Relative Responsibility for Cartel Damages

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Abstract

Cartel members are liable jointly and severally: any infringing firm may be litigated and forced to compensate victims on behalf of all. EU law stipulates that co-infringers must pay internal redress in proportion to "relative responsibility for the harm caused". We suggest to quantify this by invoking basic proportioning axioms and requiring that redress payments reflect causal links between actions and damages. This calls for application of the Shapley value. We prove that even symmetric firms may bear unequal responsibility for individual harm, characterize proportionings for linear market environments, and show that market shares typically fail to reflect relative responsibilities.

Keywords: collusion; damage proportioning; Shapley value; relative responsibility; joint liability

JEL codes: L40; C71; D04; K42

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1. Introduction

Cartel victims have a right to compensation but the pertinent legal hurdles are high. Annually up to 23.3 billion euro of damages used to remain unclaimed from EU-wide cartels according to the European Commission (SWD/2013/203/Final, recital 67). In 2014, this was key motivation for the Commission to advance the *Directive on Antitrust Damages Actions* 2014/104/EU, also known as *Damages Directive*. The position of plaintiffs has since improved and several big cases are pending.

Two provisions for the compensation of cartel victims in the Directive motivate this study. First, the members of a cartel are liable *jointly and severally*: an injured party can litigate any cartel member for the full amount of its damages; if courts confirm the claim, the defendant must compensate the plaintiff on behalf of the entire cartel. This is regardless of whether the plaintiff made its purchases from the sued firm or other ones. Similar provisions apply in Australia, Japan, the UK, or the US.

Second, the sued cartel member is later entitled to *internal redress*. Such a rule of contribution existed in EU member states before (incl. the UK) but details differed. It contrasts with the no contribution rule in federal US antitrust cases (cf. Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981), and intermediate arrangements exist elsewhere.

According to the Directive, cartelists' internal obligations in compensating any external claimant must reflect *"... their relative responsibility for the harm caused by the infringement of competition law"* (Article 11(5)). The Directive is not specific on how this should be operationalized. Our goal is to quantify relative responsibility for cartel damages in an economically sound way.[2]

The analysis focuses on the assessment of economic damage contributions, but is based on the canonical causal conception of legal and moral responsibility for harm. Feinberg (1970, p. 195f) provides a lucid discussion of its three parts: firstly, the defendant was at fault in acting. This clearly applies if, for instance, firm i's manager illegally coordinated its production of some commodity with competitors over dinner, violating antitrust laws. Secondly, the faulty act caused the harm: these conversations resulted in a price increase for the customer. And, finally, the faulty aspects of the act were relevant to its causal connection to the harm: illegal coordination by the managers – not, perhaps, just the reaction of commodity investors to observing the meeting – caused the increase. All three parts call for appropriate verification in practical applications.

After this, a systematic approach is warranted to determine each firm's contribution to a given harm. Cost or product asymmetries of cartel members

[2]The issue of how alternative norms, such as the no contribution rule in the US, affect incentives for cartel formation, whistleblowing, settlements, etc. is left aside. See, for instance, Landes and Posner (1980) or Hviid and Medvedev (2010).

can translate very differently into asymmetric turnover, revenues, or profits. So, the simple idea to proportion a compensation payment by market shares involves a high degree of arbitrariness. Sound methods should reflect considerations of the following kind: first, a firm has responsibility and should contribute to compensating a given customer only if this customer's damages would have been lower had the firm refused to participate in the cartel. How much lower the respective damage would have been if the firm had stayed legal (and then possibly some others, too) ought to, second, determine the level of the contribution. Third, if cartel membership of two firms had identical effects on harm, both should contribute the same. Finally, a victim's compensation should be proportioned in a way that neither depends on the unit of account nor on whether multiple damages are dealt with separately or jointly, whether interest has accumulated, etc.

These properties translate into mathematical conditions that are well-known in cooperative game theory as the null player, marginality, symmetry, efficiency and linearity axioms. Classical results by Shapley (1953a) and Young (1985) then imply that the Shapley value of an appropriately defined game provides the best way to a posteriori split a compensation payment according to the relative responsibility of the co-infringers. The Shapley value is an accepted tool for allocating costs or profits in joint ventures.[3] Its use for the division of cartel damages was initially proposed by Schwalbe (2013) and Napel and Oldehaver (2015) to law audiences. We are the first to analyze the pertinent quantitative aspects.

The key feature of Shapley proportionings is that they impute individual responsibility from the ability to influence prices. We revisit the underlying reasoning and a little-known formula that can simplify computations (Section 3) after detailing the problem at hand (Section 2). We prove that even firms with identical technologies and symmetric market positions generally have asymmetric obligations with respect to individual claimants (Section 4). For linear market environments, we derive how Shapley proportionings are linked to demand and cost parameters and deduce useful bounds (Section 5). In Section 6, we compare Shapley allocations to ad hoc proportioning based on sales, profits, or a flat per head assignment. We find that corresponding suggestions by legal practitioners mostly clash with proportioning by relative responsibility.

2. Cartel Damages and Relative Responsibility

Cartel customers usually suffer two kinds of damage. First, there is the visible loss: each unit that was purchased involved an overcharge ('damnum

[3]See, e.g., Shubik (1962), Hart and Moore (1990), Young (1994), or Moretti and Patrone (2008). Dehez and Ferey (2013, 2016) and Huettner and Karos (2017) apply it to sequential liability problems, Lando and Schweizer (2021) to negligence of multiple injurers.

emergens'). Second, customers who would have made (additional) purchases and enjoyed surplus on these if prices had been competitive, failed to do so. Such deprived gains ('lucrum cessans') are acknowledged by legal theory and the Damages Directive explicitly calls for compensation that "*...shall place a person who has suffered harm in the position in which that person would have been had the infringement of competition law not been committed*" (Article 3(2)). This also includes interest on any compensation that is due for overcharges and forgone surplus.

The information burden of proving deprived gains from reduced purchase volume is high, however, and compounded by procedural tactics when the litigant used cartel goods as a production or retail input.[4] Confirming the observation by Argenton et al. (2020, recital 6.131), Weber (2021, p. 455) concludes in a recent survey that "[i]n contrast with economics and also not in line with the Directive's goal of full compensation, the volume effect has not so far received any major attention in cartel cases." Our analysis will hence focus mostly on overcharge damages caused by a hardcore cartel. If a victim should be granted compensation also for deprived gains in future cases then responsibility-based internal redress for the respective payment follows in direct analogy to the computations below.[5] The suggested approach could moreover be adapted also to other collective antitrust infringements (e.g., collusion aimed at excluding rivals; cf. Argenton 2019).

A cartel member i having responsibility for damages of a given claimant k requires that k's damages are causally linked to i's cartel membership, i.e., their scale, scope or distribution would have differed without i's illegal action. Identifying the causal links between anticompetitive conduct and harm is generally difficult (see, e.g., Lianos 2015). What makes analysis of responsibility particularly interesting, though, is that even symmetric cost and demand structures may generate asymmetric links to the harm of a specific victim. Namely, price effects of cartel membership differ across cartelists as long as own-price and cross-price elasticities of the respective demands

[4] Asserting that fewer units were purchased and sold to the next level of the value chain is inconsistent with arguing that the claimant's own sales prices were unaffected by the infringement. Admitting that part of the overcharge on purchased units was passed on reduces the respective overcharge compensation and can risk acknowledgment of harm altogether. Even for retail customers, where the passing-on defense is moot, the benefits to claiming deprived gains are likely to be second-order because of substitution effects: realized surplus from purchased units of non-cartel substitutes reduces forgone surplus from non-purchased units.

[5] Linearity of the Shapley value allows to proportion applicable compensation payments – regardless of whether they reflect damnum emergens, lucrum cessans, or interest – separately or jointly. This also pertains to damages that accrued over multiple time periods. For instance, harm is often quantified by monthly but-for estimations (e.g., Bernheim 2002) and so might responsibility if membership, demand, or costs varied over time.

differ. Firms that are symmetric players in the pertinent market model can be non-symmetric players in the proportioning problems that derive from the compensation of individual customers.

