Unintended Size Effects of Granting Redress to Victims of Umbrella Pricing

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Abstract

It is generally presumed that bigger scale and scope of private antitrust enforcement promotes effective competition. This has motivated several North American and European courts to uphold redress claims not only from clients of a detected cartel but also plaintiffs who were exposed to equilibrium price adjustments by other firms ('umbrella pricing'). The paper shows that beneficial deterrence effects of an obligation to compensate aggrieved customers of non-infringing firms can be dominated by adverse cartel size effects. Liability for umbrella damages constrains the prices set by small partial cartels and improves the comparative profitability and stability of large ones. More encompassing cartels can form, prices rise, and welfare falls.

Keywords: cartel deterrence; cartel formation; effective competition; umbrella effects; redress payments; cartel size

JEL codes: L40; K21; D43

1. Introduction

Victims of antitrust infringements have a right to act against a detected cartel and to reclaim damages. Forward-looking firms anticipate the applicable redress obligations in their decisions to form cartels and to fix prices. Private antitrust enforcement thus shapes both the distribution and creation of economic surplus.

The legal discussion of private antitrust action is widely presuming that redress on a greater part of total harm is desirable not only on principle and for reasons of justice but that it generates beneficial deterrence effects. This has been acknowledged explicitly in a 2014 decision by the EU Court of Justice that extended legal standing from the customers of cartel members to customers of non-members who were harmed indirectly by an increased industry price level:

> "The right of any individual to claim compensation for such a[n indirect] loss actually strengthens the working of the European Union competition rules, since it discourages agreements or practices, frequently covert, which are liable to restrict or distort competition, thereby making a significant contribution to the maintenance of effective competition in the European Union . . . " (Case C-557/12 *Kone AG v ÖBB-Infrastruktur AG*, ECLI:EU:C:2014:1317, rec. 23).

Victims of 'umbrella effects', understood here as covering detrimental equilibrium reactions by non-infringing firms to cartel activities, are entitled to compensation also in Canada (as affirmed by the Supreme Court in 2019) and in the US depending on the competent court.[1] Leon Higginbotham Jr., former judge at the US Court of Appeals for the Third Circuit, noted in a well-cited case in 1979 (judgement 596 F.2d 573 3rd. Cir.): "Allowing standing [for customers of cartel outsiders] would also encourage enforcement, and thereby deter violation, of the antitrust laws."

Similar views are expressed in scholarly publications. For instance, Blair and Maurer (1982) hold that "[i]t is obvious that the prospect of recovery by purchasers from noncolluding competitors should have a greater deterrent effect than recovery

[1]The US Supreme Court has not yet ruled on the issue. Negative decisions include 600 F.2d 1148 5th Cir. 1979; positive ones 62 F. Supp. 2d 25 1999. See Blair and Durrance (2018), as well as Inderst et al. (2014) and Holler and Schinkel (2017). Laitenberger and Smuda (2015) show that umbrella losses constituted a big part of damages suffered by customers of the European detergent cartel in Germany. Bos et al. (2019) find suggestive evidence for umbrella effects in some but not all considered industries.

limited to direct purchasers, assuming a constant probability of detection". Blair and Durrance (2018) conclude that awarding compensation for umbrella losses "... further deters illegal price-fixing behavior".

This paper demonstrates that a pro-competitive assessment of extended legal standing is warranted only if a cartel's size is fixed. The stability of differently encompassing cartels that might form in a given market depends on how umbrella victims are treated: an obligation to compensate umbrella losses reduces optimal prices and the expected profitability of partial cartels. Small cartels can thereby become unstable, while large or all-encompassing ones – with less or no competitors whose customers could claim compensation – remain or newly become stable in the face of umbrella compensation. We show that for non-negligible sets of market configurations, extending standing to indirect cartel victims increases the incidence of larger cartels. These charge higher prices and generate lower welfare. Supposing a constant probability of detection, like Blair and Maurer (1982), a regime with umbrella compensation can even raise total unatoned overpayments, not just deadweight losses.

