr on $\mathcal{P}(A)^n$, we refer to the expected value of $\sigma(r|\mathbf{w}(\mathbf{P}), P_i)$ as player i's *a priori success* in committee $(N, A, r|\mathbf{w})$. In particular, we define the *top choice index*

$$TCI_i(N, A, r|\mathbf{w}) := \mathbb{E}[s(r|\mathbf{w}(\mathbf{P}), P_i)] = \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot s(r|\mathbf{w}(\mathbf{P}), P_i), \tag{4}$$

and, analogously, the *average rank index* $ARI_i(N, A, r|\mathbf{w}) := \mathbb{E}[\tilde{s}(r|\mathbf{w}(\mathbf{P}), P_i)]$ as the two success indicators of our interest. TCI_i equals the probability that collective decisions match player i's top-ranked alternative. ARI_i is an inverse measure of the average rank of decisions according to i's preferences: a value of $x \in [0, 1]$ means that collective decisions on average correspond to the $\big(m-(m-1)\cdot x\big)$-th best alternative from player i's perspective. $TCI_i = ARI_i = 1$ if and only if i is a dictator and $TCI_i \equiv ARI_i$ when $m = 2$.

In case of three alternatives, which many of our later computations focus on, working with TCI_i and ARI_i is without loss of generality. This is because the probability of the collective choice matching i's second-ranked alternative evaluates to $2\cdot(ARI_i - TCI_i)$ and that for i's bottom rank is $1 + TCI_i - 2\cdot ARI_i$. These probabilities and TCI_i suffice to compute a priori success $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$ for any rank-based success function[10] and one can also determine $\mathbb{E}[u_i(r|\mathbf{w}(\mathbf{P}))]$ for any cardinal utility function $u_i \colon A \to \mathbb{R}$ that represents i's preferences over lotteries on $A = \{a_1, a_2, a_3\}$.

Concerning the probability distribution over preference profiles $\mathbf{P} \in \mathcal{P}(A)^n$ that defines expectations, a popular default is the *impartial culture (IC)* assumption: all players' preferences $P_1, \ldots, P_n \in \mathcal{P}(A)$ are taken to be independent and drawn at random. Then

$$\Pr(\mathbf{P}) = (m!)^{-n}. \tag{5}$$

The IC distribution is underlying the Penrose-Banzhaf voting power index and has served as the starting point for many computations in the analysis of voting. See Klahr (1966), Fishburn (1971), Merrill (1984) or Nurmi and Uusi-Heikkilä (1985) for pioneering assessments of voting paradoxes, and Gehrlein and Lepelley (2017) for many more recent findings.

The most prominent alternative to IC is the *impartial anonymous culture (IAC)*, which is underlying the Shapley-Shubik voting power index. The IAC model is impartial regarding all rankings $\pi \in \mathcal{P}(A)$, just like IC, but assumes positive correlation across players. The respective probability distribution is given by[11]

$$\Pr(\mathbf{P}) = \left[\binom{m! + n - 1}{n} \cdot \binom{n}{n_1^{\mathbf{P}}, \ \ldots, \ n_{m!}^{\mathbf{P}}}\right]^{-1}. \tag{6}$$

Because IC presumes all players to have independent preferences, it typically yields an upper bound for success in adversarial scenarios where i's preferences are

[10]For instance, success could also mean avoiding one's worst option, i.e., $\hat{s}(a^*, P_i) = 1 \Leftrightarrow a^*$ is not ranked bottom in P_i. Proposition 1 below would then call for a method known as *anti-plurality voting*.

[11]See, e.g., Berg (1985) or Kurz et al. (2021) for details. The respective Pólya urn generalization could easily be accommodated also in this study.

negatively correlated to those of others. By contrast, IAC provides a more consensus-oriented outlook. It assumes some similarity in how players rank options and bounds individual success for potentially even greater preference affiliation from below.

Preferences in a real shareholder meeting or hiring committee will typically violate the IC or IAC assumptions. Working with the probabilities in equations (5)–(6) means doing thought experiments that assess voting from behind a 'veil of ignorance'. One disregards historical preference patterns, recent alliances, logrolling deals, etc. partly for lack of adequate data but also purposely in order to obtain a neutral constitutional evaluation of voting rules. The resulting assessments – e.g., that rule r and weights $\mathbf{w}$ make player i twice as successful than player j a priori – reflect the (un)levelness of the playing field for decision-making. Corresponding numbers typically differ from the players' actual (a posteriori) voting success in a committee since just a few preference configurations determine the latter and players' interaction involves social, political or financial dimensions that are orthogonal to voting rules.

4.2 Illustration

For illustration, let us evaluate a priori success when our stylized shareholders with voting weights of $\mathbf{w} = (45\%, 35\%, 20\%)$ choose between candidates $A = \{a, b, c\}$. The six possible individual rankings in $\mathcal{P}(A) = \{abc, acb, bac, bca, cab, cba\}$ give rise to $6^3 = 216$ different preference profiles that may obtain for a particular decision. Table 3 shows a selection of them, the respective winners $r|\mathbf{w}(\mathbf{P})$ implied by voting rules $r \in \{r^P, r^{PR}, r^C, r^B\}$ and associated success values $s(r|\mathbf{w}(\mathbf{P}), P_i)$.

For instance, the highlighted profile $\mathbf{P} = (cab, bca, abc)$ implies that c is selected under plurality and Borda rule. In contrast, b is selected under plurality with a runoff and a under Copeland rule. So, at that profile, shareholder 1 is (fully) successful under plurality and Borda rule. Additionally, half success would be attributed to player 1 by success function $\tilde{s}$ under Copeland rule (indicated by $\underline{0}$ in Table 3).

The corresponding expected success is shown at the bottom of Table 3 for the IC and IAC assumptions. The preference similarity reflected by IAC raises a priori success for all players relative to the preference independence assumed by IC. By definition, success figures for $ARI(\cdot)$ are greater than those for $TCI(\cdot)$.

The benchmark success of an independent outsider or 'dummy player' who has no say in the collective decision is 1/3 and 1/2 for the TCI and ARI, respectively.[12] A

[12]For general m, a dummy player d with independent preferences has a $TCI_d(\cdot) = 1/m$ chance to see his or her top choice win and must expect an outcome exactly in the middle, implying $ARI_d(\cdot) = 1/2$.

Table 3: Illustration of success computations

	Pr($\mathbf{P}$) for		$s(\cdot, P_i)$ for player i =															
$\mathbf{P} = (P_1, P_2, P_3)$	IC	IAC	$r^P \vert \mathbf{w}(\mathbf{P})$	1	2	3	$r^{PR} \vert \mathbf{w}(\mathbf{P})$	1	2	3	$r^C \vert \mathbf{w}(\mathbf{P})$	1	2	3	$r^B \vert \mathbf{w}(\mathbf{P})$	1	2	3
abc, abc, abc	$\frac{1}{216}$	$\frac{1}{56}$	a	1	1	1	a	1	1	1	a	1	1	1	a	1	1	1
abc, abc, acb	$\frac{1}{216}$	$\frac{1}{168}$	a	1	1	1	a	1	1	1	a	1	1	1	a	1	1	1
abc, abc, cab	$\frac{1}{216}$	$\frac{1}{168}$	a	1	1	$\underline{0}$	a	1	1	0	a	1	1	0	a	1	1	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
cab, bca, abc	$\frac{1}{216}$	$\frac{1}{336}$	c	1	$\underline{0}$	0	b	0	1	$\underline{0}$	a	$\underline{0}$	0	1	c	1	$\underline{0}$	0
cab, bca, acb	$\frac{1}{216}$	$\frac{1}{336}$	c	1	$\underline{0}$	$\underline{0}$	c	1	$\underline{0}$	$\underline{0}$	c	1	$\underline{0}$	$\underline{0}$	c	1	$\underline{0}$	$\underline{0}$
cab, bca, bac	$\frac{1}{216}$	$\frac{1}{336}$	b	0	1	1	b	0	1	1	b	0	1	1	c	1	$\underline{0}$	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
cba, cba, cba	$\frac{1}{216}$	$\frac{1}{56}$	c	1	1	1	c	1	1	1	c	1	1	1	c	1	1	1
Sum total	1	1		168	120	120		144	144	120		136	136	136		147	129	111
$TCI_i(\cdot)$ for IC				$\frac{168}{216}$	$\frac{120}{216}$	$\frac{120}{216}$		$\frac{144}{216}$	$\frac{144}{216}$	$\frac{120}{216}$		$\frac{136}{216}$	$\frac{136}{216}$	$\frac{136}{216}$		$\frac{147}{216}$	$\frac{129}{216}$	$\frac{111}{216}$
			≈	.78	.56	.56	≈	0.67	.67	.55	≈	.63	.63	.63	≈	.68	.60	.51
$TCI_i(\cdot)$ for IAC				$\frac{264}{336}$	$\frac{216}{336}$	$\frac{216}{336}$		$\frac{240}{336}$	$\frac{240}{336}$	$\frac{216}{336}$		$\frac{232}{336}$	$\frac{232}{336}$	$\frac{232}{336}$		$\frac{243}{336}$	$\frac{225}{336}$	$\frac{195}{336}$
			≈	.79	.64	.64	≈	.71	.71	.64	≈	.69	.69	.69	≈	.72	.67	.58
$ARI_i(\cdot)$ for IC				$\frac{180}{216}$	$\frac{144}{216}$	$\frac{144}{216}$		$\frac{162}{216}$	$\frac{162}{216}$	$\frac{156}{216}$		$\frac{162}{216}$	$\frac{162}{216}$	$\frac{162}{216}$		$\frac{177}{216}$	$\frac{162}{216}$	$\frac{142.5}{216}$
			≈	.83	.67	.67	≈	0.75	.75	.72	≈	.75	.75	.75	≈	.82	.75	.66
$ARI_i(\cdot)$ for IAC				$\frac{282}{336}$	$\frac{246}{336}$	$\frac{246}{336}$		$\frac{264}{336}$	$\frac{264}{336}$	$\frac{258}{336}$		$\frac{264}{336}$	$\frac{264}{336}$	$\frac{264}{336}$		$\frac{282}{336}$	$\frac{267}{336}$	$\frac{238.5}{336}$
			≈	.84	.73	.73	≈	0.79	.79	.77	≈	.79	.79	.79	≈	.84	.79	.71

Note: This table illustrates success computations when voters $N = \{1, 2, 3\}$ decide on options $A = \{a, b, c\}$ by rule $r|\mathbf{w}$ for $\mathbf{w} = (45\%, 35\%, 20\%)$ and $r \in \{r^P, r^{PR}, r^C, r^B\}$. Cases where $s(\cdot, P_i) \neq \tilde{s}(\cdot, P_i) = \frac{1}{2}$ are indicated by $\underline{0}$.

figure of, e.g., $TCI_3(\cdot) = 111/216 \approx 0.51$ under Borda rule shows that shareholder 3's voting rights clearly improve the chances to get what he or she wants. Taking decisions by pairwise majority voting further raises these chances: Copeland rule is the best from shareholder 3's perspective (lilac highlights), no matter which preference culture or success function is considered. By contrast, the large shareholder 1 is, a priori, most successful if the plurality rule is used. Shareholder 2's individual success maximizer differs for an all-or-nothing conception of success (plurality runoff) and an average rank perspective (Borda). From either perspective, we find that player-specific voting weights lead to player-specific answers to the question of which voting rule reflects one's personal preferences the best a priori.

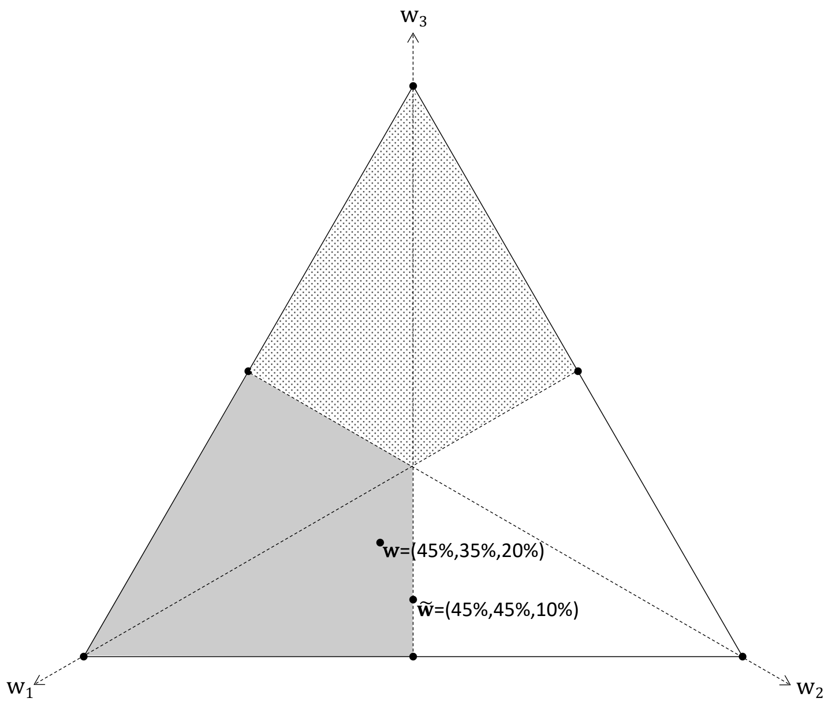


Figure 1: Simplex of all distributions of relative voting weights for $n = 3$

5 General Success Evaluations

The above illustration concerned one of the many conceivable distributions of voting weights among three players. Suppose that shareholder 3 sells a 10% stake to shareholder 2, resulting in the initial ownership structure $\tilde{\mathbf{w}} = (45\%, 45\%, 10\%)$ of Apple. How would the switch from $\mathbf{w} = (45\%, 35\%, 20\%)$ change 2's and 3's prospects for implementing their respective preferences? How is shareholder 1 affected?