For illustration consider n otherwise identical firms on a Salop circle. Think of cement plants that are equally spaced on the shores of an unshippable lake. They sold cement at inflated prices to local construction companies around the lake. The cartel was busted and a customer of firm h sues. The relative responsibilities of this customer's home firm h and of a distant firm j are tied to the counterfactual price that the customer would have paid had h or respectively j refused to participate. Unless transportation costs are zero, cartel membership of the northernmost vendor has smaller effect on overcharges faced by customers in the south than does membership of southern vendors, and vice versa (see, e.g., Levy and Reitzes 1992). The more intensely two firms would have competed in the absence of the cartel, the greater the effect of their collusion. The counterfactual prices that the customer would have paid if h or if j had not joined the cartel, but just best-responded to its practices, thus vary by location. And so do their responsibilities for the customer's harm.

Of course, a symmetric market structure implies that obligations which h and j have in compensating each others' customers are the same. So, mutual redress claims cancel if all constructors sue (or a uniform measure of them). However, they do *not* cancel in almost all other situations – e.g., if just some construction companies in the south go to court. A general proportioning method hence requires that responsibility can be attributed to cartel members for the price increase on each single product in the cartel's portfolio. Asymmetries in cost or demand make this even more important.

The described problem of proportioning compensation for cartel damages extends to customers of cartel outsiders who best-responded to the infringement. The respective *umbrella losses*[6] are legally acknowledged in the EU (CJEU C-557/12 2014) and have also been claimed successfully before several US courts. Their compensation is not linked to transactions with any cartelist but still a joint liability. Moreover, a firm's relative responsibility stays relevant if litigants settle with some infringing firm: claims against co-defendants are reduced by the settling defendant's responsibility for harm (Damages Directive, recital 51).[7]

[6]See Inderst et al. (2014), Holler and Schinkel (2017) and Napel and Welter (2022) on the theory of umbrella losses.

[7]The Shapley value φ has the attractive consistency property that it induces the same firm-specific payments φ_i irrespective of whether φ is applied to the original setting involving all co-infringers or a reduced one where some firms have already contributed their due shares. Respective reduced games $(T, \tilde{v}_T)$ for any proportioning method Φ involve the subset T of firms that are left. $\tilde{v}_T(T)$ equals the original damage $v(N)$ minus the payments $\Phi_j(N, v)$ of firms $j \in \overline{T} := N \setminus T$. Remaining counterfactual damages $\tilde{v}_T(S)$ for $S \subset T$, required for assessing responsibility of $i \in T$, follow by subtracting due shares of infringers $j \in \overline{T}$ from the

3. The Shapley Value as a Tool for Proportioning Damages

A damage proportioning problem will in the following be formalized as a cooperative game with transferable utility, denoted (N, v), in which a compensation payment $v(N)$ that was forced on some litigated cartel member j is to be divided among all jointly liable members $N = \{1, \ldots, n\} \ni j$ according to their relative responsibility for harm.

A court's verdict that the infringers must pay $v(N)$ to the respective claimant presupposes an assessment of the claimant's position that would have prevailed without the infringement. It hinges on the presumed competitive market benchmark and rests on various assumptions about the relevant players in this market, their strategy spaces, information structure, objectives, and an appropriate way to estimate counterfactual but-for outcomes from these data. In the words of the Damages Directive "*...quantifying harm means assessing how the market in question would have evolved had there been no infringement. This assessment implies a comparison with a situation which is by definition hypothetical ...*" (recital 46).

Estimating but-for market outcomes and a victim's damages can be difficult tasks, which have already been completed when a payment $v(N)$ is made and the litigated cartel member later seeks internal redress in line with Article 11(5). One needs to dive more deeply into counterfactual situations when relative responsibilities for the diagnosed damage are quantified: responsibility arises because the damage would have differed if some firms had refused to collude and responsibility is greater, the smaller the corresponding damage would have been. So, estimates are needed of how the market and a victim's associated losses $v(S)$ would have evolved if only a subset or – in the language of cooperative game theory – a coalition $S \subset N$ of the actual cartel members had participated in the infringement, while others had complied with antitrust laws. The logic of comparing actual to hypothetical outcomes thus extends from the task of quantifying harm to that of quantifying contributions to harm. Naturally, $v(S) = 0$ if the set S of collaborators had been empty or comprised but a single firm, i.e., if $|S| = 1$.

The more firmly a litigant's compensation $v(N)$ is grounded on a structural model of how choices of the relevant players determine market outcomes, the better damage estimates $v(S)$ can be obtained also for sub-cartels $S \subset N$. Corresponding market simulation analysis is well-established in merger control (cf. Budzinski and Ruhmer 2010). There, parameters of a price or quantity competition model are estimated based on pre-merger observables. They generate equilibrium predictions that simulate what happens if a subset of firms (or some of their subsidiaries) merge and internalize profit externalities,

original counterfactual damages, i.e. $\tilde{v}_T(S) = v(S \cup \overline{T}) - \sum_{j \in \overline{T}} \varphi_j(S \cup \overline{T}, v)$. φ is not unique to satisfy $\Phi_i(T, \tilde{v}_T) = \Phi_i(N, v)$ for all $T \subseteq N$ but is the only such method that is also efficient and symmetric (Hart and Mas-Colell 1989).

similar to a cartel. This permits to assess welfare in respective merger scenarios a priori. Analogous a posteriori analysis of cartels is rarer but exists (see de Roos 2006). Calibrations can draw not only on pre-cartel (like pre-merger) observables but also on observations during and after a cartel's operation. Former members may have an incentive to disclose information if they expect lower contributions than under ad hoc proportioning of $v(N)$.

We take no stance on how sophisticated estimates $v(S)$ ought to be in practice, except that the underlying model should take up or at least be consistent with the assessment by the court that determined compensation $v(N)$. The usual trade-offs between attention to detail and tractability apply. For instance, many cartels wielded market shares below 100%. The set M of players in the pertinent market model then constitutes a superset of the set N of infringing firms, who share responsibility and are the only players in cooperative game (N, v). Prices, quantities or other variables may have been coordinated for multiple products per firm and in various sub-markets, so that compensation $v(N)$ may reflect supracompetitive prices for goods from a set $\mathcal{M}$ with far greater cardinality than M.

Ideally, a fully specified model would predict prices for all goods $j \in \mathcal{M}$ and – if $v(N)$ should go beyond the overcharges on documented purchases and interest – also the hypothetical quantities demanded at these prices by the victim in question. Such an ideal model would reflect appropriate assumptions about players' strategic options and objectives (e.g., transaction costs and stability requirements for collusion; whether cartel members can make side-payments; if they maximize joint profit or follow other rationales, such as lowering quantities to increase individual profits by a fixed target percentage; etc.). It would also match the applicable distribution of information and timing of interaction (e.g., delays until exit of a cartel member affects prices; whether non-members move simultaneously or as Stackelberg followers); whether the agents who collude – sometimes rather mid-level managers in their firms – internalize the fines and compensation payments that will apply to their principals in case of detection; possible bias in their assessments; and so forth.

The illustrations below will keep things comparatively simple. There are reasons, however, to expect that even simpler modeling can still improve on naïve proportioning by market shares.[8] We will below restrict attention to the benchmark where firms $N = \{1, \ldots, n\}$ have formed an industry-wide cartel. So, the set of players in the relevant market, M, coincides with the player set of cooperative game (N, v). We moreover let each firm i produce just a single good i and in Sections 4 – 6 focus on overcharge damages in line with legal practice and the related literature (cf. also Harrington 2004, Katsoulacos et al. 2020, and Laborde 2021). This entails some loss of generality but there are

[8]See Napel and Welter (2021) on the extent to which even *binary* approximations $\tilde{v}$ of an unknown characteristic function v can identify responsibility better than relative sales, revenues, etc.

mainly expositional hurdles to extending the derivations in Sections 4 – 6 to other settings.[9]

In the following, each number $v(S)$ with $i \notin S$ reflects a scenario for how the market and damages of the compensated victim might have evolved if there had been no infringement by firm i. It is both possible that firm $j \neq i$ would then have joined the cartel anyhow ($j \in S$) or that it would have stayed legal too ($j \notin S$). In assessing i's due share of $v(N)$, the weights that are put on these distinct scenarios – and the related ones concerning other cartel members – are a key aspect of any proportioning rule. They are intimately linked to the economic properties that this rule shall satisfy.