We illustrate this counterintuitive implication of extended legal standing first by a simple and static numerical example (Section 2). Then we study a dynamic market model that allows to contrast beneficial deterrence effects for fixed cartel size with detrimental structural effects stemming from increased internal stability of large and decreased external stability of small cartels (Section 3). The observation that greater scope for private enforcement can have negative welfare implications complements results by Bos and Harrington (2015), who showed greater scale of public antitrust actions to have ambiguous effects (viz. higher fines levied on detected cartels).

2. Illustration by a Static Numerical Example

Consider a standard Bertrand oligopoly with n symmetric firms facing demand $D(p) = 10 - p$. Unit costs are zero while each firm i faces a capacity constraint $q_i \leq k$ on its output. Let k be smaller than the monopoly output of $q = 5$ but big enough for $n - 1$ firms to serve the market at cost, $(n - 1)k \geq 10$. Take the corresponding zero-profit Nash equilibrium as competitive benchmark.

Now let $s \leq n$ firms contemplate forming a cartel. If formed, the cartel will be detected with probability $0 < \alpha \leq 1$. It then faces fines of $\tau > 0$ times its profits and, additionally, a share $0 < \beta \leq 1$ of all customers with legal standing successfully reclaim overcharge damages. Suppose that respective payments are split symmetrically.[2]

If the cartel forms, its s members choose their uniform price p_s to maximize expected joint profits and non-cartel members simultaneously best-respond. In particular, the $n-s$ outsiders optimally increase prices and profits under the 'umbrella' of the cartel. They will undercut p_s by the smallest feasible amount if $p_s > 0$.[3] Assume efficient rationing in this case, so that cartel members make a profit only if their residual demand $D_s^R(p_s) = \max\left\{10 - p_s - (n-s)k, 0\right\}$ is positive. For a cartel of $s = n-1$ firms, the single outsider produces at capacity while each cartelist supplies $0 < D_s^R(p_s)/s < k$. Whether smaller cartels with $s < n-1$ could serve positive demand and whether this would in expectation be profitable depends on individual capacity k and anti-trust enforcement parameters α, β and τ.

In our 'default regime' only customers of the s cartel members have legal standing and with probability β gain redress for their overcharge damages. A cartel member's expected profit then is

$$\pi_s(p_s) = \Big(1 - \alpha(\beta + \tau)\Big) p_s D_s^R(p_s) \Big/ s. \tag{1}$$

Alternatively, let *all* customers have legal standing and also umbrella losses must be compensated with probability β ('umbrella regime'). Expected profits are then reduced to

$$\pi_s^u(p_s) = \Big[(1 - \alpha\tau) p_s D_s^R(p_s) - \alpha\beta p_s D(p_s)\Big] \Big/ s. \tag{2}$$

The profit function of a cartel outsider, marginally undercutting any anticipated cartel price p_s, is independent of the legal regime. It approximately equals

$$\tilde{\pi}(p_s) = p_s k. \tag{3}$$

Note that equilibrium prices p_s^* and p_s^{*u} in the two regimes will generically differ since profit function $\pi_s^u(p_s)$ is no increasing transformation of $\pi_s(p_s)$ or vice versa.

[2]Detected cartel members are jointly liable in most jurisdictions. It makes no difference to our analysis whether this comes with an EU-style rule of contribution where total redress payments are shared, or a US-style no-contribution rule that creates a risk for each infringer to end up paying for all. See Easterbrook et al. (1980) for a general discussion of contributions and Napel and Welter (2020) on aligning them to relative responsibility for harm among asymmetric firms.

[3]Suppose a smallest currency unit $\varepsilon > 0$ and prices $p \in \left\{k \cdot \varepsilon : \ k \in \mathbb{Z}_+\right\}$.