Such questions could be answered by redoing the computations illustrated in Table 3 case by case. The identification of rule-specific weight equivalence classes by Kurz et al. (2020), however, facilitates the determination of a player's success for *all* possible distributions of voting weights: one needs to conduct the computations behind Table 3 for only one representative of each equivalence class. We will here cover all 6, 7, 4 and 51 classes that exist for plurality, plurality runoff, Copeland and Borda rule if $n = m = 3$.[13]

[13]For $m = 4$ options there are 6 plurality, 7 plurality runoff, 4 Copeland and 505 Borda equivalence classes, corresponding to committees that differ structurally rather than just nominally. Having only $n \leq 3$ relevant players may be unrealistic for big corporations and parliaments but fits many private firms, startups, joint ventures or party alliances and government coalitions.

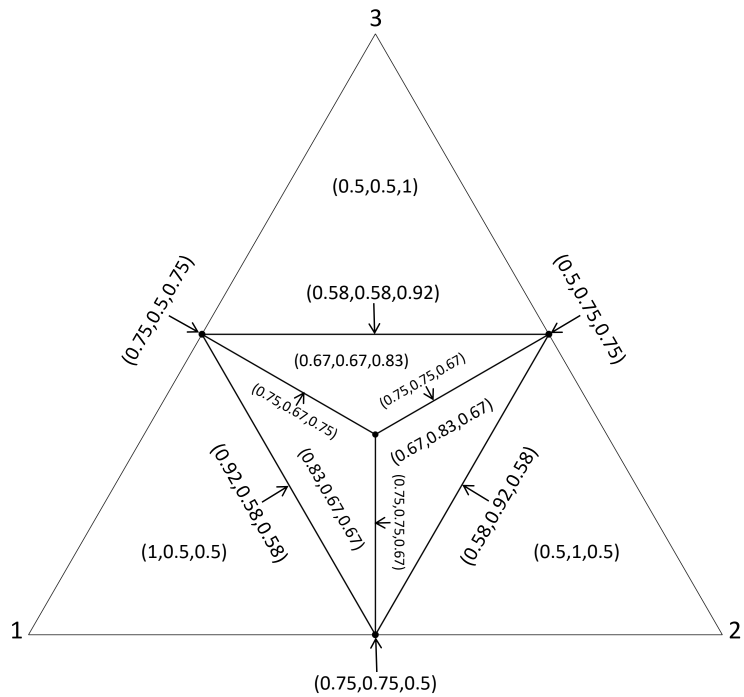


Figure 2: $ARI(\cdot)$ for plurality rule r^P and all weight distributions under IC when $n = m = 3$; $ARI_i = 0.72$ for all players if $\mathbf{w}^* = (1/3, 1/3, 1/3)$

5.1 Success for Three Players with Arbitrary Voting Weights

To present our results, we use the standard projection of the three-dimensional simplex of relative voting weights into the plane. It is illustrated in Figure 1: vertices give 100% of voting weight to the indicated player, e.g., player 1 in the bottom left corner; the midpoint corresponds to symmetric weights of $(1/3, 1/3, 1/3)$. Player 1 (2; 3) wields a plurality of votes in the shaded (blank; dotted) quadrangle.

Figures 2–4 respectively show all achievable $ARI(\cdot)$-vectors under IC for plurality with or without runoff and the Copeland method for $m = 3$ options, rounded to two decimal places.[14] Considerably more equivalence classes and associated success levels exist under the Borda rule. Figure 5 indicates the success values for player 1 by different colors – coded from red for a dummy player ($ARI_1(\cdot) = 0.5$) to blue for a dictator ($ARI_1(\cdot) = 1$). The values for player 2 or 3 correspond to player 1's success for the permuted distributions (w_2, w_1, w_3) and (w_3, w_2, w_1).

We can see that a switch from $\mathbf{w} = (45\%, 35\%, 20\%)$ to $\tilde{\mathbf{w}} = (45\%, 45\%, 10\%)$ increases player 2's a priori voting success only under plurality and Borda rule

[14] Analogous figures for IAC, $TCI(\cdot)$ or $m = 4$ are available from the authors upon request.

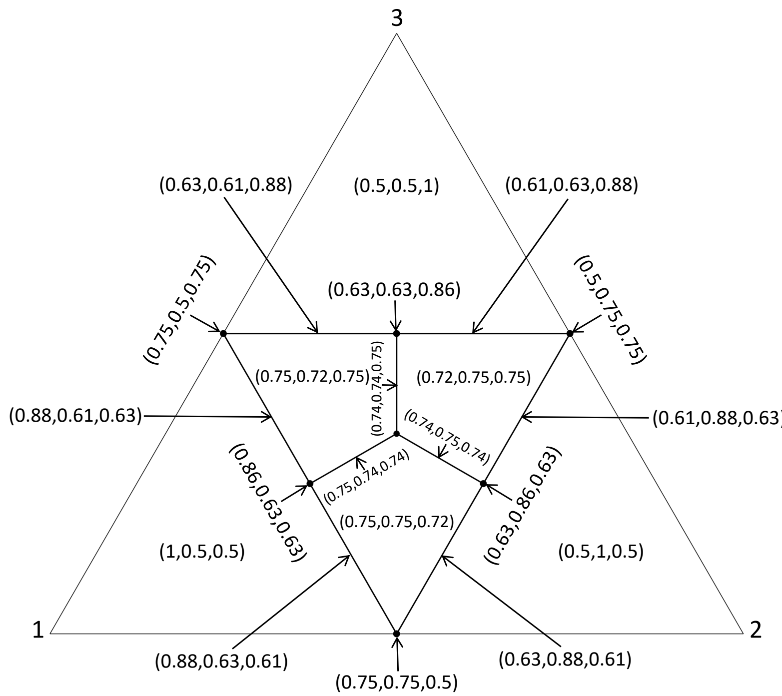


Figure 3: $ARI(\cdot)$ for plurality runoff rule r^{PR} and all weight distributions under IC when $n = m = 3$; $ARI_i = 0.75$ for all players if $\mathbf{w}^* = (1/3, 1/3, 1/3)$

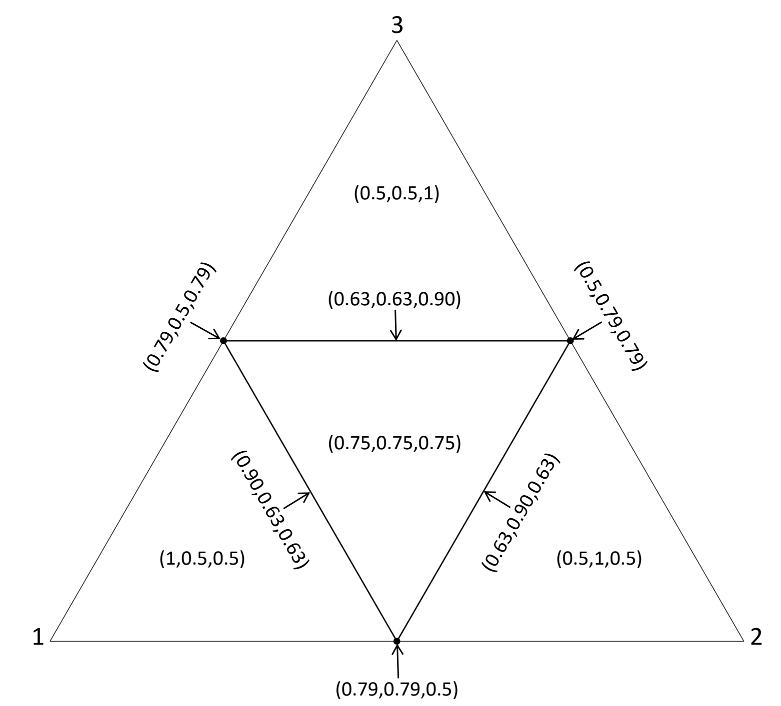


Figure 4: $ARI(\cdot)$ for Copeland rule r^C and all weight distributions under IC when $n = m = 3$

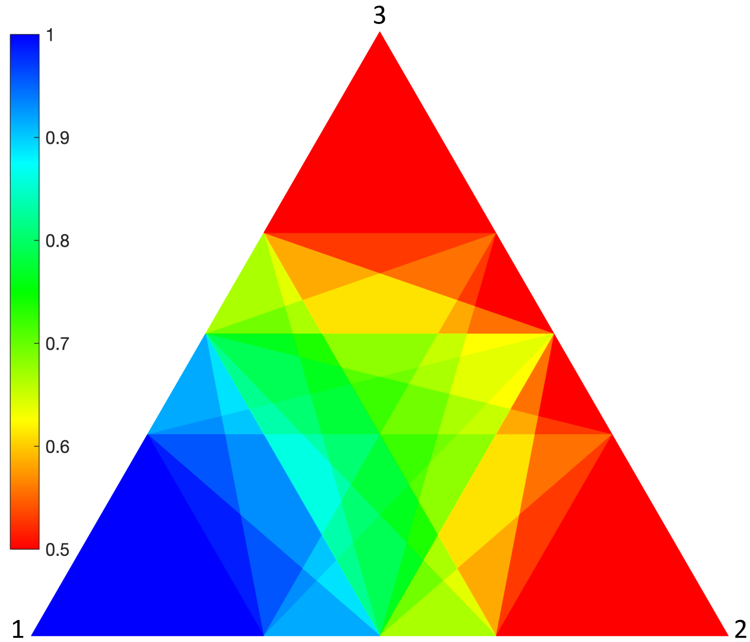


Figure 5: $ARI_1(\cdot)$ for Borda rule r^B and all weight distributions under IC when $n = m = 3$

(with increases from about 0.67 to 0.75, and 0.75 to 0.79, respectively). At the same time, expected preference satisfaction remains constant for all players under plurality runoff and Copeland rule. If a package of just 9% had been traded, the vote change would have only made a difference under Borda rule.

Figures 6 and 7 summarize which of the considered voting rules maximize player 1's success, $ARI_1(N, A, r|\mathbf{w})$ or $TCI_1(N, A, r|\mathbf{w})$, for any given distribution of voting weights among three players deciding on three or four alternatives.[15] Tongue in cheek, the figures provide a map for any self-interested member of a committee with a say on its default voting rule, such as a shareholder in a corporation with few co-owners. The figures can also help others prevent foul play.

5.2 Success for an Arbitrary Number of Symmetric Players

We can see in Figures 6 and 7 that plurality and Borda rule respectively maximize the 'all-or-nothing' top choice success $TCI_1(\cdot)$ and average-rank success $ARI_1(\cdot)$ for an equal distribution of voting weight. This finding extends to arbitrary numbers of

[15]Again, the success-maximizing rules for voters 2 and 3 can be inferred from considering the permuted distributions (w_2, w_1, w_3) and (w_3, w_2, w_1). Some focal lines or points in the figures are manually enlarged for better visibility.