3.1. Desirable Properties of Responsibility-Based Allocations

With damages $v(N)$ in a factual cartel scenario and related counterfactuals $v(S)$, $S \subset N$, described by (N, v) based on a given market model, a *damage proportioning rule* is a mapping Φ from any conceivable cartel damage problem (N, v) to a vector $\Phi(N, v) \in \mathbb{R}^n$, i.e., it is a *value* of the corresponding cooperative game. The main restriction that the cartel context imposes is that $v(\{i\}) = 0$ for all $i \in N$. As prices of substitute goods are usually higher for bigger cartels, one can take v to be monotonic in S.[10] Component $\Phi_i(N, v)$ denotes the part of the compensation for damages $v(N)$ which cartel member $i \in N$ must contribute.

That a proportioning rule reflects relative responsibilities can be translated into three formal properties of Φ. The first one is straightforward. Suppose that participation or not of a particular firm i would never have made a difference to the damage in question. That is, removing player i if $i \in S$ or adding player i if $i \notin S$ does not change $v(S)$. Then given that i's conduct has no effect on damage, the canonical conditions for i being responsible are not met (see Feinberg 1970). Hence, no responsibility-based obligations to contribute follow. A player i for whom $v(S) = v(S \setminus \{i\})$ for every $S \subseteq N$ is known as a *null player*. The first requirement for rule Φ to be based on relative responsibility hence is the *null player property*:

$$\Phi_i(N, v) = 0 \text{ whenever } i \text{ is a null player in } (N, v). \tag{NUL}$$

Presumably, supply and demand conditions in real markets are rarely compatible with a convicted cartel member being a null player. But (NUL)

[9] If, for instance, each firm i produced $t > 1$ varieties, the strategy choices $y_i \in \mathbb{R}$ considered in Section 4 would need to be replaced by vectors $y_i = (y_i^1, \ldots, y_i^t) \in \mathbb{R}^t$. Then symmetry condition A1 would apply to these vectors; inequalities in A2 must hold strictly for each $i \neq j$ for at least one variety and weakly for all others; first order conditions in A3 pertain to all variety-specific derivatives.

[10] See Davidson and Deneckere (1984) and Deneckere and Davidson (1985). *Superadditivity* and *convexity* of v can be natural assumptions, too: to ensure that new members want to join a cartel and existing members accept them, v must be superadditive. For cartels that included all big players, incentives to join have plausibly increased in size (convexity).

conducts a valid thought experiment. It also formalizes a certain robustness to misspecification of the relevant market. For instance, a large cartel may have caused additive damages in several regions with independent costs and unrelated demand. If a firm is accidentally included as 'player' in a region where it had no role, (NUL) ensures it need not contribute there.

As responsibility derives from the causal links between cartel membership and the harm suffered, a second straightforward requirement is that i's damage share should be determined by these links – and these links alone. Namely, presuming that v correctly describes factual damages as well as the relevant counterfactuals, $\Phi_i(N, v)$ shall be a function only of i's *marginal contributions* $v(S) - v(S \setminus \{i\})$ in (N, v). The corresponding formal property of *marginality*, introduced by Young (1985), is

$$\Phi_i(N, v) = \Phi_i(N, v')$$
$$\text{whenever } v(S) - v(S \setminus \{i\}) = v'(S) - v'(S \setminus \{i\}) \text{ holds for all } S \subseteq N. \qquad \text{(MRG)}$$

Marginality does not pin down *how* $\Phi_i(N, v)$ should depend on the differences that i makes to various coalitions. For instance, (MRG) does not imply (NUL).

If contributions of two firms i and j to the generation of a victim's damages are the same, i.e., $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition $S \subseteq N \setminus \{i, j\}$, they are symmetric players in proportioning problem (N, v). As equal contributions to harm call for equal contributions to compensation, Φ should satisfy *symmetry*:[11]

$$\Phi_i(N, v) = \Phi_j(N, v) \text{ whenever } i \text{ and } j \text{ are symmetric in } (N, v). \qquad \text{(SYM)}$$

Irrespective of whether a division of damages reflects responsibility of the involved players or follows alternative principles, firms' contributions should add up to $v(N)$. In the context of cooperative games, this is called *efficiency* of a value:

$$\sum_{i \in N} \Phi_i(N, v) = v(N). \qquad \text{(EFF)}$$

Efficiency and symmetry imply $\Phi_1(N, v) = \Phi_2(N, v) = v(N)/2$ for $N = \{1, 2\}$ given $v(\{1\}) = v(\{2\}) = 0$, i.e., participants to any 2-firm cartel must contribute equally. This may at first seem counterintuitive when market shares, costs, or profits are asymmetric. But exit by either firm would have restored duopolistic competition. So, while overcharges and deprived gains may differ widely in absolute level across the products of a 2-firm cartel, both firms bear the same relative responsibility for them.[12]

[11] Deviations from symmetry may be necessary if firms played highly asymmetric roles in the organization of the cartel (e.g. as ringleader) or when leniency provisions provide liability exemptions. One can then turn to *weighted Shapley values* (Shapley 1953b; Kalai and Samet 1987).

[12] The 'it takes (at least) two to tango'-aspect of collusion that is reflected by $v(\{i\}) = 0$ for all $i \in N$ is a key distinction to other antitrust infringements as well as joint liability for collective negligence (e.g. Lando and Schweizer 2021). $v(\{i\}) > 0$ if one spark is enough to burn down

Firms' shares should not depend on whether damages are proportioned for one or many units of a single or multiple products, expressed in US dollar or euro, whether they are trebled, already include interest, etc. If the same cartel N caused harm to customers in several markets – reflected by a characteristic function v for market 1, v' for market 2, etc. – then the total contribution of firm $i \in N$ should not depend on whether the proportioning rule is applied to damages in one market at a time or simultaneously. Different 'markets' could here refer to different regions or time periods, different products in the cartel's portfolio, or just distinct quantities of the same product. These requirements amount to *linearity*:

$$\Phi(N, \lambda \cdot v + \lambda' \cdot v') = \lambda \cdot \Phi(N, v) + \lambda' \cdot \Phi(N, v') \tag{LIN}$$

for any scalars $\lambda, \lambda' \in \mathbb{R}$ and any characteristic functions v, v'.

3.2. *Shapley Value and Decomposition by Average Damage Increments*

Above properties imply that a responsibility-based proportioning method must lead to contributions $\Phi_i(N, v)$ that equal

$$\varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \left[v(S) - v(S \setminus \{i\})\right] \tag{1}$$

for each $i \in N$ and $s = |S|$. $\varphi(N, v)$ is the *Shapley value* of (N, v). Shapley (1953a) showed that any allocation rule that satisfies (NUL), (SYM), (EFF) and (LIN) is equivalent to φ. Young (1985) proved that the same is true if (MRG), (SYM) and (EFF) are satisfied.[13] Formula (1) may look unwieldy but weights $(s-1)!(n-s)!/n!$ on marginal contributions and, implicitly, the underlying counterfactual membership scenarios are a mathematical consequence of the desired properties.

It is little-known – but will below be very practical – that an equivalent way of writing eq. (1) is

$$\varphi_i(N, v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=1}^{n-1} \left[\bar{v}^i(s) - \bar{v}^{\not i}(s)\right] \tag{2}$$

where

$$\bar{v}^i(s) := \binom{n-1}{s-1}^{-1} \sum_{S \ni i, |S|=s} v(S)$$

captures the *average damages* caused by coalitions of size s which *include* firm i

the barn. Then $v(\{1\}) > v(\{2\})$ is possible and would call for $\Phi_1(\{1,2\}, v) > v(N)/2 > \Phi_2(\{1,2\}, v)$.