Cartel size s	p_s^*	$\pi_s(p_s^*)$	$\tilde{\pi}(p_s^*)$	$\mathcal{I}_s$	$\mathcal{E}_s$	p_s^{*u}	$\pi_s(p_s^{*u})$	$\tilde{\pi}(p_s^{*u})$	$\mathcal{I}_s^u$	$\mathcal{E}_s^u$
0	0	0	0	n.a.	n.a.	0	0	0	n.a.	n.a.
2	0.5	0.09	1.5	+0.09	+0.57	0	0	0	n.a.	n.a.
3	2	0.93	6	−0.57	+3.86	1.14	0.31	3.43	+0.31	+1.78
4	3.5	2.14	10.5	−3.86	+7	3.07	1.65	9.21	−1.78	+5.71
5	5	3.5	n.a.	−7	n.a.	5	3.5	n.a.	−5.71	n.a.

Table 1: Internal and external stability in default and umbrella regime (rounded)

Adopting a static stability perspective à la D'Aspremont et al. (1983), a cartel with s members is *internally stable* if respective cartel profits from charging p_s^* exceed the profits of a freeriding cartel outsider facing price p_{s-1}^*. This translates into condition

$$\mathcal{I}_s := \pi_s(p_s^*) - \tilde{\pi}(p_{s-1}^*) \geq 0 \tag{4}$$

when eligibility for compensation is restricted to cartel customers. The cartel is *externally stable* if an outsider's profits facing p_s^* exceed cartel profits associated with p_{s+1}^*, i.e.,

$$\mathcal{E}_s := \tilde{\pi}(p_s^*) - \pi_{s+1}(p_{s+1}^*) \geq 0. \tag{5}$$

We say the cartel is *(structurally) stable* if $\min\left\{\mathcal{I}_s, \mathcal{E}_s\right\} \geq 0$. Analogous conditions apply to the umbrella regime.

Table 1 summarizes Nash equilibrium outcomes and the pertinent stability indications when s out of $n = 5$ firms form a cartel with capacities $k = 3$, detection probability $\alpha = 1/5$, fine multiplier $\tau = 1/2$, and proportion $\beta = 1$ of eligible victims receiving compensation. In the default regime, a cartel is stable iff it comprises $s = 2$ firms. When also customers who suffered umbrella losses must be compensated in case of detection, a cartel of two firms ceases to be stable. However a cartel comprising $s' = 3$ firms, which failed to be internally stable before, can form successfully. Prices then go up and welfare falls.

The general mechanism at play is simple: anticipated umbrella redress obligations constrain the optimal overcharges imposed by partial cartels, especially those with many non-members. This makes freeriding on a small cartel less attractive compared to membership of larger cartels. The latter's internal stability increases; external stability of the former falls. The same logic applies to cartels of all sizes but the effect is stronger, the greater the market share of non-infringing firms.

For the parameters at hand, giving standing to umbrella victims raises the size of the only stable cartel from $s = 2$ to $s' = 3$ and respective (umbrella) prices paid by customers increase to (just below) $p_3^{*u} \approx 1.14$ compared to $p_2^* = 0.5$. So the expected uncompensated damage for customers in the default scenario comprises an overcharge of (approximately) 0.5 for all $3k$ units purchased from outsiders plus $D_2^R(0.5) = 0.5$ units from the cartel that remain uncompensated with probability $1 - \alpha = 4/5$. That makes $0.5 \cdot (9 + 4/5 \cdot 0.5) = 4.7$ in total. In contrast, an overcharge of 1.14 on 8.86 units arises in the umbrella scenario, yielding an expected uncompensated overcharge damage of $4/5 \cdot 10.12 = 8.1$. Extended standing therefore not just increases deadweight losses via $p_3^{*u} > p_2^*$ but raises total unatoned over-payments by almost 75%.

We note that if the anticipated detection probability α were not fixed exogenously, as our analysis will continue to assume, but varied in cartel size then α would need to rise by about one third in order for default and extended legal standing to generate identical uncompensated harm (from 20% for $s = 2$ to 26.46% for $s' = 3$).