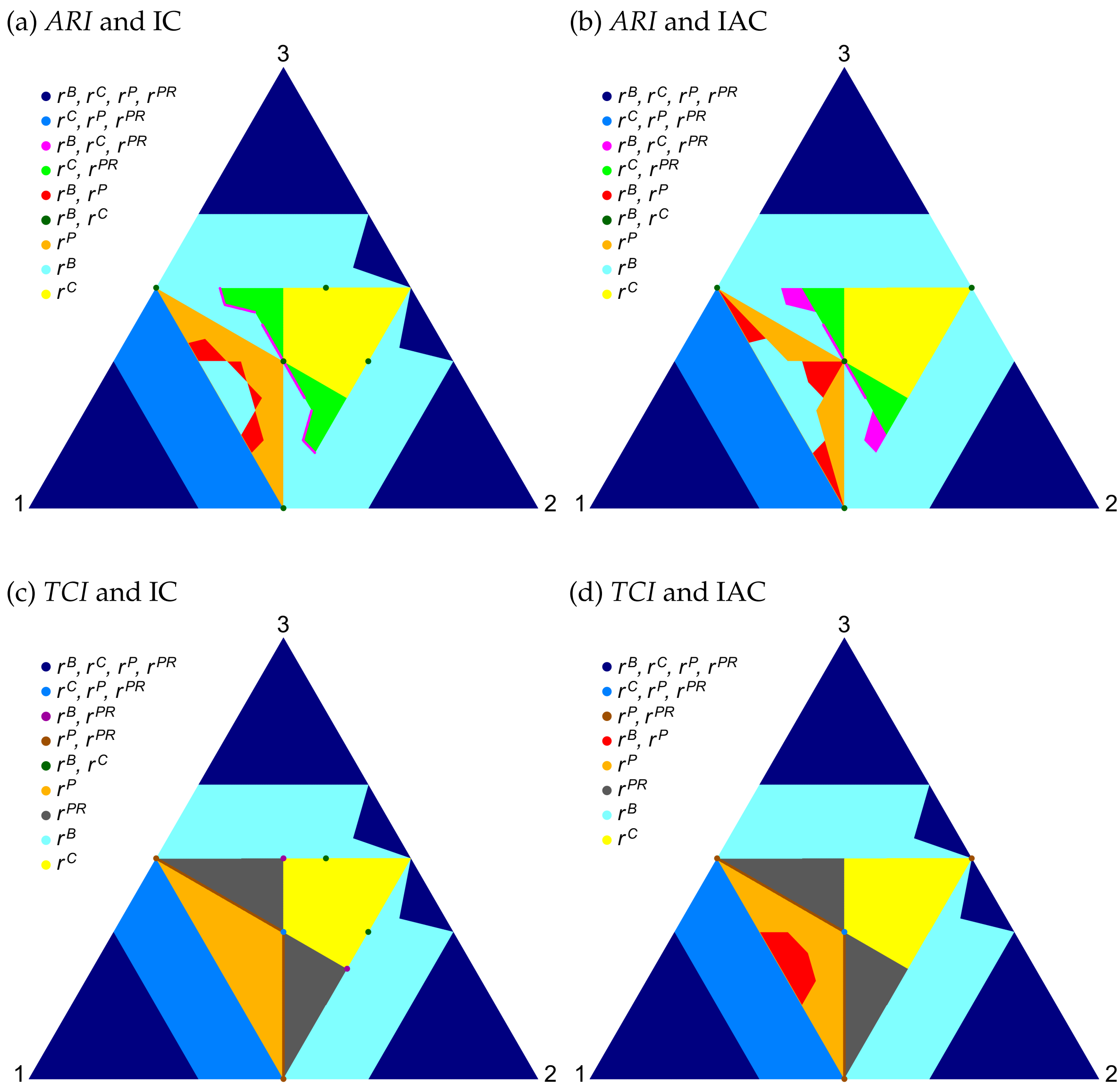


Figure 6: Maximizers of player 1's voting success for $n = m = 3$

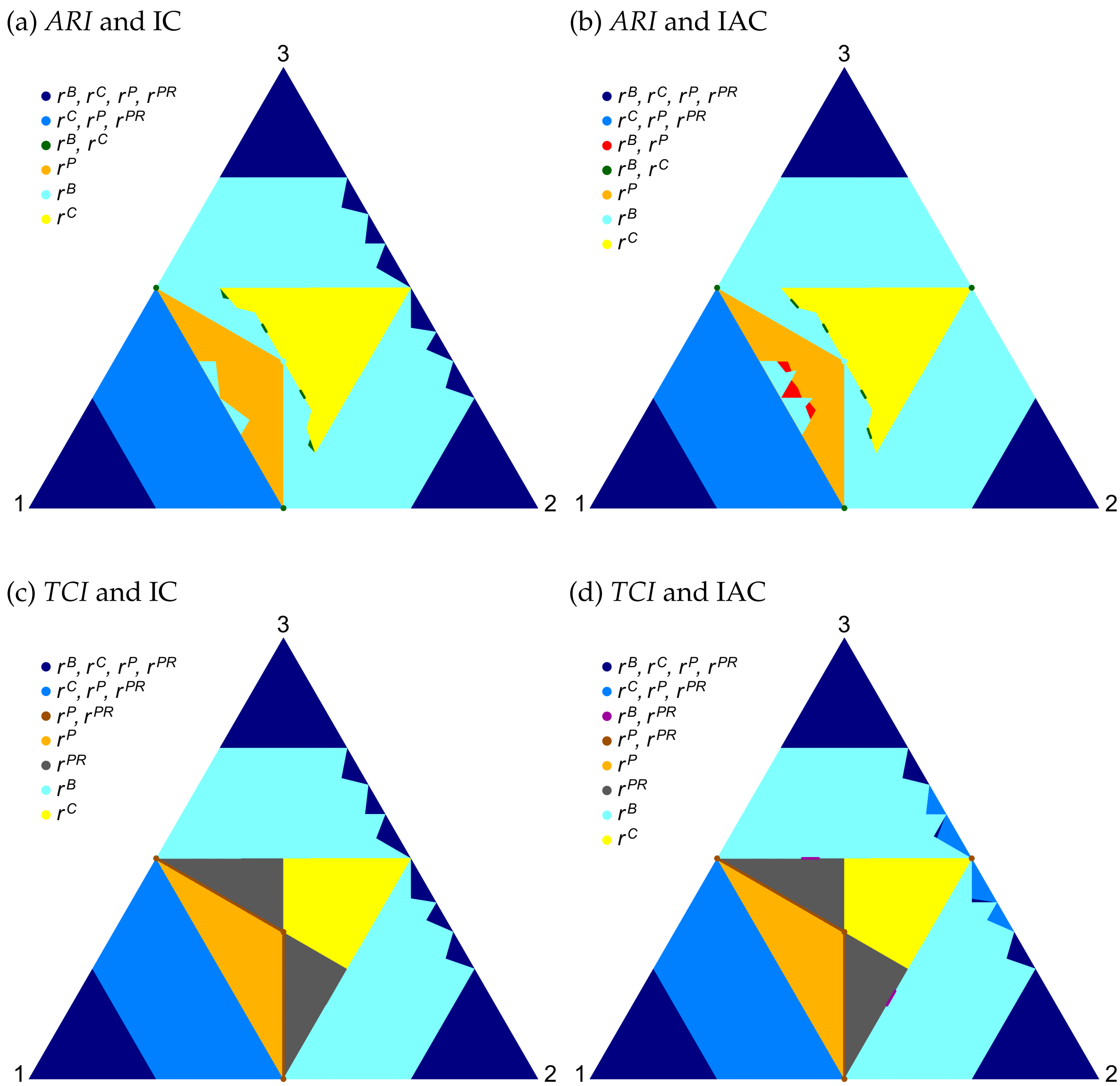


Figure 7: Maximizers of player 1's voting success for $n = 3$ and $m = 4$

players or alternatives, and analogous statements apply for other success functions. Namely, we have the following general recommendation for which voting rule r to use if voters are symmetric, i.e., all have voting weight $w_i = 1/n$ and their preferences are statistically exchangeable a priori, such as for IC or IAC:

Proposition 1. *Consider symmetric voters $N = \{1, \ldots, n\}$ whose preferences over $A = \{a_1, \ldots, a_m\}$ are exchangeable random variables a priori and a success function $\sigma^{\mathbf{s}}$ such that $\sigma^{\mathbf{s}}(a^*, P_i) = s_k$ with $1 = s_1 \geq s_2 \geq \ldots \geq s_m = 0$ whenever P_i ranks a^* in k-th place. Then any player i's a priori success $\mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_i)]$ is maximized by the scoring rule $r^{\mathbf{s}}$ that selects*

$$r^{\mathbf{s}}(\mathbf{P}) \in \arg\max_{a \in A} \sum_{i=1}^{n} \sum_{k=1}^{m} s_k \cdot \chi_k^{P_i}(a)$$

where $\chi_k^{P_i}(a) = 1$ if P_i ranks a in k-th place and 0 otherwise. In particular, $TCI_i(\cdot)$ is maximized by plurality rule r^P and $ARI_i(\cdot)$ is maximized by Borda rule r^B.

Proof. Exchangeable preferences imply $\mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_i)] = \mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_j)]$ for any players $i, j \in N$. Maximization of $\mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_i)]$ with respect to r is hence equivalent to the maximization of

$$\sum_{j=1}^{n} \mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_j)] = \sum_{j=1}^{n} \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \sigma^{\mathbf{s}}(r(\mathbf{P}), P_j)] = \sum_{j=1}^{n} \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \sum_{k=1}^{m} s_k \cdot \chi_k^{P_j}(r(\mathbf{P})) \tag{7}$$

$$= \sum_{\mathbf{P} \in \mathcal{P}(A)^n} \Pr(\mathbf{P}) \cdot \Big[\sum_{j=1}^{n} \sum_{k=1}^{m} s_k \cdot \chi_k^{P_j}(r(\mathbf{P})) \Big]. \tag{8}$$

By definition, $r^{\mathbf{s}}(\mathbf{P})$ maximizes the bracketed term in equation (8) for every $\mathbf{P} \in \mathcal{P}(A)^n$. Hence $r^{\mathbf{s}}$ maximizes $\mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_i)]$. It remains to note that $1 = s_1 > s_2 = \ldots = s_m = 0$ for all-or-nothing success function $s(\cdot)$ and that then $r^{\mathbf{s}} = r^P$. Similarly, $s_k = (m-k)/(m-1)$ holds for $k = 1, \ldots, m$ for the more gradual function $\tilde{s}(\cdot)$. Then, $r^{\mathbf{s}} = r^B$ because rescaling Borda scores $b_i(a, P)$ by $1/(m-1) > 0$ leaves the score maximizers and thus the selected outcomes unchanged. ∎

The intuition behind Proposition 1 is straightforward: plurality rule is defined as maximizing the number of voters who see their top choice win. In a perfectly symmetric world, this is equivalent to maximizing the probability that a fixed voter sees its top choice win. Similarly, Borda rule picks the option with the highest average rank among all voters, which entails maximizing the expected rank assigned to the outcome by any fixed voter under symmetry. Changing the order of summation in equations (7) and (8) formalizes just this but we are unaware of explicit previous statements of this generalization of the Rae-Taylor theorem for binary collective decisions (Rae 1969; Taylor 1969).

It is not straightforward but important that even the slightest deviation from perfect symmetry can destroy the optimality of r^P and r^B identified in Proposition 1. To see this, move slightly to the northeast or northwest of the midpoint of the simplex, e.g., in Figure 6(d): r^P immediately stops being TCI_1-optimal. Proposition 1 is therefore a fragile result. Equal voting weights represent a knife-edge situation and quick extrapolation of conclusions holding under symmetry should be met with suspicion. In particular, the intuitive advantage of plurality rule (Borda rule) in yielding high top choice success (average rank success) is proven to be frail by Figures 6 and 7.

There is no monotonic pattern for which voting rules are optimal for player 1 if the symmetry is broken. Consider Figure 6(d) and $\mathbf{w} = (w_1, (1-w_1)/2, (1-w_1)/2)$ for illustration: all rules make 1 a dummy player for $w_1 = 0$ but r^P and r^{PR} maximize 1's success given the correlation with players 2 and 3's preferences;[16] r^C is the unique TCI_1-maximizer for $w_1 \in (0, 1/3)$; then r^C, r^P and r^{PR} are success maximizing for $w_1 = 1/3$; only r^P maximizes TCI_1 for $w_1 \in (1/3, 3/7]$ and $w_1 = 1/2$, while r^P is tied with r^B for $w_1 \in (3/7, 1/2)$; again r^C, r^P and r^{PR} are optimal for $w_1 \in (1/2, 2/3]$; and finally all four rules induce $TCI_1 = 1$ for $w_1 \in (2/3, 1]$. A similar back and forth can also be seen in the other panels of Figures 6 and 7.

Based on these observations we should expect the theoretical result in Proposition 1 to provide little to no practical guidance for shareholder voting at publicly traded companies, parliaments, etc. unless all of the relevant stakeholders have equal voting weights. Quite surprisingly, the findings obtained in Section 6 for prominent US share distributions will disprove this conjecture.

5.3 Other Evaluation Criteria

The premise motivating our investigation is that a given player i cares only about its own success, not some greater good. Let us nonetheless comment on two additional and normatively appealing aspects of voting rules: the total of individual success values and the extent to which voting success correlates with voting weight.

[16] Note that r^P and r^{PR} are equivalent if $n = 2$ or if only two players have positive weight. Also, all considered methods amount to simple majority voting à la May (1952) if $m = 2$.

5.3.1 Aggregate Success of All Players

By-laws or rules of procedure should appeal also to future investors, not just the founders or current owners of a company. It may be unclear who will be a large or a small shareholder. Instituting a rule that implies a high sum of individual success values then is attractive. High aggregate a priori success is also desirable from the welfare perspective of a regulator. It is good to know, therefore, that plurality (Borda) rule maximizes the sum of all voters' top choice (average rank) success. This is a direct corollary of Proposition 1 for symmetric voters but extends to any asymmetric weights if we treat each share equally, i.e., consider the weighted sum of individual success values:

Proposition 2. *Under the conditions of Proposition 1, the scoring rule $r^{\mathbf{s}}$ maximizes the $\mathbf{w}$-weighted total success $\sum_{j=1}^{n} w_j \mathbb{E}[\sigma^{\mathbf{s}}(r|\mathbf{w}(\mathbf{P}), P_j)]$ for any given $\mathbf{w}$. In particular, $\sum_{j=1}^{n} w_j TCI_j(\cdot)$ is maximized by plurality rule r^P, and $\sum_{j=1}^{n} w_j ARI_j(\cdot)$ is maximized by Borda rule r^B for any distribution of voting weights.*

The proof follows directly from replacing $\sum_{j=1}^{n} \mathbb{E}[\sigma^{\mathbf{s}}(r(\mathbf{P}), P_j)]$ by $\sum_{j=1}^{n} w_j \mathbb{E}[\sigma^{\mathbf{s}}(r|\mathbf{w}(\mathbf{P}), P_j)]$ in equation (7) in the proof of Proposition 1. For Proposition 2, one may even drop the exchangeability condition that is needed for Proposition 1.

5.3.2 Transparency

The extent to which larger shareholdings imply greater success is another aspect that investors and authorities may care about. Voting rights are the distinguishing feature of common shares. Thus, differences in ownership should go with differences in how well the respective shareholder preferences are reflected in corporate decisions.