[13] See, e.g., Maschler et al. (2013, ch. 18). To verify that Shapley's uniqueness result extends to the class of damage proportioning problems, note that cartels in which $i \in T \subseteq N$ produce perfect substitutes with competitive price $p^* = 0$ and cartel price $p^c = 1$ while all $j \notin T$ operate in unrelated markets define the required carrier games (N, u_T).

and

$$\bar{v}^{\not\ni i}(s) := \binom{n-1}{s}^{-1} \sum_{S \not\ni i,\, |S|=s} v(S).$$

those which *exclude* firm i.[14] Abbreviating $\kappa(s) := (s-1)!(n-s)!/n! = \binom{n-1}{s-1}^{-1}/n$, eq. (2) follows from

$$\begin{aligned}
\varphi_i(N,v) &= \sum_{S \subseteq N} \kappa(s) \cdot \left[v(S) - v(S \setminus \{i\}) \right] = \sum_{\substack{S \subseteq N \\ S \ni i}} \kappa(s) v(S) - \sum_{\substack{S \subseteq N \\ S \not\ni i}} \kappa(s+1) v(S) \\
&= \kappa(n) v(N) + \sum_{s=1}^{n-1} \left[\sum_{\substack{S \ni i \\ |S|=s}} \kappa(s) v(S) - \sum_{\substack{S \not\ni i \\ |S|=s}} \kappa(s+1) v(S) \right] \\
&= \frac{v(N)}{n} + \frac{1}{n} \sum_{s=1}^{n-1} \left[\bar{v}^i(s) - \bar{v}^{\not\ni i}(s) \right].
\end{aligned}$$

Equation (2) simplifies further because a 'cartel' of size $s = 1$ leaves prices constant, i.e., $\bar{v}^i(1) = \bar{v}^{\not\ni i}(1) = 0$ for each $i \in N$. We thus obtain:

Shapley proportioning Rule *Firm i must contribute*

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=2}^{n-1} \left[\bar{v}^i(s) - \bar{v}^{\not\ni i}(s) \right] \tag{3}$$

to compensation of a given cartel damage $v(N)$ in order to reflect i's relative responsibility for harm and to share it in a scale-invariant additive way.

Proportioning by relative responsibility of the infringers – formalized by (NUL), (MRG) and (SYM), plus (EFF) and (LIN) – thus means: start out with *equal shares per head*; then add an n-th of the *average size-specific damage increments* that arise due to a given firm i's participation. This summand accounts for asymmetric effects on harm, which can arise even in symmetric market environments (cf. Section 4).

Equation (3) provides a useful perspective on φ_i but can also facilitate its calculation: possible symmetry among players reduces the sum of 2^n differences in eq. (1) to less than n ones in (3). This extends when asymmetries are such that i's increments for specific coalition sizes s can be expressed as a function of 'aggregate asymmetry' among other firms (see Subsection 5.3). The calculation is further simplified if cartel sizes s below some threshold $\tilde{s}$ are unstable (Bos and Harrington 2010), or if exchangeability of firms implies that $\bar{v}^i(s) - \bar{v}^{\not\ni i}(s)$ is zero. For instance, the summand vanishes in a homogeneous Bertrand or Cournot oligopoly or when $n = 2$; then equal shares follow.

[14]The decomposition in eq. (2) is distinct from those suggested by Kleinberg and Weiss (1985) and Rothblum (1988). It may here be stated for the first time.

4. Unequal Responsibility of Symmetric Differentiated Firms

By contrast, unequal responsibility and Shapley shares in compensation follow when $n > 2$ symmetric firms produce differentiated goods. This holds even for a very strong form of symmetry, where differentiation generates greater own-price than cross-price effects but firms are otherwise identical.

Specifically, let firms $1, \ldots, n$ simultaneously choose strategies $y_1, \ldots, y_n \in \mathbb{R}$ that jointly determine prices p_i and profits Π_i for all $i \in N$. $p = (p_1, \ldots, p_n)$ and $\Pi = (\Pi_1, \ldots, \Pi_n)$ are taken to be smooth functions of $y = (y_1, \ldots, y_n)$. The two focal cases are differentiated price competition where $p_i(y) \equiv y_i$ and quantity competition where y_i denotes firm i's output. However, y_i might also refer to some marketing activity, product characteristic, firm i's geographic radius of operation, etc. We make the following general assumptions:

A1. Price p_i and profits Π_i are affected identically by own strategy y_i for all firms $i \in N$ and also by all strategy choices y_j of the respective other firms $j \neq i$, that is

$$p_i(y_1, \ldots, y_n) \equiv p_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}) \text{ and } \Pi_i(y_1, \ldots, y_n) \equiv \Pi_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}) \tag{I}$$

for each $i \neq j$ and all permutations $\varrho \colon N \to N$ with $\varrho(i) = j$ and $\varrho(j) = i$.[15]

A2. For all $i \neq j \in N$,

$$\left|\frac{\partial p_i}{\partial y_i}\right| > \left|\frac{\partial p_i}{\partial y_j}\right| \tag{IIa}$$

and

$$\frac{\partial \Pi_j}{\partial y_i} \cdot \frac{\partial p_i}{\partial y_i} > 0. \tag{IIb}$$

Condition (IIa) is trivially satisfied for price competition and otherwise formalizes that inverse demand responds more to changes of the quantity (or product characteristic, delivery range, etc.) of the variety in question than to that of others. Condition (IIb) requires y_i to change own price p_i and other firms' profits Π_j in the same direction: e.g. for quantity competition, greater output y_i lowers p_i as well as the profits of firms $j \neq i$; for price competition, higher prices $y_i = p_i$ raise profits of j.

A3. For all $S \subseteq N$ there exists a unique collusion outcome $y^S = (y_1^S, \ldots, y_n^S)$ such that

[15] For instance, each variety i could be the personal favorite of an equal share of consumers who regard varieties $j \neq i$ as equally close substitutes. The symmetry in A1 is stronger than in the Salop model: some permutation ϱ with $\varrho(i) = j$ and $\varrho(j) = i$ satisfies (I) there, but not all do.

– $y_i^S = y^c$ if $i \in S$, where y^c solves the first order condition

$$\frac{d\Pi_S}{dy_i} = \sum_{j \in S} \frac{\partial \Pi_j}{\partial y_i} = 0$$

for maximization of joint profit $\Pi_S(y) = \sum_{k \in S} \Pi_k(y)$ by cartel S;

– $y_i^S = y^o$ if $i \notin S$, where y^o solves the first order condition

$$\frac{\partial \Pi_i}{\partial y_i} = 0$$

for individual profit maximization by an outsider to cartel S.

Sufficient conditions for the equilibrium in A3 to exist are provided in Section 5.

Proposition 1. *Given A1–A3, let $p^i(s)$ ($p^{\not i}(s)$) equal the equilibrium price for good i if firm i is (is not) part of a cartel with $s \in \{2, \ldots, n-1\}$ members. Then $p^i(s) > p^{\not i}(s)$.*

Proof. Inequality (IIb) implies that firms' strategies either lower their own prices, $\partial p_i/\partial y_i < 0$, and have a negative externality on each other's profits, $\partial \Pi_j/\partial y_i < 0$, as for quantity competition; or that $\partial p_i/\partial y_i > 0$ and $\partial \Pi_j/\partial y_i > 0$. In the former case, internalization of the negative profit externality in a cartel with $s \in \{2, \ldots, n-1\}$ members implies a smaller individual action or output choice $y^c < y^o$ for cartel members than outsiders (see A3); otherwise $y^c > y^o$.

We first address $y^c < y^o$ with $\partial p_i/\partial y_i < 0$ and $\partial \Pi_j/\partial y_i < 0$. Let $S = \{1, \ldots, s\}$ w.l.o.g. and consider the straight line L which connects $\hat{y} = (y^o, y^c, \ldots, y^c, y^o, \ldots, y^o, y^c)$ to $y^S = (y^c, y^c, \ldots, y^c, y^o, \ldots, y^o, y^o)$ in the space of output choices. L can be parameterized by

$$r(t) = (\underbrace{y^o - t, y^c, \ldots, y^c}_{s \text{ terms}}, \underbrace{y^o, \ldots, y^o, y^c + t}_{n-s \text{ terms}})$$

with $t \in [0, y^o - y^c]$, i.e., we simultaneously decrease firm 1's action and increase firm n's action by identical amounts as we move along L. The gradient $\nabla p_n = \big(\partial p_n/\partial y_1, \ldots, \partial p_n/\partial y_n\big)$ of function p_n can be used in order to evaluate the price change caused by switching from $\hat{y}$ to y^S. In particular, the (Stokes) gradient theorem for line integrals (see, e.g., Protter and Morrey 1991, Thm. 16.15) implies

$$\begin{aligned} p_n(y^S) - p_n(\hat{y}) &= \int_L \nabla p_n \, dr = \int_0^{y^o - y^c} \nabla p_n(r(t)) \cdot r'(t) \, dt \\ &= \int_0^{y^o - y^c} \Big(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\Big)\bigg|_{y=r(t)} \cdot \Big(-1, 0, \ldots, 0, 1\Big) \, dt \\ &= \int_0^{y^o - y^c} \left[\frac{\partial p_n(r(t))}{\partial y_n} - \frac{\partial p_n(r(t))}{\partial y_1}\right] dt < 0. \end{aligned} \tag{4}$$

The inequality follows from (IIa): firm n's own strategy changes have bigger price effects than changes by competitor firm 1.