3. Dynamic Analysis of Compensation for Umbrella Losses

Giving legal standing to victims of umbrella pricing has detrimental size effects also in richer dynamic settings. We investigate three necessary conditions for formation of a stable cartel in a symmetric version of the Bertrand-Edgeworth competition model developed by Bos and Harrington (2010, 2015). The conditions are also sufficient if cartel formation is friction-less and driven only by profit opportunity. We hence leave aside potential moral considerations, fear of personal criminal sanctions, or transaction costs for coordination in the following.

The first condition for establishment of a cartel with s members is obviously (i) its *profitability*, i.e., that positive markups generate sufficient residual demand despite cheaper sales by non-members with a joint capacity $(n - s)k$. Formation of a cartel next presupposes its (ii) *dynamic stability*. As customary, we consider a cartel to be dynamically stable if strategies involving a suitable markup and reversion to static Nash play after a defection form a subgame perfect equilibrium. In particular, a one-off deviation by a cartel member must not raise the present value of its profit stream. Finally, (iii) *structural stability* requires that none of the s members would permanently prefer to be a non-member of a dynamically stable cartel involving $s-1$ firms, nor any non-member could be better off as member of a dynamically stable cartel of $s+1$ firms. We address conditions (i)–(iii) after introducing the basic setup. We will see that compensation of umbrella losses tightens them more for small cartels than larger ones. This ends up raising prices and lowering welfare for various market configurations.

3.1. Setup

Let n symmetric firms repeatedly engage in simultaneous price setting for a homogeneous good. Each firm i faces an exogenous capacity constraint $q_i \leq k$ on period production and maximizes the present value of profits for a common discount factor $\delta \in (0,1)$ with infinite time horizon.

Market demand is described by $D(p) = a - bp$ with $a, b > 0$ for simplicity.[4] Prices must be integer multiples of a small unit of account $\varepsilon > 0$ and consumers buy at the lowest available price à la Bertrand. Demand is rationed efficiently when a firm i's capacity is exhausted and (residual) demand is split equally if several firms post identical prices.

Constant unit costs are normalized to zero.[5] Individual capacity k is assumed to be less than monopoly demand but big enough for $n-1$ firms to serve the market at cost, i.e.,

$$\frac{a}{n-1} \leq k < \frac{a}{2}. \tag{A1}$$

This requires $n \geq 4$ and gives rise to a symmetric static Nash equilibrium with zero profits. It is also an equilibrium for all firms to choose $p = \varepsilon \approx 0$ and we therefore suppose cartel prices $p \geq 2\varepsilon$.

As Bos and Harrington (2010, 2015), we allow at most one cartel to operate. Its $2 \leq s \leq n$ members are assumed to use stationary strategies that do not condition on past behavior of non-cartel members but permanently revert to the static zero-profit equilibrium after a deviation (the harshest possible punishment). So non-members will at any point in time maximize their static period profits and undercut the anticipated uniform cartel price p by ε (cf. Bos and Harrington 2010). This leaves

[4]Extensions of our results to non-linear demand are described in Appendix A3.

[5]For positive unit costs one can interpret p as the price markup: consider $c > 0$, prices $\tilde{p} = c + p$ and demand $\tilde{D}(\tilde{p}) = \tilde{a} - b\tilde{p} = a - bp$ with $a = \tilde{a} + bc$. Then maximizing $(\tilde{p} - c)\tilde{D}(\tilde{p})$ amounts to maximizing $p(a - bp)$. Also see Appendix A3.

a residual demand of $D_s^R(p) = \max\left\{D(p) - (n-s)k, 0\right\}$ for the cartel.

In any period t of cartel operations, the infringement is detected with probability $\alpha \in (0,1)$. We take α to be fixed, i.e., it depends neither on the legal regime as such, nor on a cartel's size or price choices (Katsoulacos et al. 2015, 2020).[6] Whether our findings extend to variable detection rates would obviously depend on the specific functional relation between α and membership, cartel markups, or redress rules.