A simple measure of how transparently a priori voting success is aligned to voting rights is their correlation. For instance, under Borda rule, IC and $m = 3$, the correlation coefficient for the weight distribution $\mathbf{w} = (45\%, 35\%, 20\%)$ and the success distribution $ARI(\cdot) = (177/216, 162/216, 142.5/216)$ in Table 3 is 0.9992. This number is considerably higher than the respective figures for plurality and plurality runoff run (0.8030 and 0.9177); the uniform success values for Copeland are entirely uncorrelated with the given weights.

Figure 8 shows which method maximizes the respective correlation coefficient for all conceivable voting weight configurations among three players who decide on three alternatives. The winner is mostly Borda rule – no matter if one considers *ARI* or *TCI* success vectors for the IC or IAC preference distribution. An analogous

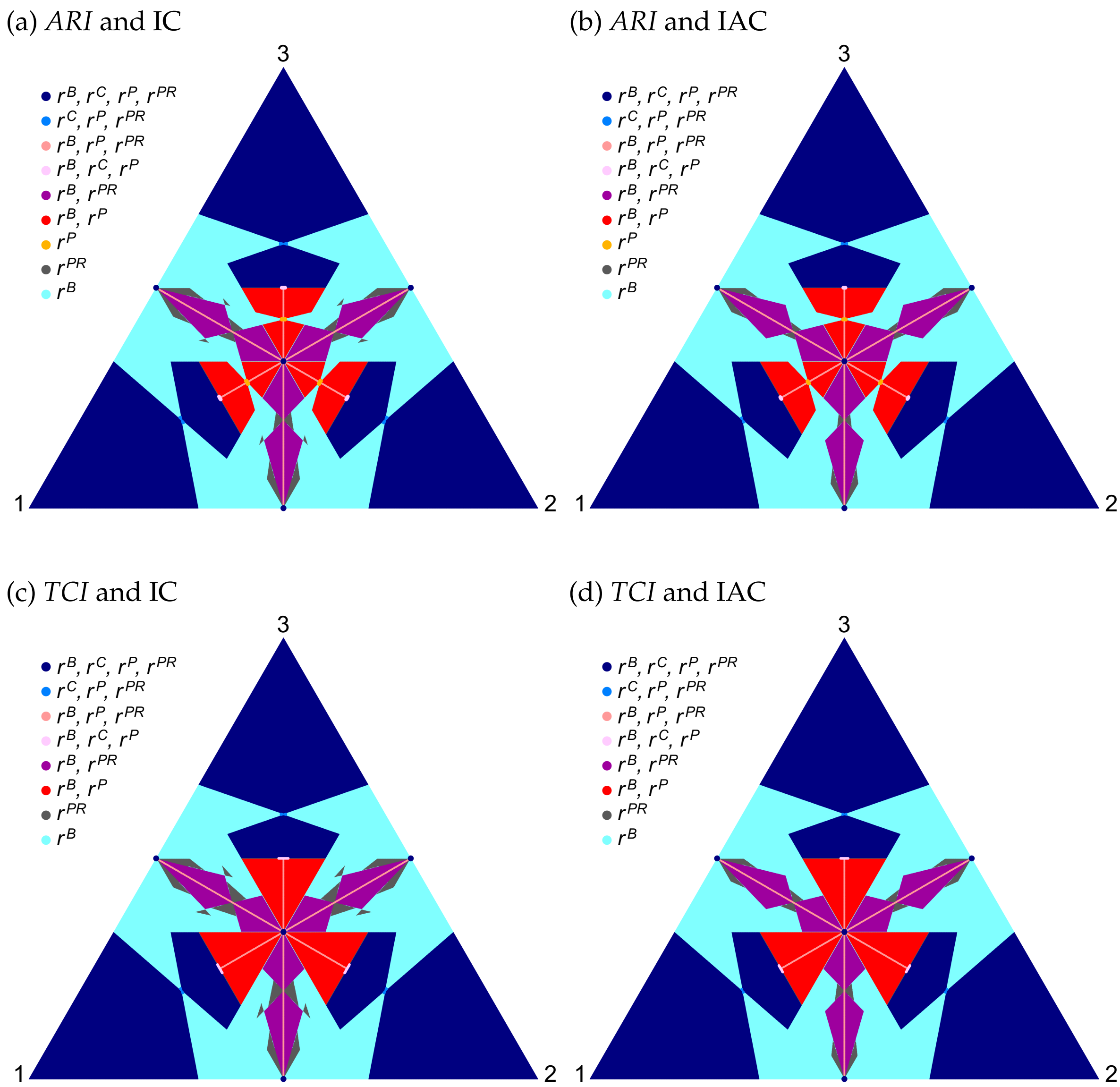


Figure 8: Maximizers of correlation between voting weights and success when $n = m = 3$

evaluation using the Kendall rank correlation coefficient gives similar results (with Borda rule being even more dominant). The underlying reason is that the Borda method comes with many more weight equivalence classes than the other methods (cf. Kurz et al. 2020). So, weight variations are more likely to make a difference. We also confirmed that Borda rule translates the large shareholdings in S&P 100 corporations, which we study next, into success the most transparently.

6 Application to S&P 100 Corporations

As we saw above, the individual optimality of the plurality and Borda voting rules can break down even for small deviations from symmetric voting weights. We therefore apply our success measures to a range of actual distributions of voting weights in order to assess the practical relevance of our findings (cf. Leech 1988). We consider the ownership structure of the companies in the S&P 100 stock index as composed at the beginning of 2022. To achieve reasonable computation times, we focus on $m = 3$ alternatives and the ten largest shareholdings of each index constituent. The remaining shares are treated as a homogeneous, perfectly divisible 'ocean' of free float.[17]

Under the IC assumption, the law of large numbers induces a uniform distribution of the float's cumulative weight across the $m! = 6$ possible rankings. We evaluate success in each of the resulting $6^{10} \approx 60$ mio. distinguishable preference configurations. The situation is more involved under the IAC assumption: positive correlation between shareholders – both large and small – generates infinitely many relevant configurations. We therefore approximate a priori success values under IAC in an extensive Monte Carlo simulation.[18]

[17] As a robustness check, we alternatively ignore all but the ten largest shareholders and evaluate their 6^{10} possible preference profiles. Ignoring smaller shareholders is in line with Azar et al. (2018), who eliminated all holdings below 0.5%. Disregarding the float does not change any results for IC, while corresponding findings for IAC become somewhat more similar to the IC findings.

[18] The exchangeable preferences under IAC can – in line with de Finetti's theorem – be simulated by first drawing common preference inclinations $\mathbf{p} = (p_1, \ldots, p_{m!})$ uniformly from the $(m!-1)$-dimensional unit simplex and then determining individual preferences via (conditionally) independent single draws from the $\mathbf{p}$-multinomial distribution (cf., e.g., Berg 1985). We approximate TCI_i (ARI_i) by averaging 350 000 iterations of the following steps: (1) draw $\mathbf{p}$; (2) draw P_i for $i = 1, \ldots, 10$; (3) shortcut draws for float shareholders by dividing the float's total weight in proportion to $(p_1, \ldots, p_{m!})$; (4) determine winner a^*; (5) evaluate $s(a^*, P_i)$ ($\tilde{s}(a^*, P_i)$) for $i = 1, \ldots, 10$.

Table 4: Descriptive statistics for shareholdings in the 92 included S&P 100 corporations

	Percentage values for the i^{th} largest shareholder					
Shareholdings	$i =$ 1	2	3	...	10	11 − ...
Mean	10.76	7.23	5.38	...	1.14	62.33
Mean (cumulative)	10.76	18.00	23.38	...	37.67	100.00
Standard devation	7.19	1.24	1.26	...	0.34	8.46
Standard devation (cumulative)	7.19	7.15	7.13	...	8.50	0.00
Maximum	48.87	15.14	9.11	...	2.29	72.68
Maximum (cumulative)	48.87	53.38	56.65	...	68.92	100.00
Minimum	6.96	4.52	3.27	...	0.44	31.08
Minimum (cumulative)	6.96	13.24	17.90	...	27.32	100.00

6.1 Data

The considered shareholder data comes from the Thomson Reuters Global Ownership database as of January 19, 2022, accessed via Refinitiv Eikon. The data combines information from various sources, such as mandatory disclosures, and has previously been used, e.g., by Bushee and Noe (2000), Azar et al. (2018) or Backus et al. (2021). We consider only ordinary shares with equal voting rights and exclude corporations with dual or multi-share classes that have distinct voting rights (cf. Backus et al. 2021). This leaves 92 constituents of the S&P 100 index in our sample. We consolidated the shareholdings of all BlackRock entities in analogy to Backus et al. (2021) or Ben-David, Franzoni, Moussawi and Sedunov (2021).

The descriptive statistics in Table 4 provide an overview of the consolidated share distributions. The largest shareholders of the S&P 100 constituents hold ≈ 7% to almost 50% of the corporate stock with a standard deviation (std. dev.) of 7.19% and a mean of 10.76%. The ten largest shareholders hold a cumulative stake of 37.67% on average. The remaining holdings define the respective free float mentioned above.

For the independent preferences assumed by IC, a sixth of the float's voting weight can be associated with each of the six alternative rankings of $m = 3$ options. Then, despite not wielding the required majority formally, the largest shareholder of three S&P 100 firms (Oracle, T-Mobile US, Walmart) can effectively dictate collective choices under any of our voting rules with a top choice and average rank index value of $TCI_1 = ARI_1 = 1$ (and $TCI_i = 1/3$ or $ARI_i = 1/2$ for shareholders $i \geq 2$). The three corporations are not included in the IC analysis, and so we report results for altogether 89 (92) out of 100 corporations included in the S&P 100 for IC (IAC).

Table 5: Success maximizers for S&P 100 shareholders

Success index and Prob. distr.	Success maximizers for the i^{th} largest shareholder r	1	2	3	4	5	6	7	8	9	10
ARI	Borda	76	85	56	84	71	76	78	70	73	70
IC	Copeland	2	4	33	3	11	7	7	9	4	13
$n = 89$	Plurality	11	0	0	0	0	0	0	0	0	0
	Plurality runoff	0	0	0	2	7	6	4	10	12	6
ARI	Borda	87	92	92	92	92	92	92	92	92	92
IAC	Copeland	0	0	0	0	0	0	0	0	0	0
$n = 92$	Plurality	5	0	0	0	0	0	0	0	0	0
	Plurality runoff	0	0	0	0	0	0	0	0	0	0
TCI	Borda	0	10	7	6	8	9	8	9	9	9
IC	Copeland	0	0	19	0	1	0	0	0	0	2
$n = 89$	Plurality	89	50	60	78	61	70	63	62	66	58
	Plurality runoff	0	29	3	5	19	10	18	18	14	20
TCI	Borda	0	4	3	3	3	3	3	3	3	3
IAC	Copeland	0	0	0	0	0	0	0	0	0	0
$n = 92$	Plurality	92	88	89	89	89	89	89	89	89	89
	Plurality runoff	0	0	0	0	0	0	0	0	0	0

6.2 Results

Table 5 summarizes the success maximizers for the included S&P 100 companies. The striking message is that the theoretical conclusions of Proposition 1 for a perfectly symmetric distribution of voting weight extend to the considered voting share distributions. Respective shareholdings are highly asymmetric but, with preference correlation à la IAC, Borda is the best rule for all except 5 of the largest shareholders if success is evaluated according to the shareholder's average ranking of collective choices (ARI_i).[19] Similarly, the plurality rule maximizes success in overwhelmingly many cases if a shareholder i cares only about the probability with which it obtains its top choice (TCI_i).

Under IC, plurality rule maximizes the TCI_1 value for all 89 of the largest shareholders, the respective TCI_2 number for 50 of the second-largest shareholders, the TCI_3 level for 60 of the third-largest and so on. We obtain similar findings for the Borda rule and the average rank index: Borda is the ARI_1 ($ARI_2, ARI_3, \ldots, ARI_{10}$) maximizer for 76 (85, 56, ..., 70) of the respective largest (second-largest, third-largest, ..., tenth-largest) shareholders out of the included 89 companies. These results

[19]The exceptions arise when the largest shareholder owns $w_1 \geq 20\%$ of shares.

are totally unexpected given the distribution sensitivity and the non-monotonicities observed in Section 5 for only three voters.

A robust rule of thumb emerges for most shareholders: make a case for deciding by Borda rule if your objective is to maximize the ARI_i value, and for plurality rule if you want to maximize TCI_i. This is essentially independent of whether you have the largest, second-largest, etc. holding in a corporation and whether the IC or IAC preference distribution is deemed more relevant for a personal a priori assessment. The same recommendation holds for shareholders with small or infinitesimal holdings subsumed in the float[20] and, as shown in Proposition 2, Borda and plurality rule also maximize the weighted total of all shareholders' success values.

Copeland and plurality runoff rule maximize a player i's ARI_i and TCI_i levels only in a few cases. The key exceptions are ARI_3 numbers for the third-largest shareholders (Copeland is optimal in 33 cases) and TCI_2 values for the second-largest shareholders (plurality runoff is optimal for 29 of them) if all preferences are considered independent.