A1 then implies

$$p^1(s) := p_1(y^c, y^c, \ldots, y^c, y^o, \ldots, y^o, y^o) = p_n(y^o, y^c, \ldots, y^c, y^o, \ldots, y^o, y^c) \tag{5}$$
$$= p_n(\hat{y}) > p_n(y^S) = p_n(y^c, y^c, \ldots, y^c, y^o, \ldots, y^o, y^o) := p^{\not n}(s).$$

That is, the price $p^1(s)$ of good 1 when its producer is one of s exchangeable cartel members exceeds the price $p^{\not n}(s)$ of good n when firm n is *not* part of a cartel with s members. And, also by A1, we have $p^{\not 1}(s) = p^{\not n}(s)$ and $p^1(s) = p^n(s)$. So, we can conclude $p^1(s) > p^{\not 1}(s)$ from (5). The same applies to any other firm i, and we obtain $p^i(s) > p^{\not i}(s)$ for all $s \in \{2, \ldots, n-1\}$ as claimed.

For $y^c > y^o$, the specific case $p_i(y) = y_i$ directly implies the claim. The general case of $\partial p_i/\partial y_i > 0$, $\partial \Pi_j/\partial y_i > 0$ is analogous to $y^c < y^o$: reversed orientation of the integral from $t = 0$ to $y^o - y^c < 0$ in (4) and the reversed sign of integrand $\partial p_n/\partial y_n - \partial p_n/\partial y_1$ cancel. □

Recall that linearity of the Shapley value allows to deal with damages on different products or quantities, applicable interest payments, possible trebling of damages, etc. in a straightforward way. We can therefore focus on the per unit overcharge damage $v^h(N)$ that accrued to a customer who bought her home product h and paid cartel price $p^c = p_h(y^N)$ instead of competitive price $p^* = p_h(y^\varnothing)$. The counterfactual average damages implied by partial cartels of size s that include and exclude firm h then are $\bar{v}^h(s) = p^h(s) - p^*$ and $\bar{v}^{\not h}(s) = p^{\not h}(s) - p^*$, respectively. Proposition 1 implies

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) > 0 \ \text{ for any } \ s = 2, \ldots, n-1.$$

So, from eq. (3) we can conclude

Proposition 2. *Given A1–A3, consider an overcharge damage $v^h(N)$ that was suffered on purchases from firm $h \in N$ after $n \geq 3$ symmetric producers of differentiated goods formed cartel N. Then*

$$\varphi_i(N, v^h) \begin{cases} > \frac{v^h(N)}{n} & \text{if } i = h, \\ < \frac{v^h(N)}{n} & \text{if } i \neq h. \end{cases} \tag{6}$$

That is, firm h is always responsible for more than 1/n-th of harm to its own (home) customers.

This finding readily generalizes to situations where $v^h(N)$ denotes compensation for the full welfare loss – including deprived gains – instead of only overcharge damage. Namely, let claimant k be a 'home customer' to firm h in the sense that k's welfare is a non-increasing function $W^h(p_1, \ldots, p_n)$ with $\partial W^h/\partial p_h < \partial W^h/\partial p_j \leq 0$ for $j \neq h$ and $p \in [p^*, p^c]^n$. Considering $h = 1$ (w.l.o.g.) we would then have

$$\bar{v}^h(s) = W^1(p^*, \ldots, p^*) - W^1\Big(\underbrace{p^h(s), \ldots, p^h(s)}_{s \text{ entries}}, \underbrace{p^{\not h}(s), \ldots, p^{\not h}(s)}_{n-s \text{ entries}}\Big)$$

and

$$\bar{v}^{\not h}(s) = W^1(p^*, \ldots, p^*) - W^1\Big(\underbrace{p^{\not h}(s), p^h(s), \ldots, p^h(s)}_{s \text{ entries}}, \underbrace{p^{\not h}(s), ..., p^{\not h}(s), p^h(s)}_{n-s \text{ entries}}\Big)$$

with $\bar{v}^h(s) - \bar{v}^{\not h}(s) > 0$ implied by $p^h(s) > p^{\not h}(s)$ for any $s = 2, \ldots, n-1$. This again yields asymmetric responsibility for harm, as summarized by eq. (6).

5. Proportioning by Responsibility in Linear Market Settings

Shapley proportionings require estimates of counterfactual damages for all conceivable sub-cartels (see, e.g., de Roos 2006). We illustrate this here for benchmark situations in which the costs and demand for differentiated goods are linear. We conjecture that parameter restrictions in analogy to, e.g., the proportionality condition of Epstein and Rubinfeld (2001) could reduce the data requirements in practical cases sufficiently to be applicable. If the producers of differentiated products face at most one kind of asymmetry, closed-form expressions for the Shapley shares can be derived via eq. (3).[16] This is often impossible in other applications of the Shapley value. The parametric solutions allow to derive upper and lower bounds for the responsibility-based contribution by a firm to harm of its own and of other firms' customers, respectively. They also facilitate assessing the degree to which, e.g., cartel-period revenue shares might serve as proxies of relative responsibility in Section 6.

5.1. Model

Consider a cartel of $n \geq 3$ suppliers where each firm $i \in N = \{1, \ldots, n\}$ produces a single good. Firm i's costs for output $q_i \in \mathbb{R}_+$ are given by

$$C_i(q_i) = \gamma_i q_i \text{ for } \gamma_i \geq 0. \tag{7}$$

Demand at price vector $p = (p_1, \ldots, p_n) \in \mathbb{R}^n_+$ is described by

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \;\text{ for } a_i > \gamma_i,\ d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i. \tag{8}$$

We presume $D_i(\gamma) > 0$, i.e., demand is positive when all firms price at cost. This amounts to assuming $a + (n-1)b\gamma > d\gamma$ in the symmetric case with $\gamma_i = \gamma$, $a_i = a$, $d_i = d$ and $b_{ij} = b$ for all $i \neq j \in N$. Firms set prices simultaneously à la Bertrand. If some group $S \subseteq N$ forms a cartel, outsiders $j \notin S$ are assumed to somehow become aware of this and to best-respond to cartel prices, which is already anticipated by S.

[16] Quadratic costs do not change findings much: Proposition 3 below then involves cost parameter γ but remains independent of a. Later expressions get significantly more unwieldy, however.

We assume that members of $S \subseteq N$ maximize the sum of their profits

$$\Pi_S(p) = \sum_{i \in S} (p_i - \gamma_i) D_i(p)$$

with corresponding first-order conditions

$$\frac{\partial \Pi_S(p)}{\partial p_j} = D_j(p) + \sum_{i \in S} (p_i - \gamma_i) \frac{\partial D_i(p)}{\partial p_j} \text{ for all } j \in S.$$

Analogous expressions hold if j is a cartel outsider. Implicitly, above specification of $\Pi_S(p)$ supposes that side-payments are feasible and that the relevant agents in infringing firms fail to internalize fines and compensation payments faced by their principals in case the cartel is detected. Both is motivated by simplicity considerations but not unrealistic.[17] It is then sufficient for existence and uniqueness of a corresponding collusion equilibrium that a uniform increase of all prices and a unilateral increase of any single price respectively decrease individual and total demand.[18] Formally, this requires $\sum_{j=1}^{n} \partial D_i / \partial p_j < 0$ and $\sum_{j=1}^{n} \partial D_j / \partial p_i < 0$, i.e., we will assume

$$\alpha_i := d_i / \sum_{j \neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N. \tag{9}$$

Above inequalities are equivalent if $b_{ij} = b_{ji} = b$ but we may generally have $\sum_{j \neq i} b_{ij} \neq \sum_{j \neq i} b_{ji}$. Condition (9) simplifies to $\alpha := d / ((n-1)b) > 1$ in the symmetric case.

Products are relatively good substitutes when α_i is small; then price increases by one firm significantly raise profits for other firms. A cartel internalizes this. So, the price p_i set by cartel member i will be the higher, the smaller α_i.

For any $S \subseteq N$, vector p^S collects the equilibrium prices p_i^S assuming firms in S coordinate and the rest acts competitively. The per unit overcharge suffered by a customer who bought product i is denoted by $v^i(N) = p_i^N - p_i^{\emptyset}$.