In case of detection, each active cartel member must pay a fine of $\tau > 0$ times its period t profit (or revenue) and compensate a share $\beta \in (0,1]$ of eligible customers for the associated overcharges.[7] This gives rise to an individual expected profit for a cartel member of

$$\pi_s(p) = p \cdot \underbrace{(1 - \alpha(\beta + \tau))D_s^R(p)/s}_{:= D_s^*(p)} = pD_s^*(p) \tag{6}$$

in the default regime and

$$\pi_s^u(p) = p \cdot \underbrace{\Big((1 - \alpha\tau)D_s^R(p) - \alpha\beta D(p)\Big)/s}_{:= D_s^{*u}(p)} = pD_s^{*u}(p) \tag{7}$$

in the umbrella regime. We refer to $D_s^*(p)$ and $D_s^{*u}(p)$ as *net demand* of a cartel member, subtracting all units that cover expected fines and applicable redress payments from its share of demand. Private antitrust enforcement with parameter β is equivalent to public action with multiplier $\tau' = \beta + \tau$ in the default regime but not the umbrella case. We assume

$$1 - \alpha(\beta + \tau) > 0 \tag{A2}$$

to ensure that an all-encompassing cartel that chooses $p = 2\varepsilon$ would face positive net demand $nD_n^*(p) = nD_n^{*u}(p) \approx \Big(1 - \alpha(\beta + \tau)\Big)D(0)$ and thus be profitable.

A detected cartel can and will immediately resume its activities in period $t+1$, provided the applicable profitability and stability conditions are met. However, cartel

[6] To have a ballpark figure in mind: Bryant and Eckard (1991) estimated the annual probability of getting indicted by federal authorities in the US at between 13% and 17%. Combe et al. (2008) present comparable results for a European sample.

[7] Setting $\beta > 1$ to, e.g., reflect treble damages is fine as long as condition (A2) below is satisfied.

activities are ended for good if a member deviates in any way from the agreed price in period t and all firms revert to $p = 0$ from $t + 1$ on. The best deviation of a cartel member is either to match the non-members' price $p - \varepsilon$ or to undercut them with $p - 2\varepsilon$, depending on outsiders' joint capacity. In either case the dominant effect is to raise the respective firm's net demand from $D_s^*(p)$ or $D_s^{*u}(p)$ to k, and we equate the one-off deviation profit with approximately pk.[8] Before characterizing discount factors δ that make this dynamically unattractive, we check which cartel sizes s would be profitable (besides $s = n$ as guaranteed by (A2)).

3.2. *Profitability*

Consider a partial cartel of $s < n$ members. Its profitability, i.e., $D_s^*(p) > 0$ or $D_s^{*u}(p) > 0$ for $p \geq 2\varepsilon$, depends on aggregate outside capacity $(n - s)k$ in a straightforward way:

Lemma 1. *A cartel of size $s < n$ has positive net demand for $p = 2\epsilon \approx 0$ iff*

(i) $(n - s)k < a$ *when only cartel customers are eligible to seek redress and*

(ii) $(n - s)k < \mu \cdot a$ *when umbrella victims are eligible, too,*

where $\mu := \frac{1-\alpha(\beta+\tau)}{1-\alpha\tau} \in (0, 1)$.

Constant μ will be referred to as *umbrella coefficient*. It scales down the maximum capacities compatible with positive cartel profits in the umbrella regime. Lemma 1 directly implies a minimum size for profitable cartels:

Proposition 1. *A cartel of s members is profitable iff*

(i) $s > n - \frac{a}{k}$ *when only cartel customers are eligible to seek redress and*

(ii) $s > n - \mu \cdot \frac{a}{k}$ *when umbrella victims are eligible, too.*

Size $\underline{s}^u := \lceil n - \mu \cdot \frac{a}{k} \rceil$ *of the smallest profitable cartel in the umbrella regime is never smaller than the corresponding size* $\underline{s} := \lceil n - \frac{a}{k} \rceil \leq n - 2$ *in the default regime; it is greater iff* $\left\lfloor \frac{a}{k} \right\rfloor > \left\lfloor \mu \cdot \frac{a}{k} \right\rfloor$.[9]

For instance, in the parametric example considered in Section 2,[10] we have $3 = \left\lfloor \frac{10}{3} \right\rfloor > \left\lfloor \frac{7}{9} \cdot \frac{10}{3} \right\rfloor = 2$ and the minimum profitable cartel size rises from $\underline{s} = 5 - 3$ to $\underline{s}^u = 5 - 2$.