Having such exceptions may, of course, make it worthwhile not to rely on a rule of thumb but to assess voting methods individually and case by case. We can, for instance, zoom in on the success maximizers for the largest institutional investors: Vanguard and BlackRock. Both are among the ten largest shareholders of all examined S&P 100 companies and usually in the top three. It turns out that the advantage of Borda (plurality) in maximizing their ARI_i (TCI_i) success level is pronounced for both of them.[21]

Figure 9 illustrates the mean values of ARI_i and TCI_i for the ten largest shareholders under the applicable voting rule. On the one hand, the figure corroborates the above rule of thumb: the interpolated lines for Borda (plurality) lie consistently above those of the other three voting rules when considering ARI_i (TCI_i).[22] On the other hand, comparing the spaces between the lines to their slopes highlights that, non-surprisingly, the voting advantage that an investor derives from having more shares than another tends to matter more than whether this or that rule applies.

We use a linear regression model in order to quantify expected voting rule dif-

[20] Positive correlation under IAC aligns the interests of a dummy player with the average interests of the shareholders in Table 5, while all rules yield identical dummy success $TCI_d = 1/3$ and $ARI_d = 1/2$ under IC. We, moreover, cross-checked the findings reported in Table 5 by repeating the analysis with an additional shareholder 11 owning 0.1% of shares: all conclusions continued to hold.

[21] Exceptions arise under IC, with results for $TCI_{\text{BlackRock}}$ similar to those for the second largest shareholders in Table 5.

[22] Using medians instead of means leads to the same inferences.

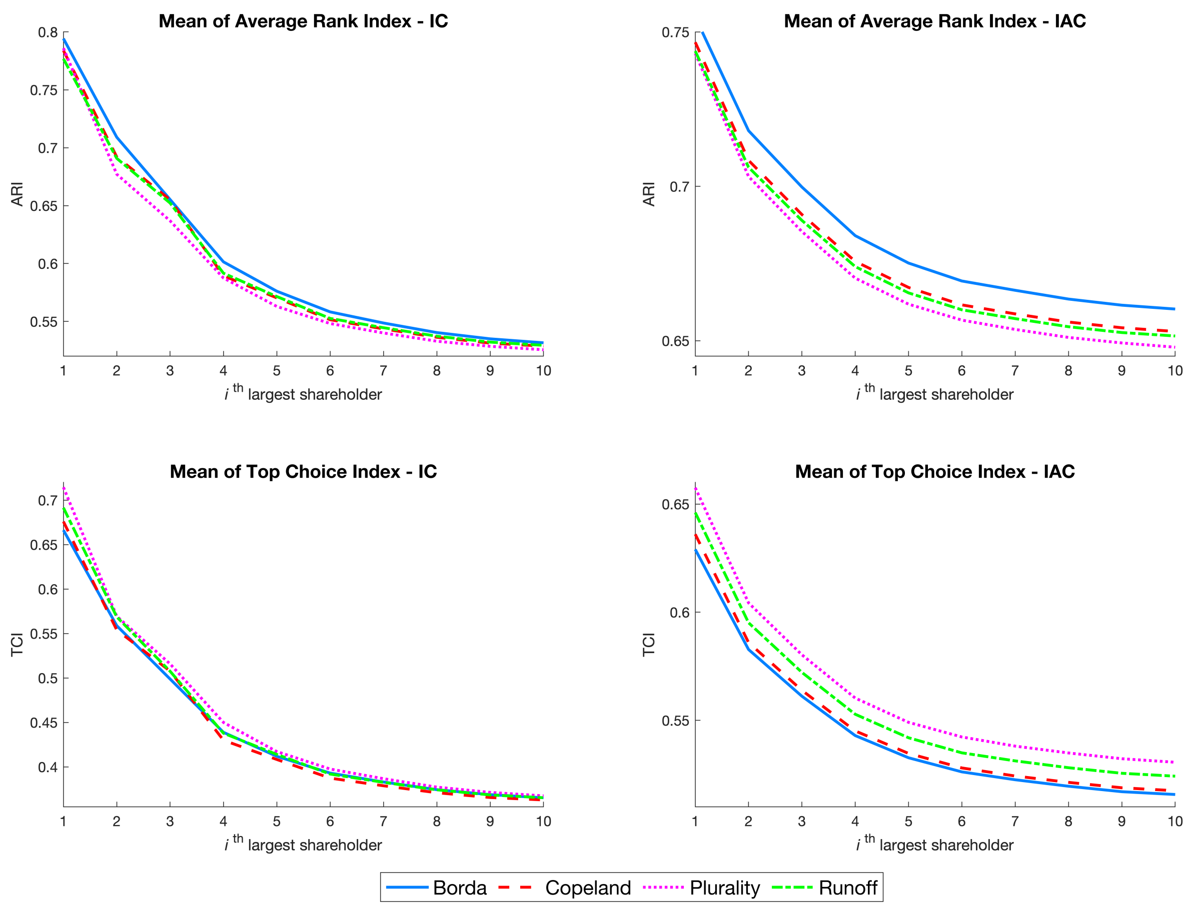


Figure 9: Success indices averaged per shareholding rank across S&P 100 companies

ferences and to determine their statistical significance. Since it is well-known from the analysis of binary votes that it is not higher voting weight as such that gives an advantage, but how this weight facilitates the formation of winning majorities (or blocking minorities) together with other voters, we opted to include proxies of shareholders' voting power instead of the shareholdings themselves. To respect the underlying correlation pattern, we draw on the Penrose-Banzhaf index (*PBI*) for IC and the Shapley-Shubik index (*SSI*) value for IAC.[23] We then estimate the following regression model using an ordinary least squares (OLS) regression with heteroscedasticity robust standard errors (White 1980):[24]

$$VotingSuccess_{ir} = \alpha_0 + \beta_1 PowerIndex_i + \sum_{j=2}^{4} \beta_j VotingRules_r + \varepsilon_{ir}, \tag{9}$$

where *VotingSuccess* is the shareholder's success index ARI_i (TCI_i) and *VotingRules* are *Copeland*, *Plurality* and *PluralityRunoff* (*Borda*, *Copeland* and *PluralityRunoff*) indicator variables, respectively. *PowerIndex* represents either shareholder *i*'s *PBI* or *SSI* value, namely PBI_i for IC and SSI_i for IAC.

We estimate the regressions for 3 560 (3 680) voting rule-shareholder observations consisting of the ten largest shareholders of the 89 (92) S&P 100 companies for our four voting rules considering the IC (IAC) distribution. We present the corresponding results in Table 6. One can see that the adjusted R^2 values are very high. The shareholders' voting powers – captured heuristically by indices that assume binary voting – strongly drive both success indexes, *ARI* and *TCI*. This finding is especially pronounced under the IC assumption but the fit is good also for IAC. These observations retrospectively support the many previous investigations that employed the Penrose-Banzhaf or Shapley-Shubik indices of binary voting power also in contexts where collective choices are not from the start simple 'yes'-or-'no' affairs, but at some stage require a selection from multiple candidates, motions or bills prior to an official vote.

The results in columns (1) and (2) of Table 6 confirm the inferences we drew from

[23] The respective power index computations assume binary decisions by shareholders $i = 1, \ldots, 10$ with a 50% majority quota. Dedicated power measures for non-binary decisions (Kurz et al. 2021) are more challenging to compute and scale down β_2, β_3 and β_4 because some rule effects are captured already. Nevertheless, coefficients β_2, β_3 and β_4 remain statistically different from zero.

[24] As a robustness check, we alternatively control for the holding percentage of shareholder *i* and/or the ownership concentration among shareholders $j \neq i$ measured by the Herfindahl index (cf. Demsetz and Lehn 1985 and Ajinkya, Bhojraj and Sengupta 2005). The model then loses some explanatory power but results are qualitatively unchanged. Including shareholder rank fixed effects does not alter our conclusions either, nor does using tobit regressions for censored dependent variables.

Table 6: Multivariate regression results

	Dependent variable: Success index for given preference distribution			
	ARI		*TCI*	
	IC	IAC	IC	IAC
	(1)	(2)	(3)	(4)
Variable	Coefficient	Coefficient	Coefficient	Coefficient
PBI	0.4950***		0.6166***	
SSI		0.3249***		0.4195***
Borda			-0.0108***	-0.0180***
Copeland	-0.0068***	-0.0080***	-0.0126***	-0.0154***
Plurality	-0.0124***	-0.0130***		
PluralityRunoff	-0.0071***	-0.0098***	-0.0065***	-0.0078***
Intercept	0.5078***	0.6528***	0.3356***	0.5210***
Adjusted R^2	0.9924	0.8644	0.9885	0.8797
n	3 560	3 680	3 560	3 680

Note: This table presents the coefficients and significance levels of our four OLS regressions with heteroscedasticity robust standard errors (White 1980). The superscripts *, ** and *** represent significance levels of 10%, 5% and 1% (two-tailed t-tests), respectively. We estimate the following regression models: $VotingSuccess_{ir} = \alpha_0 + \beta_1 PowerIndex_i + \sum_{j=2}^{4} \beta_j VotingRules_r + \varepsilon_{ir}$ where *VotingRules* are *Copeland*, *Plurality* or *PluralityRunoff* indicators in columns (1) and (2) and *Borda*, *Copeland* or *PluralityRunoff* indicators in columns (3) and (4). The dependent variable *VotingSuccess* is the shareholder's success index *ARI* in columns (1) and (2) and *TCI* in columns (3) and (4), respectively. The success indexes are calculated assuming IC in columns (1) and (3) and IAC in columns (2) and (4) as probability distribution over preference profiles. Consequently, the Penrose-Banzhaf index (*PBI*) represents the *PowerIndex* in columns (1) and (3) the Shapley-Shubik index (*SSI*) is used in columns (2) and (4).

Table 5 and Figure 9: individual voting success as measured by *ARI* is significantly lower when the voting rules *Copeland*, *Plurality* or *PluralityRunoff* are applied instead of Borda rule, which is the baseline in these regressions. Similarly, columns (3) and (4) show significantly lower success indexes for *Borda*, *Copeland* and *PluralityRunoff* in comparison to the benchmark plurality rule considering *TCI*. All differences are statistically significant at the 1% level.

Using the optimal voting rules on average leads to 1 to 3% higher success rates. This may not seem a big difference but in a majority of cases the tenth largest shareholders (with mean holdings of 1.14%) would benefit more from switching from the worst to the best voting rule than from switching their holdings with the respective seventh largest shareholders (with mean holdings of 1.78%). Considering only IAC, the effects are yet more pronounced: the tenth largest shareholders in more than 80% (50%) of the firms would benefit more from a rule change than from swapping their holdings with those of the sixth (fifth) largest shareholders.[25] Recall also that a priori figures include many preference configurations with broad agreement among voters. If, a posteriori, one focuses on those situations where disagreement has called for an explicit act of voting, the success gap between the individually best vs. worst rule can be significantly higher.

7 Concluding Remarks

The key takeaway from this investigation is that the adopted voting procedure matters from an a priori and day-to-day perspective – not only in selected textbook examples or under specific historical circumstances (cf. Leininger 1993, Tabarrok and Spector 1999, Maskin and Sen 2016, but also Darmann and Klamler 2023 or Lachat and Laslier 2024). Voting rules entail different prospects of seeing one's most preferred option win and a different expected personal ranking of collective outcomes. This can be quantified, predicted for one's preferred benchmark distribution of preferences and reacted to in the design of by-laws, statutes or other governance instruments.

Of course, our findings have practical and theoretical limitations. First, a wide range of voting rules exist, and we have focused on just four. For instance, after a plurality vote without a majority winner, one may delete only the alternative with the lowest support and vote again; a chairperson may put just specific pairwise comparisons on the agenda; there are many scoring methods that evaluate the candidate positions in individual preference rankings differently from Borda or plurality. The rules investigated here include prominent representatives from the three main classes of single-winner methods (Condorcet methods, scoring rules and runoff rules), but there are ample opportunities for follow-up work. The Borda rule is probably the least frequently used of the voting procedures that we have looked at. It comes with particularly compelling axiomatic properties, however,[26] and Ambuehl and Bern-

[25]Considering Apple Inc., for instance, the ARI_{10}-gain of $0.6619-0.6497 = 0.0121$ from replacing plurality rule by Borda rule exceeds the difference between $ARI_{10} = 0.6497$ and $ARI_5 = 0.6616$ under plurality rule, which reflect holdings of 0.66% vs. 2.07%. An acquisition of 2.07%−0.66% = 1.41% of Apple stock would have cost around USD 38 billion in Jan. 2022.

[26]Borda rule is closely connected to both May's and Arrow's classical axioms of rational collective choice. As discovered by Maskin (2024), the rule is unique in satisfying anonymity, neutrality, responsiveness, unrestricted domain, a Pareto ranking condition and modified independence of irrelevant alternatives. This comes on top of the desirable consistency properties that distinguish Borda rule from, e.g., pairwise voting (cf. Young 1974).

heim (2024) found that Borda winners have better ex post support than plurality or pairwise winners in allocation experiments. Our findings that Borda rule yields higher individual ranks also for practically relevant asymmetry and links success more transparently to voting weights strengthens the case for using it more.

Second, we are aware that very many votes by boards or general assemblies are 'yes'-or-'no' decisions with little dissent. The existing sets of options are often whittled down to singletons without documented votes long before any official shareholder or board meeting. However, decisions can involve disagreement. Then it is advantageous to know how different procedures translate into different success expectations. The informal mechanisms that help to generate a single consensual proposal can implicitly involve multiple pairwise comparisons, a runoff-like focus on the two options with the most initial support or the accumulation of individual scores. In such cases, using the right method can pay off individually and raise aggregate success just as much as with explicit votes.