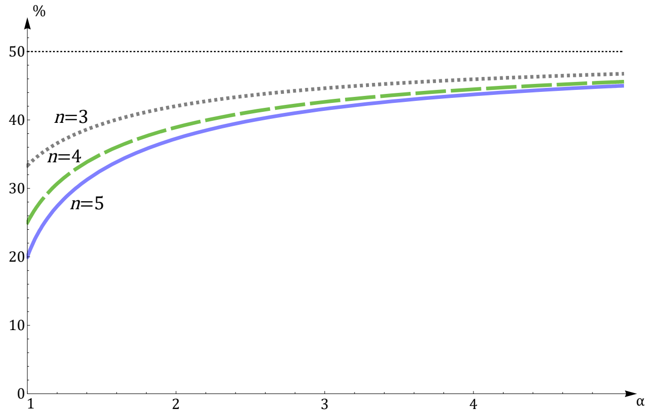
5.2. *Symmetric Case*

Own and cross price elasticities for the considered goods vary even under symmetry given $\alpha > 1$. We hence distinguish the home firm h that produced the good for which a fixed customer suffered harm from those cartel members $j \neq h$ that were not part of their transaction. We focus on the per unit overcharge $v^h(N)$. After solving for the equilibria p^S implied by (7) – (9) for all $S \subseteq N$, the percentages of $v^h(N)$ for which firms h and j are respectively responsible, $\rho_h^{h*} := \varphi_h(N, v^h)/v^h(N)$ and $\rho_j^{h*} := \varphi_j(N, v^h)/v^h(N)$, can be determined in closed

[17]For instance, Smuda (2014) analyzes 191 overcharge estimates and fails to find evidence for any reaction of cartel prices in European markets to changes in antitrust provisions. See Leslie (2018) on side payments.

[18]See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6).

Figure 1: Share ρ_h^{h*} of overcharge damages on good h attributed to firm h for given differentiation parameter α (assuming $d = 2$, $b = 2/[(n-1)\alpha]$)



form (see the Appendix):

Proposition 3. *Suppose firms are symmetric in the linear market environment defined by equations (7), (8) and (9). Let h be the producer of the good for which overcharge damages are to be proportioned, and j be any of h's $n-1$ competitors. The relative responsibilities for harm then are*

$$\rho_h^{h*} = \frac{1}{n} + \frac{n-1}{n} \sum_{s=2}^{n-1} \frac{(s-1)\cdot(4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n-1)^2 - 2(n+s-3)(n-1)\alpha + s(n-s) - 2(n-1)}$$

and $\rho_j^{h*} = (1-\rho_h^{h*})/(n-1)$.

The common unit cost γ and demand intercept a have no effect on h's Shapley share. It is determined only by the ratio α of own and cross-price parameters. If this degree of differentiation α is low, cartel participation by all firms is important. In the limit, each firm is essential for maintaining an overcharge and affects damage equally:

$$\lim_{\alpha \to 1} \rho_h^{h*} = \frac{1}{n} \quad \text{and} \quad \lim_{\alpha \to 1} \rho_j^{h*} = \frac{1}{n} \text{ for } j \neq h.$$

If, in contrast, firms produce highly differentiated goods, we have

$$\lim_{\alpha \to \infty} \rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} (s-1) = \frac{1}{2}.$$

One can check that ρ_h^{h*} is strictly increasing in α. We therefore obtain:

Corollary 1. *If $v^h(N)$ is the compensation obtained by a customer of firm $h \in N$, then proportioning by relative responsibility calls for firm i to contribute*

$$\varphi_i(N, v^h) \in \begin{cases} \left(\frac{v^h(N)}{n}, \frac{v^h(N)}{2}\right) & \text{if } i = h, \\ \left(\frac{v^h(N)}{2(n-1)}, \frac{v^h(N)}{n}\right) & \text{if } i \neq h. \end{cases}$$

Figure 1 illustrates the behavior of ρ_h^{h*} for intermediate degrees of differentiation.

5.3. *Asymmetric Case*

The bounds in Corollary 1 provide guidance for mildly asymmetric markets by continuity. When firms are sufficiently heterogeneous, though, it is possible that home firm h has *lower* responsibility for harm of its customers and will be assigned a *smaller* share of compensation than its competitors, i.e., $\varphi_h(N, v^h) < v^h(N)/n$. This happens when the cross-price effects involving firm h are sufficiently smaller than those between other cartel members. We can, e.g., have three firms such that demands of firm 1 and 2 involve high mutual cross-price reactions b_{12} and b_{21}, while there are only small linkages b_{i3} and b_{3i} with firm 3 ($i \neq 3$). Firm 3's cartel participation matters for overcharges on p_1, p_2 and p_3 but a significant increase of p_3 would have occurred even if firm 3 had not been part of the cartel and had just best-responded. This part of $v^3(N)$ is caused by price increases on goods 1 and 2, which are mostly driven by shutting down competition between firms 1 and 2. The latter hence had greater influence on $v^3(N)$ than firm 3 itself.[19] Therefore, asymmetry in cross-price effects does not come with useful bounds on responsibilities.

Asymmetry in demand parameters a_i or costs γ_i can be dealt with better, although calculations become tedious. For instance, supposing $\gamma = 0$ and that firm-specific demand intercepts a_i are the only asymmetry at hand, we have:

Proposition 4. *Suppose firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (7), (8) and (9) with $\gamma = 0$. Firm h's Shapely share then is*

$$\rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)} \cdot \sum_{s=2}^{n-1} \frac{(s-1)\Big[6\alpha(n-1) + (s+4-n) + \Big(4\alpha^2(n-1)^2 + \tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big](\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big)\Big[(3\alpha + \frac{2-n}{n-1}) + (2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]}$$

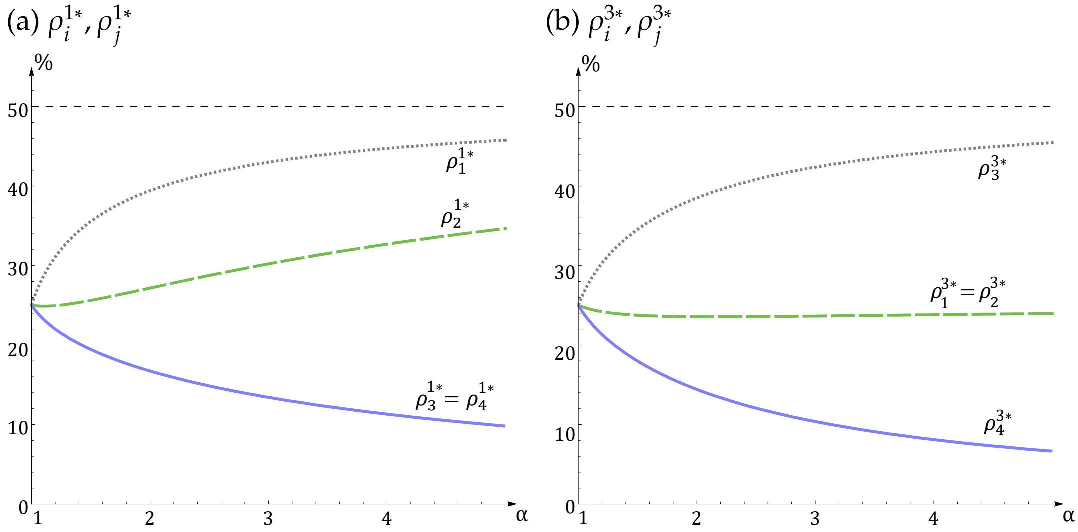
with $\bar{a}_{-h} := \sum_{i\neq h}^{n} a_i/(n-1)$, $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$.

Ratio $\bar{a}_{-h}/a_h$ relates the market sizes of firm h and its competitors: a large ratio means firm h is comparatively small, a ratio close to zero that h's market is big. It can be checked that ρ_h^{h*} is maximized when (*i*) firms produce highly differentiated goods, i.e, $\alpha \to \infty$, and (*ii*) when firm h's market size is massive, that is, $\lim a_h/\bar{a}_{-h} \to 0$. Then firm h is responsible for half of the damage to its customers. Contrary, ρ_h^{h*} is minimized when goods are close substitutes and firm h's market size is small. Then, firm h is responsible for around $1/n$-th of its own customers' damage. Similar reasoning for firms $j \neq h$ yields

Corollary 2. *Suppose firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (7), (8) and (9)*

[19] For instance, assume $a_i = 10$, $d_i = 3$, $\gamma_i = 0$ for $i \in \{1, 2, 3\}$, $b_{12} = b_{21} = 2$, $b_{13} = b_{23} = b_{31} = b_{32} = 0.5$ and consider $v^3(N)$. Then Shapley shares evaluate to $\rho_1^{3*} = \rho_2^{3*} \approx 35.1\% > \rho_3^{3*} \approx 29.8\%$.