[8] This assumes that only active cartel members are fined and liable for redress in case of detection (in line with, e.g., Motta and Polo 2003 or Katsoulacos et al. 2015, 2020).

[9] $\lceil x \rceil$ denotes the smallest integer not smaller than x and $\lfloor x \rfloor$ the greatest integer not greater than x. $\underline{s} \leq n - 2$ is implied by (A1).

[10] Namely for $n = 5$, $a = 10$, $b = 1$, $k = 3$, $\alpha = 1/5$, $\beta = 1$, and $\tau = 1/2$.

3.3. *Dynamic Stability*

A profitable cartel with s members is dynamically stable if the present value of profits that accrue from serving net demand $D_s^*(p)$ or $D_s^{*u}(p)$, respectively, at cartel price $p \geq 2\varepsilon$ is at least as great as realizing the one-off deviation profit of approximately pk and then reverting to competition. For any given p with positive net demand, this is equivalent to the requirement that firms' discount factor $\delta \in (0, 1)$ is not smaller than the critical discount factor $\delta_s(p)$ determined by

$$\frac{pD_s^*(p)}{1 - \delta_s(p)} = pk \qquad \Leftrightarrow \qquad \delta_s(p) = 1 - \frac{D_s^*(p)}{k} \tag{8}$$

in the default regime with

$$D_s^*(p) = \frac{(1 - \alpha(\beta + \tau))(a - (n - s)k - bp)}{s}, \tag{9}$$

and analogously

$$\delta_s^u(p^u) = 1 - \frac{D_s^{*u}(p^u)}{k} \tag{10}$$

in the umbrella regime. Computations for the latter are simplified by observing that $D_s^{*u}(p)$ can be written as

$$D_s^{*u}(p) = \frac{(1 - \alpha(\beta + \tau))(a - {(n - s)k}/{\mu} - bp)}{s}, \tag{11}$$

i.e., a cartel member's net demand (profit, etc.) with umbrella compensation equals default net demand (profit, etc.) after virtually scaling up non-members' capacities by the inverse $\mu^{-1} = \frac{1-\alpha\tau}{1-\alpha(\beta+\tau)} > 1$ of the umbrella coefficient.

So focus on the choice of p in the default regime. The unrestricted maximizer of cartel profits $sD_s^*(p)p$ is $p_s^* = \frac{1}{2b}\Big(a - (n - s)k\Big)$. This price can be sustained only if $\delta \geq \delta_s(p_s^*)$. In particular, substituting p_s^* into (8) identifies

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1 - \alpha(\beta + \tau))(a - (n - s)k))}{2sk} \tag{12}$$

as the lowest δ which allows the cartel to stabilize its unrestricted profit maximizer p_s^*.

If otherwise $\delta < \bar{\delta}_s$, cartel members maximize $sD_s^*(p)p$ subject to the constraints $\delta_s(p) = \delta$ and $p \geq 2\varepsilon$. These constraints cannot jointly be satisfied, i.e., dynamic stability of collusion is incompatible with profitability, if $\delta < \underline{\delta}_s$ where

$$\underline{\delta}_s := \delta_s(2\varepsilon) \approx \delta_s(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k))}{sk}. \tag{13}$$