Third, we have focused on sincere voting. This is a restrictive assumption despite relatively scant empirical support for its most compelling alternative: strategic voting. Assuming the latter comes with a pervasive non-uniqueness of the resulting voting outcomes and entails limitations of its own. Still, we present a detailed investigation of strategic voting equilibria in Appendix B. It shows that many of the weight-specific maximizers identified in Section 5 continue to maximize success at least for one plausible way to select from non-singleton sets of equilibria.

Notwithstanding these limitations, our analysis is a reminder that deciding by vote is more complex than meets the eye. Accounting for the combinatoric properties of weighted voting is non-trivial already for binary options. Assessing which procedure best reflects the preferences of an individual for more than two alternatives is yet more cumbersome, but rewarding.

Appendices

Appendix A: Robustness to Random Tie Breaking

Lexicographic tie breaking assumes some fixed ordering $a_1 <_L a_2 <_L \ldots <_L a_m$ of the alternatives such that if options $a_{i_1}, \ldots, a_{i_k}$ receive the same plurality score (Borda score, etc.) at a given preference profile $\mathbf{P}$ then a_{i^*} with $i^* = \min\{i_1, \ldots, i_k\}$ is selected as the unique winner $r|\mathbf{w}(\mathbf{P})$.

To demonstrate that the tie-breaking assumption is innocuous for our analysis, consider the *set-valued version of a given voting rule r* like r^P (r^B, etc.). This maps each preference profile $\mathbf{P}$ to the non-empty set $\hat{r}|\mathbf{w}(\mathbf{P}) = A^* \subseteq A$ of all alternatives that have the highest plurality score (Borda score, etc.). In contrast to our point-valued baseline, the respective set-valued version of r^P (r^B, etc.) is neutral, i.e., $\hat{r} = \hat{r}^P$ ($\hat{r}^B$, etc.) satisfies $\hat{r}(\rho(\mathbf{P})) = \rho(\hat{r}(\mathbf{P}))$ for any permutation $\rho\colon A \to A$ and $\mathbf{P} \in \mathcal{P}(A)^n$.

Now take an arbitrary alternative-based tie breaking method. It can be described by a family $\{\beta_B\}_{B \in 2^A \setminus \varnothing}$ of probability distributions that assign winning probabilities $\beta_B(a)$ to all $a \in B$ with $\sum_{a \in B} \beta_B(a) = 1$ for any set of tied alternatives B. We will write $\{\beta_B\}$ for short. Lexicographic tie breaking $\{\beta_B^{\text{lex.}}\}$ amounts to $\beta_B^{\text{lex.}}(a) = 1$ iff a is the lexicographically minimal element of B. A popular alternative is uniform random tie breaking $\{\beta_B^{\text{uni.}}\}$ where $\beta_B^{\text{uni.}}(a) = 1/|B|$ for any $a \in B$. A given method $\{\beta_B\}$ might also apply uniform tie breaking if $|B| = 2$, lexicographic tie breaking if $|B| = 3$, prescribe particular B-specific probabilities if $|B| = 4$, etc. All we require is that preferences affect the outcome via the baseline voting rule while tie break probabilities are independent of $\mathbf{P}$.[27]

For a given success function $\sigma\colon A \times \mathcal{P}(A) \to \mathbb{R}$ let *σ's extension to $\{\beta_B\}$-tie breaking* $\hat{\sigma}\colon 2^A \times \mathcal{P}(A) \to \mathbb{R}$ be defined by

$$\hat{\sigma}(B, P_i) := \sum_{a^* \in B} \beta_B(a^*)\sigma(a^*, P_i). \tag{10}$$

This equals the expectation of $\sigma(a^*, P_i)$ under the pertinent tie break probabilities. With these definitions we have

Proposition 3. *Consider voters $N = \{1, \ldots, n\}$ with voting weights $\mathbf{w} = (w_1, \ldots, w_n)$ whose preferences over $A = \{a_1, \ldots, a_m\}$ are drawn from a probability distribution on $\mathcal{P}(A)^n$ that satisfies* $\Pr(\mathbf{P}) = \Pr(\rho(\mathbf{P}))$ *for any permutation $\rho\colon A \to A$. Let $\hat{r}|\mathbf{w}$ be the neutral set-valued*

[27]Making tie break probabilities a function of the preferences of, e.g., the committee's chairperson or the largest shareholder would shift the distribution of a priori success in the expected direction.

version of $r|\mathbf{w}$ and $\hat{\sigma}$ be the extension of success function σ to $\{\beta_B\}$-tie breaking. Then

$$\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)] = \mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)].$$

In particular, player i's top choice or average rank success under lexicographic tie breaking, $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$, and the respective success under any other alternative-based tie breaking method, $\mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)]$, are identical.

Proof. For any non-empty subset $B \subseteq A$ of alternatives, denote the set of preference profiles that yield a tie between the alternatives in B by

$$\boldsymbol{\mathcal{P}}_B := \Big\{\mathbf{P} \in \mathcal{P}(A)^n : \hat{r}|\mathbf{w}(\mathbf{P}) = B\Big\}. \tag{11}$$

Also denote the permutations of A that only switch elements of $B \subseteq A$ by

$$\mathcal{S}^B := \Big\{\rho\colon A \to A \;:\; [a \notin B \Rightarrow \rho(a) = a]\Big\}. \tag{12}$$

If $\mathbf{P} \in \boldsymbol{\mathcal{P}}_B$, then neutrality of $\hat{r}|\mathbf{w}$ implies that also $\mathbf{P}' = \rho(\mathbf{P}) \in \boldsymbol{\mathcal{P}}_B$ for any $\rho \in \mathcal{S}^B$. Hence $\boldsymbol{\mathcal{P}}_B$ can be partitioned into $k(B) = {}^{|\boldsymbol{\mathcal{P}}_B|}/_{|B|!}$ subsets $\boldsymbol{\mathcal{P}}_{B,1}, \dots, \boldsymbol{\mathcal{P}}_{B,k(B)}$ that each contain $|B|!$ profiles which differ only by permutations of B's elements. For any such partition element $\boldsymbol{\mathcal{P}}_{B,j}$ let us fix a 'representative' profile $\mathbf{P}^{B,j} \in \boldsymbol{\mathcal{P}}_{B,j}$ that ranks B's elements $B = \{a_{r_1}, \dots, a_{r_{|B|}}\}$ from player i's perspective by $a_{r_1} P_i^{B,j} a_{r_2} P_i^{B,j} \dots P_i^{B,j} a_{r_{|B|}}$.

Player i's success $\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)$ for $\mathbf{P} \in \boldsymbol{\mathcal{P}}_{B,j}$ equals

$$\hat{\sigma}(B, P_i) = \hat{\sigma}(B, P_i^{B,j}) = \beta_B(a_{r_1})\sigma(a_{r_1}, P_i^{B,j}) + \beta_B(a_{r_2})\sigma(a_{r_2}, P_i^{B,j}) + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P_i^{B,j}) \tag{13}$$

if $\mathbf{P} = \mathbf{P}^{B,j}$. For the related profile $\mathbf{P}' \in \boldsymbol{\mathcal{P}}_{B,j}$ where, e.g., a_{r_1} and a_{r_2} are permuted, i's success evaluates to

$$\begin{aligned}\hat{\sigma}(B, P_i') &= \beta_B(a_{r_1})\sigma(a_{r_1}, P_i') + \beta_B(a_{r_2})\sigma(a_{r_2}, P_i') + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P_i') \\ &= \beta_B(a_{r_1})\sigma(a_{r_2}, P_i^{B,j}) + \beta_B(a_{r_2})\sigma(a_{r_1}, P_i^{B,j}) + \dots + \beta_B(a_{r_{|B|}})\sigma(a_{r_{|B|}}, P_i^{B,j})\end{aligned} \tag{14}$$

and, more generally, we have $\hat{\sigma}(B, P_i') = \sum_{a \in B} \beta_B(a)\sigma(\rho^{-1}(a), P_i^{B,j})$ if $\mathbf{P}' = \rho(\mathbf{P}^{B,j})$.

This implies

$$\begin{aligned}\sum_{\mathbf{P} \in \boldsymbol{\mathcal{P}}_{B,j}} \Pr(\mathbf{P})\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) &= \sum_{\rho \in \mathcal{S}^B} \Pr(\rho(\mathbf{P}^{B,j})) \sum_{a \in B} \beta_B(a)\sigma(\rho^{-1}(a), P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \beta_B(a) \sum_{\rho \in \mathcal{S}^B} \sigma(\rho^{-1}(a), P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \beta_B(a) \sum_{\tilde{a} \in B} \frac{|B|!}{|B|} \sigma(\tilde{a}, P_i^{B,j}) \\ &= \Pr(\mathbf{P}^{B,j}) \sum_{a \in B} \frac{|B|!}{|B|} \sigma(a, P_i^{B,j}).\end{aligned} \tag{15}$$

The second equality exploits that $\Pr(\mathbf{P}) = \Pr(\rho(\mathbf{P}))$ for any permutation $\rho\colon A \to A$ and changes the order of summation. The third equality uses that, as we go over all permutations of B's elements supposing a given a prevails in the tie break, a is ranked in the location that $\tilde{a} = \rho^{-1}(a)$ has in reference ranking $P_i^{B,j}$ exactly $|B|!/|B|$ times for each $\tilde{a} \in B$. The final equality just replaces the expectation of a constant by this constant and renames $\tilde{a}$ to a.

We therefore have

$$\begin{aligned}\mathbb{E}[\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i)] &= \sum_{\mathbf{P}\in\mathcal{P}(A)^n} \Pr(\mathbf{P})\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) \\ &= \sum_{B\subseteq A}\sum_{j=1}^{k(B)}\sum_{\mathbf{P}\in\boldsymbol{\mathcal{P}}_{B,j}} \Pr(\mathbf{P})\hat{\sigma}(\hat{r}|\mathbf{w}(\mathbf{P}), P_i) \qquad (16)\\ &= \sum_{B\subseteq A}\sum_{j=1}^{k(B)} \Pr(\mathbf{P}^{B,j})\sum_{a\in B}\frac{|B|!}{|B|}\sigma(a, P_i^{B,j})\end{aligned}$$

independently of the specific family $\{\beta_B\}$ of tie break probabilities. A priori success under tie breaking method $\{\beta_B\}$ hence equals a priori success under lexicographic tie breaking $\{\beta_B^{\text{lex.}}\}$, which is $\mathbb{E}[\sigma(r|\mathbf{w}(\mathbf{P}), P_i)]$. ■

Appendix B: Robustness to Strategic Voting

The main analysis has assumed voters to express their preferences without strategic misrepresentation. From a theoretical point of view, this is restrictive: unless some voter is a dictator, the preference profiles for which sincere voting is a Nash equilibrium are a strict subset of preference domain $\mathcal{P}(A)^n$ (cf. Gibbard 1973 and Satterthwaite 1975).[28]

Strategic voting is arguably less problematic in practice than in theory. It requires information about other voters' preferences that is often unavailable or difficult to obtain. Manipulation attempts can have negative reputation effects; they may fail or even backfire. It can also be computationally expensive for a voter to evaluate which outcomes are achievable through which preference misrepresentation. This holds already if everyone else votes sincerely[29] and further complexity is added if, potentially, other voters misrepresent their preferences too. The subjects in experiments by van der Straeten, Laslier, Sauger and Blais (2010) voted strategically only if the required computations were elementary. Other authors made similar observations (see, e.g., Kube and Puppe 2009, Groseclose and Milyo 2010, Pons and Tricaud 2018, Abeler, Nosenzo and Raymond 2019 or Baujard and Lebon 2022). Even from a theory perspective it is not clear if the assumptions for a particular strategic voting equilibrium are less restrictive than for sincere voting: players must be aware of the possibility to manipulate; their costs of exercising this option must be small; and they must somehow come to correctly anticipate their adversaries' strategies even when there are many alternative equilibria.

It is nonetheless worthwhile to assess the robustness of our success comparisons with respect to strategic voting. The key difficulty in doing this is non-uniqueness of equilibrium. If, for instance, our stylized shareholders with weights $\mathbf{w} = (45\%, 35\%, 20\%)$ choose between candidates $A = \{a, b, c\}$ and have sincere preferences $\mathbf{P} = (acb, bca, cba)$, there exist 40 equilibria in pure strategies under r^P and r^{PR}, 39 under r^C and 14 under r^B.[30] If we eliminate weakly dominated strategies

[28] For instance, in Table 3's example with $\mathbf{w} = (45\%, 35\%, 20\%)$, at least one voter has an incentive to misreport her preferences for 36 (24, 24, 72) out of the 216 preference profiles for r^P (r^{PR}, r^C, r^B).

[29] Weighted votes using r^B and r^{PR} are NP-hard to manipulate for three or more alternatives, r^C for at least four alternatives. The manipulation problem has polynomial complexity for any number of alternatives only for r^P. See the survey by Conitzer and Walsh (2016).