Figure 2: Shares ρ_i^{h*} for cost leaders $i \in \{1, 2\}$ and laggards $j \in \{3, 4\}$



with $\gamma = 0$. If $v^h(N)$ reflects damages to a customer of firm $h \in N$, then

$$\varphi_i(N, v^h) \in \begin{cases} \left(\frac{v^h(N)}{n}, \frac{v^h(N)}{2}\right) & \textit{if } i = h, \\ \left(0, \frac{v^h(N)}{2}\right) & \textit{if } i \neq h. \end{cases} \tag{10}$$

The bounds in eq. (10) also apply to firms which are symmetric in all but technology. This case is illustrated in Figure 2. It considers responsibility for per unit overcharges of $v^1(N)$ and $v^3(N)$ for a cartel of two low-cost firms (1 and 2) and two high-cost producers (3 and 4) with common parameters $a = 10$, $d = 2$, and $b = 2/(3\alpha)$. No matter whether the selling firm has (a) low costs $\gamma_1 = \gamma_2 = 1$ or (b) high costs $\gamma_3 = \gamma_4 = 5$, it bears responsibility for 25% to 50% of overcharges on its product, and always the greatest share.

6. Comparison to Proportioning by Market Shares

A simple and reliable proportioning heuristic could save the effort of above calculations. Perhaps market shares, which are comparatively easy to obtain, are a good proxy for whose cartel participation is responsible for which proportion of a damage, at least under some identifiable circumstances? If yes, should we use sales or revenues? From the cartel or competitive regime? Or perhaps better use a profit measure?

We will address these questions by a range of numerical simulations. We already know from the above analysis of symmetric situations that respective (symmetric) market shares clash with firms' asymmetric responsibilities for harm of a single customer who purchased only one good (e.g. compare market share $1/n$ to ρ_h^{h*} in Figure 1 if $\alpha > 1$). So, to give proportioning heuristics a good shot we will assume that *all customers* of the detected cartel received compensation of their entire overcharge damage. Firms' over and under-contributions relative to the product-specific Shapley shares can then cancel out for a given heuristic across products. In particular, a division by heads perfectly matches relative responsibility in the aggregate if firms are symmetric.

So, let us consider asymmetric firms. Our benchmark are aggregate payments under Shapley proportioning for each firm $i \in N$,

$$\Phi_i := \sum_{j \in N} \varphi_i(N, v^j) = \sum_{j \in N} q_j^c \cdot v^j(N) \cdot \rho_i^{j*}(N, v^j),$$

and we compare this to firm i's payment H_i^ρ if the total damage

$$\mathcal{D} := \sum_{i \in N} q_i^c \cdot v^i(N),$$

is proportioned by a market share measure ρ, i.e. to $H_i^\rho := \rho_i \cdot \mathcal{D}$ (with q_i^c denoting firm i's cartel sales). Firms' respective over and under-payments are summed and normalized to give an index of *aggregate mis-allocation of damages*

$$M^\rho := \sum_{i \in N} | \Phi_i - H_i^\rho | \Big/ \mathcal{D}.$$

This index is proportional to the expected mis-allocation of compensation for a unit purchase by a randomly drawn customer, or for a customer who made purchases from all firms in proportion to their cartel sales, or when all customers go after the cartel with identical positive probability.

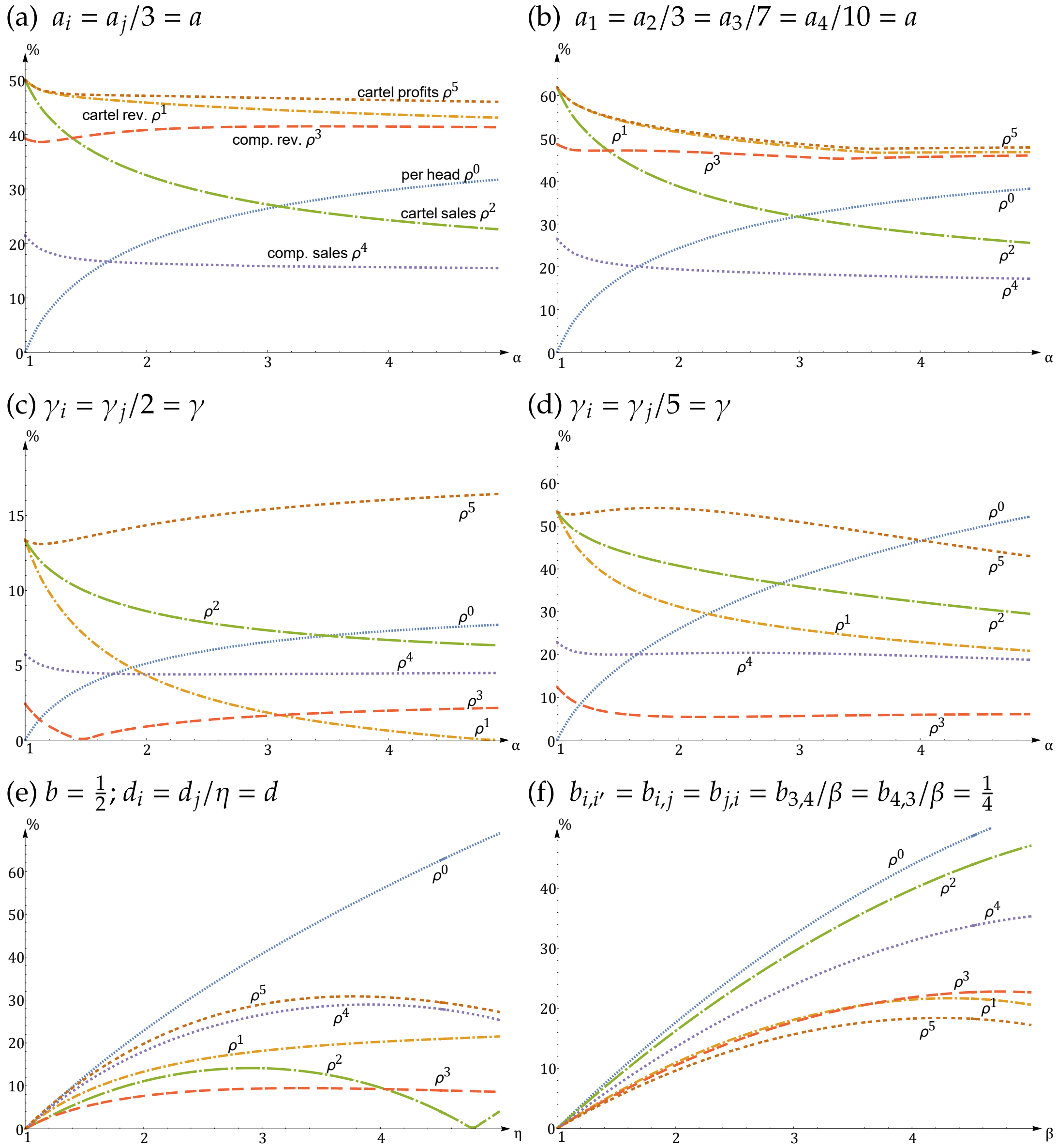
In Figure 3, we start from the baseline scenario $a = 10$, $\gamma = 1$, $d = 2$, $b = d/(3\alpha)$ and break symmetry for one parameter at a time. The two top panels consider heterogeneity in firm-specific market sizes a_i. Panel (a) involves two large and two small firms; in panel (b) all firms differ. An *equal per head allocation* ρ^0 non-surprisingly performs well when differentiation is very low. It soon loses out, though, to allocating damages in proportion to *market shares based on competitive sales* ρ^4 and to *market shares based on cartel sales* ρ^2. Market shares determined by *cartel revenues* ρ^1 or *competitive revenues* ρ^3 produce very high mis-allocations at all levels of differentiation. Only proportioning by *cartel profits* ρ^5 is worse.

Panels (c) and (d) assume an intermediate and a big cost asymmetry between firms 1 and 2 vs. firms 3 and 4. The deviations from the Shapley payments, aggregated for each firm, are significantly higher for the big asymmetry in (d) than the smaller one in (c).[20] Revenue-based market shares ρ^1 or ρ^3 and sales-based competitive market shares ρ^4 here perform the best.