For $\delta \in (\underline{\delta}_s, \bar{\delta}_s)$ a cartel of s members can stabilize some prices $2\varepsilon \leq p_s < p_s^*$. The corresponding constrained profit maximizer $p_s^\circ(\delta)$ is characterized by

$$\delta_s(p_s^\circ) = \delta \qquad \Leftrightarrow \qquad p_s^\circ(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b} \tag{14}$$

and linearly increases in δ from $p_s^\circ(\underline{\delta}) = 2\varepsilon$ to $p_s^\circ(\bar{\delta}) = p_s^*$. Overall, a cartel of s members is profitable and dynamically stable in the default regime iff $\delta \in (\underline{\delta}_s, 1)$ and the respective price overcharge is

$$p_s^\circledast(\delta) = \begin{cases} p_s^\circ(\delta) & \text{if } \delta \in (\underline{\delta}_s, \bar{\delta}_s), \\ p_s^* & \text{if } \delta \in (\bar{\delta}_s, 1). \end{cases} \tag{15}$$

Extending standing to customers of non-members of the cartel reduces both the range of discount factors δ that render collusion of s firms dynamically stable and (constrained) profit maximizers. Namely, using the observation in (11), cartel profits in the umbrella regime, $sD_s^{*u}(p)p$, are maximized by $p_s^{*u} = \frac{1}{2b}\Big(a-(n-s)k\cdot\mu^{-1}\Big)$ if $\delta \geq \bar{\delta}_s^u$ with

$$\bar{\delta}_s^u := \delta_s^u(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1}))}{2sk}. \tag{16}$$

Dynamic stability of collusion clashes with profitability if $\delta < \underline{\delta}_s$ for

$$\underline{\delta}_s^u := \delta_s^u(2\varepsilon) \approx \delta_s^u(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1}))}{sk}, \tag{17}$$

while for $\delta \in (\underline{\delta}_s, \bar{\delta}_s)$ the constrained profit maximizer equals

$$p_s^{\circ u}(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}. \tag{18}$$

So cartel prices in the umbrella regime $p_s^{\circledast u}(\delta)$ are $p_s^{\circ u}(\delta)$ if $\delta \in (\underline{\delta}_s^u, \bar{\delta}_s^u)$ and p_s^{*u} if $\delta \in (\bar{\delta}_s^u, 1)$.

Proposition 2. *For given cartel size s with $\underline{s} \leq \underline{s}^u \leq s < n$, extending legal standing to umbrella victims*

(i) increases critical discount factors to $\underline{\delta}_s^u > \underline{\delta}_s$ and $\bar{\delta}_s^u > \bar{\delta}_s$;

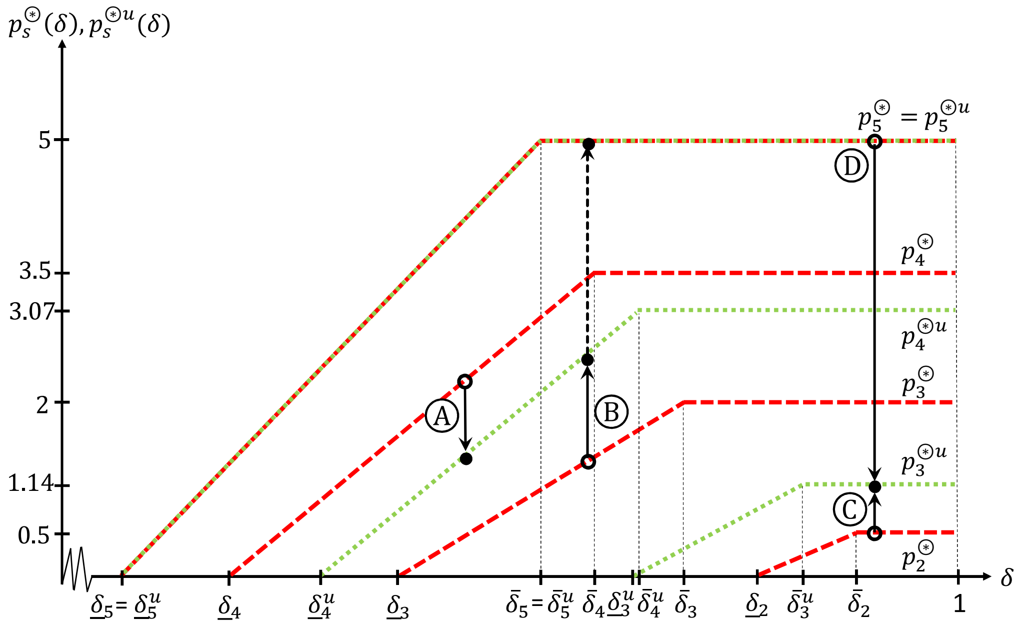


Figure 1: Optimal cartel prices that satisfy dynamic stability for cartel size $s \in \{2, \dots, 5\}$ in default (red) and umbrella (green) regime

(ii) and decreases prices to (approximately) $p_s^{\circledast} > p_s^{\circledast u}$.