[30] The numbers refer to a normal-form game with strategy sets $S_i = \{abc, \ldots, cba\}$ in which each player $i \in N$ has complete information about r and $\mathbf{P}$. For instance, strategy $s_1 = cab$ by voter 1 in a game where $P_1 = acb$ means that 1 acts like a sincere voter with preferences cab: under r^{PR}, 1 first

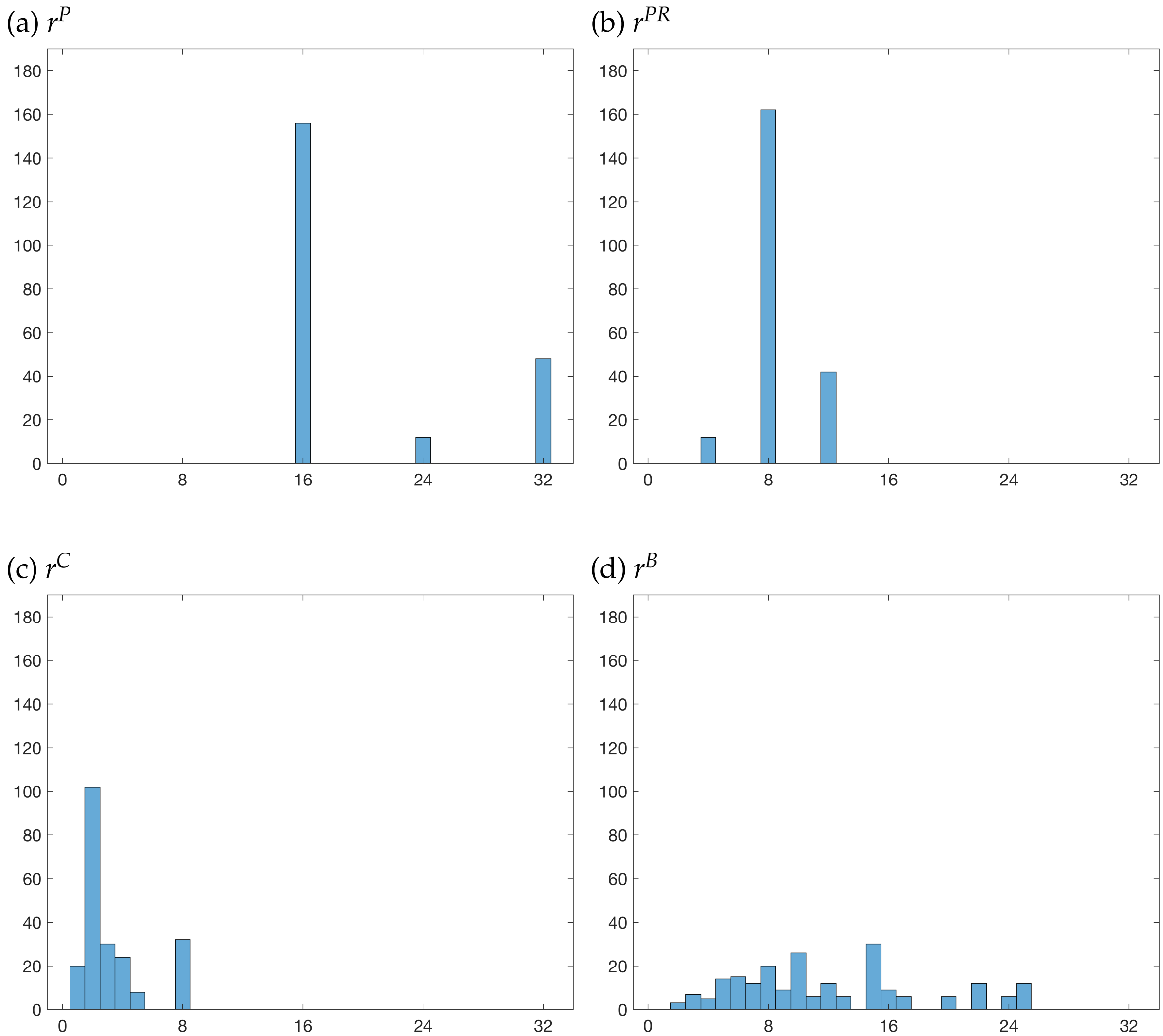


Figure A-1: Distribution of the number of Nash equilibria in undominated pure strategies for $\mathbf{w} = (45\%, 35\%, 20\%)$, $m = 3$ and $\mathbf{P} \in \mathcal{P}(A)^n$

(see Farquharson 1969), there are still 16, 8, 3 and 6 equilibria to choose from. Figure A-1 reports the distribution of the numbers of corresponding equilibria across the $(3!)^3 = 216$ preference profiles $\mathbf{P}$. For most $\mathbf{P}$, there are many equilibrium strategy choices to select from as a theorist – and to coordinate between as actual voters. Success under strategic voting is highly contingent on one's selection strategy and coordination skills.

This holds for weight distributions other than $\mathbf{w} = (45\%, 35\%, 20\%)$, too. We

votes for c and then for c (a) if there is a runoff (not) involving option c. This would, e.g., be better for 1 than the sincere strategy $s_1^\circ = acb$ if 2 and 3 play $s_2 = bca$ and $s_3 = cba$. We disregard mixed-strategy equilibria because they have weak foundations and require an extension of $\mathbf{P}$ to lotteries.

have computed the sets of pure-strategy Nash equilibria for all profiles $\mathbf{P}$ for $n = 3$ players, $m = 3$ options and all non-dictatorial weight equivalence classes of r^P, r^{PR}, r^C and r^B. We checked in each case if (i) sincere voting is a Nash equilibrium or if (ii) the winning alternative is the same as under sincere voting in some equilibrium involving undominated strategies. The results are summarized at the end of this Appendix in Tables A-3 to A-4.[31] The share of profiles $\mathbf{P}$ in which the sincere voting outcome $r|\mathbf{w}(\mathbf{P})$, and hence success $\sigma(r|\mathbf{w}(\mathbf{P}), P_i)$, are in sense (i) or (ii) consistent with strategic voters often exceeds 90%. It falls below 80% only for what essentially are 2-player tie-breaking games. Hence, the conclusions from the analysis of sincere voting are unlikely to be far off.

This can be made more precise by, for instance, adopting a cost of lying-based selection criterion and then evaluating individual success in the corresponding equilibrium outcomes. Specifically, among multiple strategic voting equilibria, let us select the one that requires the lowest number of pairwise preference misrepresentations, i.e., we identify the equilibrium strategy profile $\mathbf{s}^* \in \mathcal{P}(A)^n$ with minimal Kemeny distance to the sincere profile $\mathbf{s}^\circ = \mathbf{P}$. Then we compute player i's success $\sigma(r|\mathbf{w}(\mathbf{s}^*), P_i)$ at $\mathbf{P}$ and obtain the respective a priori success $TCI_i^*(\cdot)$ and $ARI_i^*(\cdot)$ under strategic voting by taking expectations with respect to $\mathbf{P}$.[32]

Figure A-2 shows the corresponding maximizers of $TCI_1^*(\cdot)$ and $ARI_1^*(\cdot)$ for strategic voters under the IC and IAC benchmark distributions for $n = m = 3$ in direct analogy to the results for sincere voters in Figure 6. There are visible differences. However, in most cases, at least one of the previously success-maximizing rules is still a success maximizer. Strategic voting thus helps to select between rules that promise equal a priori success under sincere voting. A few areas where r^C is the maximizer under sincere voting but now r^{PR} maximizes player 1's success, and vice versa, represent exceptions to this. Furthermore, in some cases where player 1 is a dummy under r^C, r^P and r^{PR} but not under r^B (i.e., $w_j \in (50\%, 66.67\%)$ for $j = 2$ or 3), Borda rule *minimizes* player 1's success (cf. the middle blue parts inside the mainly cyan colored area). The latter is caused by Pareto-inefficient equilibria under r^B, which reduce player 1's success below the dummy level guaranteed by r^C, r^P and r^{PR}. This once more highlights the equilibrium selection problem.

[31] We indicate each equivalence class by an integer weight distribution with minimum sum. Our example $\mathbf{w} = (45\%, 35\%, 20\%)$ is equivalent to $\mathbf{w}' = [3, 2, 2]$ under r^P, to $\mathbf{w}' = [2, 2, 1]$ under r^{PR}, to $\mathbf{w}' = [1, 1, 1]$ under r^C and to $\mathbf{w}' = [5, 4, 2]$ under r^B (cf. Kurz et al. 2020).

[32] If several equilibrium strategy profiles $\mathbf{s}^*$ minimize the Kemeny distance to $\mathbf{P}$, we pick at random and evaluate the expectation of $\sigma(r|\mathbf{w}(\mathbf{s}^*), P_i)$. For combinations of $r|\mathbf{w}$ and $\mathbf{P}$ where no pure strategy equilibrium exists, we maintain the sincere voting assumption.

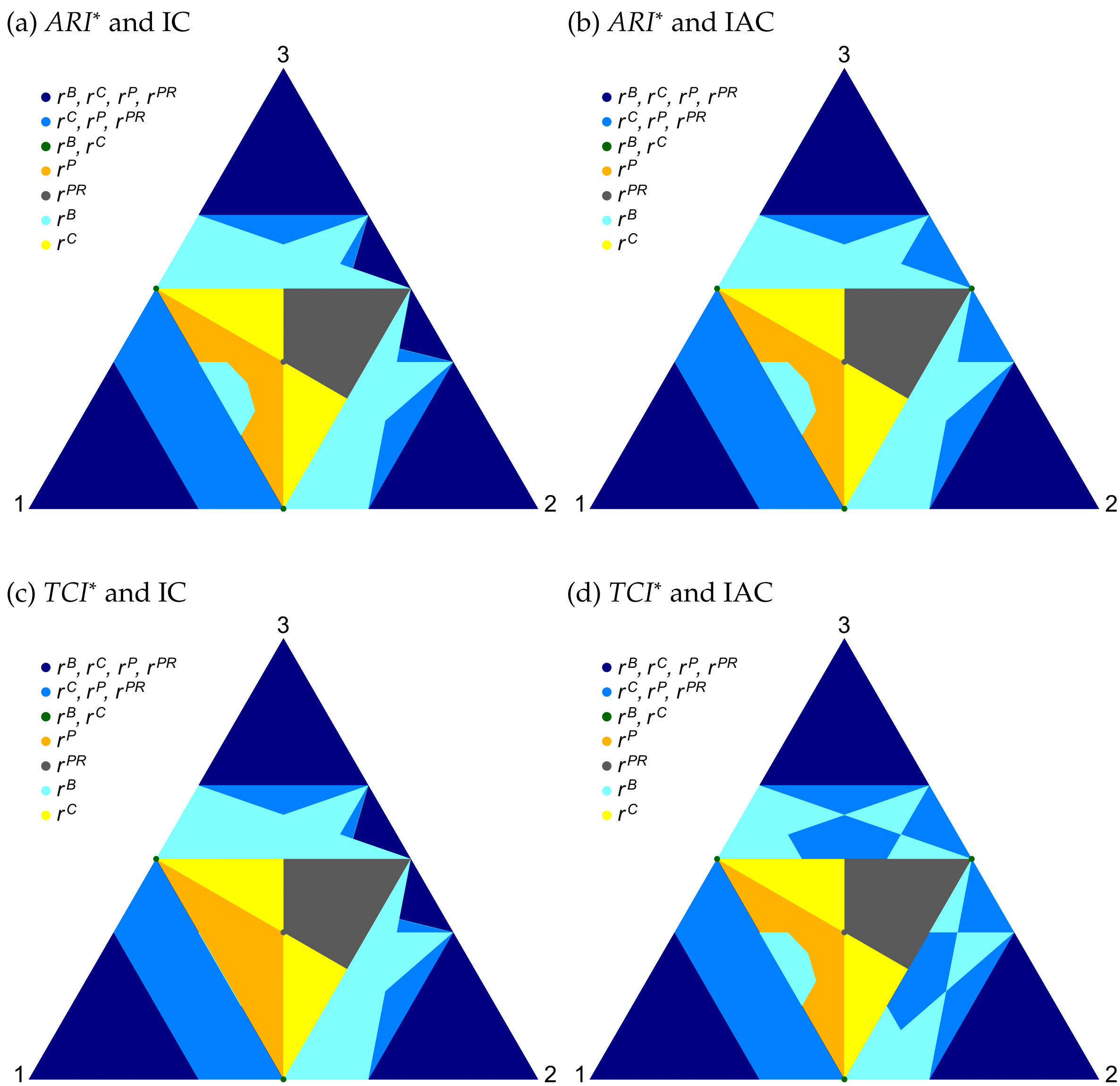


Figure A-2: Maximizers of ARI_1^* and TCI_1^* in strategic voting equilibria with minimal Kemeny distance to **P** when $n = m = 3$

As a complementing robustness check, we considered voting under uncertainty (see, e.g., Majumdar and Sen 2004, Ángel Ballester and Rey-Biel 2009 or Lu, Tang, Procaccia and Boutilier 2012) and verified if a sincere vote would maximize the expected utility of a voter with $u_i(a) = \tilde{s}(a, P_i)$ who – lacking better information – assumes that preferences of the others are independent, distributed uniformly and expressed truthfully.[33] We checked for each voter i with any fixed preferences P_i if, writing $\mathbf{P}_{-i} = (P_1, \ldots, P_{i-1}, P_{i+1}, \ldots, P_n)$,