Panel (e) assumes firms 3 and 4 face bigger own-price elasticities than firms 1 and 2. Market shares ρ^2 and ρ^3 based on cartel sales or competitive

[20]The kink that is visible in panel (c) for ρ^3 – or ρ^2 in (e) – results from cancellation of product-specific deviations at the firm level when these switch from having opposite to identical signs. We have checked that Figure 3 remains virtually unchanged if we conduct the same simulations not only for compensation of overcharges but of full welfare losses including deprived gains from volume effects.

Figure 3: Mis-allocation M^ρ by different heuristics considering $i = 1, 2$ and $j = 3, 4$



revenues then are closest to the Shapley benchmark. The final panel (f) assumes heterogeneity in cross-price effects: firms 1 and 2 face a fixed cross-price parameter of 1/4, competition between firms 3 and 4 is more intense by some factor β. In contrast to the five environments (a)–(e) in which its relative ranking was consistently low, proportioning by cartel profits ρ^5 here comes closest to reflecting the Shapley shares.

The key message of the ups and downs and, notably, the changing ranking of $M^{\rho^0}, \ldots, M^{\rho^5}$ in Figure 3 is that *no* market share heuristic provides a reliable short-cut to relative responsibilities for harm. This holds even when contributions are evaluated at the aggregate market level rather than for harm suffered by an individual litigant. So, tempting as proportioning compensation payments by cartel sales, revenues, or profits may be, we see that market shares generally fail to reflect responsibility shares.

7. Concluding Remarks

The results in this paper have been obtained under the assumption that damages in "What if some cartel members had refused to participate?"-scenarios can somehow be quantified. This is a limitation. But counterfactuals provide the basis of any causality-based ascription of responsibility, as well as of the quantification of harm and a victim's compensation in the first place (see Directive 2014/104/EU, recital 46).

Depending on the case at hand, refined estimates of per unit overcharges may be obtained from a structural market model that has been calibrated to a sufficiently rich panel of data (see de Roos 2006). The assessment by an experienced practitioner is still skeptical: "... for almost all real-life cases, such a data panel will be exceedingly difficult or downright impossible to obtain" (Bornemann 2018, recital 124).

In our view, it is nonetheless relevant to study ideal worlds with accurate assessments $v^j(S)$ for counterfactual sub-cartels S: one can gain structural insights (such as the bounds derived above) and, importantly, assess the quality of more pragmatic suggestions. Without a sound benchmark it is unclear why proportioning by "... sales of the product during the conspiracy ...", as proposed by Baker (2004, p. 388) early on, should reflect relative responsibilities any better than, say, profit shares or a division by heads. The numerical analysis in Section 6 demonstrates, alas, that *any* market share heuristic provides a blurred reflection of responsibility at best.

A possible way forward is to anyhow proportion by (an arbitrary choice of) market shares but to stop pretending that robust links to causality-based responsibility exist. Another and our preferred alternative would be to capture causal links between actions and harm by applying the Shapley value at least to first approximations of applicable counterfactuals. In a companion paper, approximations of $v^j(S)$ that partition partial cartels $S \subset N$ into binary categories (namely, S is either able to sustain significant overcharges or not) turn out to perform surprisingly well (Napel and Welter 2021). We conjecture that modestly finer classifications – such as S causing 'significant' vs. 'intermediate' vs. 'insignificant' harm to buyers of good j – are still tractable but come very close to implementing Directive 2014/104/EU's provision: when cartel victims are compensated, jointly liable co-infringers need to contribute according to their "relative responsibility for the harm caused".

A. Appendix – Proofs of Propositions 3 and 4

Proof of Proposition 3. Suppose $n \geq 3$ firms are symmetric in the linear market environment defined by equations (7), (8) and (9). The cartel price then evaluates to

$$p^c := p_i^N = \left(\frac{a}{d-(n-1)b} + \gamma\right)\Big/2$$

for each differentiated product $i \in N$.[21] Corresponding competitive prices are

$$p^* := p_i^{\varnothing} = \frac{a + d\gamma}{2d-(n-1)b} \text{ for all } i \in N.$$

This implies per unit cartel overcharges of

$$v^i(N) = p^c - p^* = \frac{a/d - \gamma(1-\frac{1}{\alpha})}{4\alpha - 6 + 2/\alpha} \quad \text{with} \quad \alpha = \frac{d}{(n-1)b} > 1$$

for each product $i \in N$.[22] They are homogeneous of degree one in (a, γ) and strictly decreasing in differentiation parameter α as well as in unit costs γ.

If there is a partial cartel S of size $s = 2, \ldots, n-1$, equilibrium prices are

$$p_i^S = \begin{cases} \dfrac{a(2d+b) + \gamma\big(2d^2 + bd(3-2s) + b^2(ns - n - s^2 + 1)\big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \in S, \\[2ex] \dfrac{a(2d - sb + 2b) + \gamma\big(2d^2 - bd(s-2) - b^2(s^2 - s)\big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \notin S \end{cases}$$

with $\eta_s = s(n-s) - 2(n-1) \geq -(n-1)$.

Comparing the price p_h^S of the home product $h \in N$ paid by a suing customer in case that the respective producer h is part of a cartel with s members, i.e., for $h \in S$, to the respective price p_h^S if h is not, i.e., for $h \notin S$, yields[23]

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) = \frac{b(s-1)\big(a + (n-1)b\gamma - d\gamma\big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} > 0.$$

Inserting this into eq. (3) gives the Shapley proportioning $\varphi_h(N, v^h)$ in absolute terms. Dividing the latter by $v^h(N)$ yields h's claimed Shapley share ρ_h^{h*}. □

Proof of Proposition 4. Suppose $n \geq 3$ firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (7),

[21] The detailed algebraic manipulations omitted here are available upon request.

[22] The associated reduction in sales is $\Delta D := D(p^*, \ldots, p^*) - D(p^c, \ldots, p^c)$ per product, which evaluates to $[b(n-1)(a-(b+d-bn)\gamma)]\big/[4d - 2b(n-1)]$. Firm i's customers lose surplus of $\Delta D \cdot (p^c - p^*)/2$ on non-purchased units. These deprived gains can be proportioned in analogy to overcharges if applicable.

[23] The three factors in the numerator are strictly positive. Invoking $s \leq n-1$ and $\eta_s \geq -(n-1)$ first, and $d > (n-1)b$ next, the denominator can be bounded below by $2d[2d - 2(n-2)b] - b^2(n-1) > 2d[2(n-1)b - 2(n-2)b] - bd = 3bd > 0$. Hence $p^h(s) - p^{\not h}(s) > 0$.

(8) and (9) with $\gamma = 0$. Then, firm h's cartel price is

$$p_h^C = \frac{a_h d - (n-2)a_h b + b(n-1)\bar{a}_{-h}}{2(b+d)(d+b-bn)}$$

with $\bar{a}_{-h} = \sum_{l=1 \neq h}^{n} a_l/(n-1)$. Firm h's corresponding competitive price is

$$p_h^B = \frac{2a_h d - (n-2)a_h b + b(n-1)\bar{a}_{-h}}{(2d+b)(2d+b-bn)}.$$

A customer's per unit cartel overcharge by the product h then is

$$v^h(N) = p_h^C - p_h^B = \frac{b(n-1)[b(3d+2b-bn)a_h + (2d^2+b^2n-b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)}.$$

It rises in the saturation level a_h of firm h's demand as well as in the average saturation quantity $\bar{a}_{-h}$ of firms $l \neq h$. The corresponding Shapley value of firm h in proportioning $v^h(N)$ is

$$\varphi_h = \frac{v^h(N)}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{b(s-1)[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(4d^2-(2n-6+2s)db+\eta_s b^2)}$$

with $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$. Dividing φ_h by $v^h(N)$ and substituting $\alpha = d/(b(n-1))$ gives

$$\rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)} \cdot \sum_{s=2}^{n-1} \frac{(s-1)\left[6\alpha(n-1) + (s+4-n) + \left(4\alpha^2(n-1)^2 + \tau_s\right)\frac{\bar{a}_{-h}}{a_h}\right](\alpha-1)(2\alpha-1)}{\left(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\right)\left[(3\alpha + \frac{2-n}{n-1}) + (2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\right]}$$

as claimed. □

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