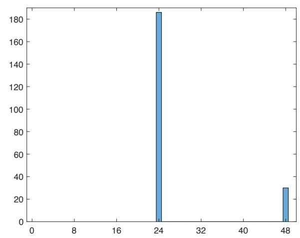
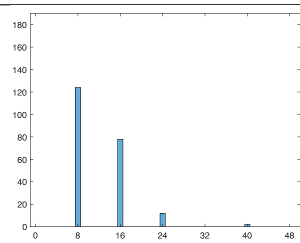
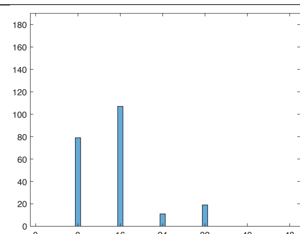
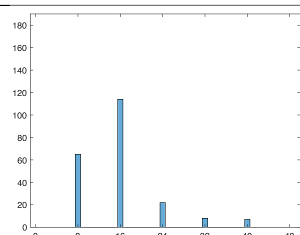
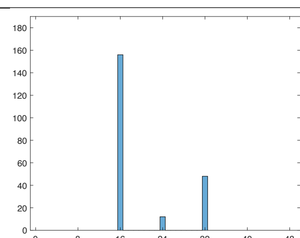
$$\sum_{\mathbf{P}_{-i} \in \mathcal{P}(A)^{n-1}} \Pr(\mathbf{P}_{-i}) \cdot \tilde{s}(r|\mathbf{w}(P_i, \mathbf{P}_{-i}), P_i) \geq \sum_{\mathbf{P}_{-i} \in \mathcal{P}(A)^{n-1}} \Pr(\mathbf{P}_{-i}) \cdot \tilde{s}(r|\mathbf{w}(P'_i, \mathbf{P}_{-i}), P_i) \quad \forall P'_i \neq P_i. \tag{17}$$

This inequality happens to hold for all voters i and preferences P_i in all plurality and plurality runoff equivalence classes when $n = m = 3$. It is also satisfied for 42 out of the 51 Borda equivalence classes and for 3 out of the 4 Copeland classes, where the remaining 9 and 1 represent non-generic distributions (i.e., locally isolated points or lines in the simplex).[34] In other words: sincere voting is typically the best strategy for a voter who applies the principle of insufficient reason to the unknown actions of other players. If we take a virtual walk through the simplex in Figure 1 and enumerate all $(1\,500 + 2)!/(1\,500! \cdot 2!) = 1\,127\,251$ games with non-negative integer voting weights $\mathbf{w} = (w_1, w_2, w_3)$ such that $w_1 + w_2 + w_3 = 1\,500$, the proportion of games where sincere voting is optimal in this sense – i.e., strategy $s_i = P_i$ satisfies inequality (17) for all i and P_i – evaluates to either $\approx 99\%$ or 100% under r^P, r^{PR}, r^C and r^B for both $m = 3$ or 4 alternatives.

[33] This is a special case of the model by Majumdar and Sen (2004). Knightian preference uncertainty was earlier considered by Moulin (1981).

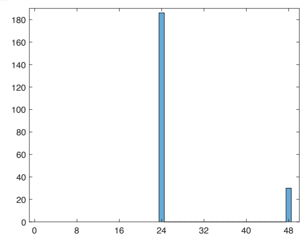
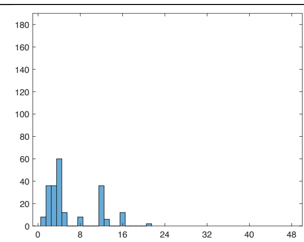
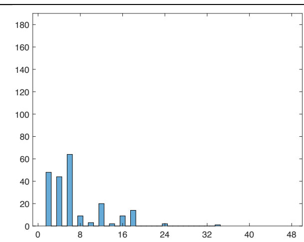
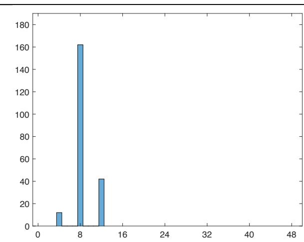
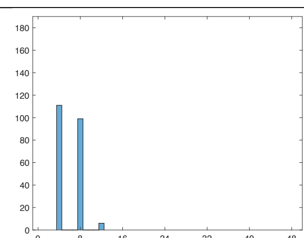
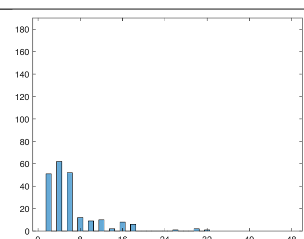
[34] For $m = 4$ options, deviating from truthful voting does not pay in 3 of the 6 plurality classes, 4 of the 7 plurality runoff classes, 3 of the 4 Copeland classes and 354 of the 505 Borda classes.

Table A-1: Number and properties of undominated Nash equilibria under plurality rule

$r^P \mid \mathbf{w}$	Distribution of # NE for $\mathbf{P} \in \mathcal{P}(A)^n$	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		210	6	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[1,1,1]		178	38	0	180/216 ≈ 0.83	180/216 ≈ 0.83
[2,1,1]		197	19	0	192/216 ≈ 0.89	196/216 ≈ 0.91
[2,2,1]		148	68	0	180/216 ≈ 0.83	188/216 ≈ 0.87
[3,2,2]		132	84	0	180/216 ≈ 0.83	204/216 ≈ 0.94

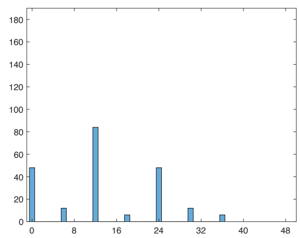
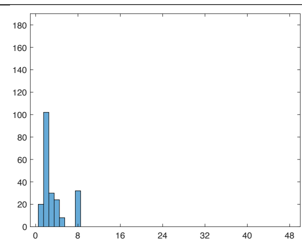
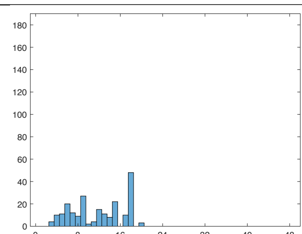
Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all five non-dictatorial r^P equivalence classes for $n = m = 3$ and all $\mathbf{P} \in \mathcal{P}(A)^3$.

Table A-2: Number and properties of undom. Nash equilibria under plurality runoff rule

$r^{PR}\|\mathbf{w}$	Distribution of # NE for $\mathbf{P} \in \mathcal{P}(A)^n$	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		210	6	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[1,1,1]		196	20	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[2,1,1]		194	22	0	198/216 ≈ 0.92	202/216 ≈ 0.94
[2,2,1]		186	30	0	192/216 ≈ 0.89	192/216 ≈ 0.89
[3,2,1]		208	8	0	196/216 ≈ 0.91	196/216 ≈ 0.91
[3,2,2]		185	31	0	192/216 ≈ 0.89	196/216 ≈ 0.91

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all six non-dictatorial r^{PR} equivalence classes for $n = m = 3$ and all $\mathbf{P} \in \mathcal{P}(A)^3$.

Table A-3: Number and properties of undominated Nash equilibria under Copeland rule

$r^C\|\mathbf{w}$	Distribution of # NE for $\mathbf{P} \in \mathcal{P}(A)^n$	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]		168	0	0	132/216 ≈ 0.61	156/216 ≈ 0.72
[1,1,1]		178	38	0	192/216 ≈ 0.89	204/216 ≈ 0.94
[2,1,1]		149	67	0	167/216 ≈ 0.77	210/216 ≈ 0.97

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all three non-dictatorial r^C equivalence classes for $n = m = 3$ and all possible preference configurations $\mathbf{P} \in \mathcal{P}(A)^3$. The outcome distribution does not sum up to 216 for $\mathbf{w} = [1, 1, 0]$ because there is no pure NE for 48 profiles of sincere preferences.

Table A-4: Number and properties of undom. Nash equilibria under Borda rule

$r^B\|\mathbf{w}$	Average # NE	# different NE outcomes for $\mathbf{P} \in \mathcal{P}(A)^n$			Share of $\mathbf{P}$ s.t. sincere voting is a NE	Share of $\mathbf{P}$ s.t. sincere outcome is a NE outcome
		1	2	3		
[1,1,0]	13.50	168	0	0	$132/216 \approx 0.61$	$156/216 \approx 0.72$
[1,1,1]	11.96	69	141	6	$165/216 \approx 0.76$	$216/216 = 1$
[2,1,0]	19.83	192	0	0	$162/216 \approx 0.75$	$186/216 \approx 0.86$
[2,1,1]	7.75	171	45	0	$176/216 \approx 0.81$	$208/216 \approx 0.96$
[2,2,1]	8.60	131	71	6	$144/216 \approx 0.67$	$198/216 \approx 0.92$
[3,1,1]	13.72	204	12	0	$178/216 \approx 0.82$	$208/216 \approx 0.96$
[3,2,0]	18.00	144	0	0	$108/216 \approx 0.50$	$144/216 \approx 0.67$
[3,2,1]	9.09	169	47	0	$161/216 \approx 0.75$	$208/216 \approx 0.96$
[4,1,1]	9.22	216	0	0	$201/216 \approx 0.93$	$213/216 \approx 0.99$
[3,2,2]	9.96	139	75	2	$169/216 \approx 0.78$	$204/216 \approx 0.94$
[3,3,1]	4.06	156	12	0	$132/216 \approx 0.61$	$168/216 \approx 0.78$
[4,2,1]	8.54	194	18	0	$168/216 \approx 0.78$	$203/216 \approx 0.94$
[3,3,2]	12.53	66	138	12	$156/216 \approx 0.72$	$216/216 = 1$
[4,3,1]	10.06	179	35	0	$138/216 \approx 0.64$	$196/216 \approx 0.91$
[5,2,1]	9.72	212	0	0	$174/216 \approx 0.81$	$202/216 \approx 0.94$
[4,3,2]	7.38	147	65	0	$155/216 \approx 0.72$	$201/216 \approx 0.93$
[5,2,2]	12.06	180	36	0	$174/216 \approx 0.81$	$216/216 = 1$
[5,3,1]	11.50	200	16	0	$150/216 \approx 0.69$	$198/216 \approx 0.92$
[4,3,3]	13.56	66	138	12	$174/216 \approx 0.81$	$216/216 = 1$
[5,4,1]	7.41	174	22	0	$130/216 \approx 0.60$	$180/216 \approx 0.83$
[6,3,1]	4.50	192	0	0	$162/216 \approx 0.75$	$186/216 \approx 0.86$
[5,3,3]	6.56	132	84	0	$168/216 \approx 0.78$	$210/216 \approx 0.97$
[5,4,2]	12.06	126	90	0	$144/216 \approx 0.67$	$207/216 \approx 0.96$
[6,4,1]	7.56	186	10	0	$123/216 \approx 0.57$	$176/216 \approx 0.81$
[7,2,2]	15.50	216	0	0	$186/216 \approx 0.86$	$210/216 \approx 0.97$
[5,4,3]	12.24	100	110	6	$159/216 \approx 0.74$	$210/216 \approx 0.97$
[7,4,1]	7.78	196	0	0	$144/216 \approx 0.67$	$182/216 \approx 0.84$
[6,5,2]	10.07	156	51	0	$138/216 \approx 0.64$	$195/216 \approx 0.90$
[7,5,1]	6.82	163	9	0	$114/216 \approx 0.53$	$160/216 \approx 0.74$
[6,5,3]	12.06	126	90	0	$144/216 \approx 0.67$	$204/216 \approx 0.94$
[7,5,2]	10.60	170	46	0	$145/216 \approx 0.67$	$203/216 \approx 0.94$
[8,5,1]	7.75	172	0	0	$117/216 \approx 0.54$	$161/216 \approx 0.75$
[6,5,4]	12.58	76	130	10	$165/216 \approx 0.76$	$214/216 \approx 0.99$
[7,5,3]	9.21	132	84	0	$156/216 \approx 0.72$	$208/216 \approx 0.96$
[7,6,2]	6.00	163	17	0	$132/216 \approx 0.61$	$174/216 \approx 0.81$
[8,5,2]	10.94	186	30	0	$156/216 \approx 0.72$	$207/216 \approx 0.96$
[7,5,4]	6.08	162	42	0	$165/216 \approx 0.76$	$197/216 \approx 0.91$
[7,6,4]	12.29	93	116	7	$150/216 \approx 0.69$	$212/216 \approx 0.98$
[8,6,3]	12.06	126	90	0	$144/216 \approx 0.67$	$210/216 \approx 0.97$
[9,6,2]	9.83	192	24	0	$138/216 \approx 0.64$	$198/216 \approx 0.92$
[8,7,3]	8.18	151	48	0	$138/216 \approx 0.64$	$189/216 \approx 0.88$
[8,6,5]	9.35	133	74	5	$168/216 \approx 0.78$	$204/216 \approx 0.94$
[10,7,2]	9.13	187	21	0	$129/216 \approx 0.60$	$188/216 \approx 0.87$
[11,7,2]	10.17	196	12	0	$132/216 \approx 0.61$	$187/216 \approx 0.87$
[9,7,5]	8.10	149	55	0	$153/216 \approx 0.71$	$193/216 \approx 0.89$
[10,8,3]	10.06	158	54	0	$138/216 \approx 0.64$	$198/216 \approx 0.92$
[11,8,2]	8.42	174	18	0	$120/216 \approx 0.56$	$174/216 \approx 0.81$
[11,9,3]	7.94	168	24	0	$132/216 \approx 0.61$	$180/216 \approx 0.83$
[13,8,2]	10.50	192	0	0	$126/216 \approx 0.58$	$174/216 \approx 0.81$
[12,9,7]	4.83	180	12	0	$162/216 \approx 0.75$	$186/216 \approx 0.86$

Note: This table reports the number and properties of Nash equilibria (NE) in undominated pure strategies for all 50 non-dictatorial r^B equivalence classes for $n = m = 3$ and all $\mathbf{P} \in \mathcal{P}(A)^3$. If the outcome distribution does not sum up to 216, there are some $\mathbf{P}$ for which no pure NE exists.

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