$\underline{\delta}_s^u$, $\underline{\delta}_s$, $\bar{\delta}_s^u$, and $\bar{\delta}_s$ are decreasing and prices $p_s^{\circledast}$ and $p_s^{\circledast u}$ are increasing in cartel size $s \in \{2, \dots, n\}$.

Proof. Using (A2), the inequalities in (i) follow directly from $\mu \in (0, 1)$ and hence $(n-s)k\mu^{-1} > (n-s)k$. The latter also implies (ii). Moreover, for $2 < s \leq n$ we have

$$\underline{\delta}_s - \underline{\delta}_{s-1} \approx \frac{(1-\alpha(\beta+\tau))(a-(n-s+1)k))}{(s-1)k} - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k))}{sk} \tag{19}$$

$$= \underbrace{(a-nk)}_{<0} \cdot \underbrace{\frac{1-\alpha(\beta+\tau)}{s(s-1)k}}_{>0} < 0 \tag{20}$$

by (A1) and (A2). $\underline{\delta}_s^u - \underline{\delta}_{s-1}^u < 0$ follows similarly. Also note $\bar{\delta}_s - \bar{\delta}_{s-1} = \frac{1}{2}(\underline{\delta}_s - \underline{\delta}_{s-1})$ and $\bar{\delta}_s^u - \bar{\delta}_{s-1}^u = \frac{1}{2}(\underline{\delta}_s^u - \underline{\delta}_{s-1}^u)$. Finally, the coefficients on s in $p_s^{\circ}(\delta)$ and $p_s^{\circ u}(\delta)$ as well as p_s^* and p_s^{*u} can all be seen to be positive. For instance, eq. (14) can be re-written as

$$p_s^{\circ}(\delta) = \frac{a-nk}{b} + s \cdot \frac{(\delta - \alpha(\beta+\tau))k}{(1-\alpha(\beta+\tau))b} \tag{21}$$

and $p_s^{\circ}(\delta) > 0$ for $\delta > \underline{\delta}_s$ implies $\delta > \alpha(\beta+\tau))k$ because $a < nk$. $\square$

All critical discount factors for the parameters considered earlier (see fn. 10) are shown on the horizontal axis of Figure 1, which traces optimal prices for a profitable cartel of s out of $n = 5$ firms as a function of δ. The highlighted situation Ⓐ involves a profitable cartel of size $s = 4$. As legal standing is extended to umbrella victims, the cartel would cease to be dynamically stable if price $p_4^{\circledast}(\delta)$ were retained. But cartel members can reduce the price to $p_4^{\circledast u}(\delta)$ and continue to operate profitably. Extended legal standing here raises welfare and lowers expected uncompensated overcharges: all customers suffer smaller harm and more of them can gain redress.

But scenario Ⓑ is an alternative possibility: a cartel of $s = 3$ can no longer stabilize profitable collusion in the umbrella regime. It is then conceivable that collusion ends and firms revert to competitive pricing. But it is equally conceivable that membership rises from $s = 3$ to $s' = 4$ or 5 instead, i.e., cartel operations become more encompassing. Whether these – or the original cartel of $s = 3$ members – are structurally stable will be studied below. But note already that $s' = 4$ or 5 comes with prices $p_4^{\circledast u}$ or $p_5^{\circledast u}$ that are considerably greater than $p_3^{\circledast}(\delta)$. This generally lowers welfare and here also raises the expected uncompensated overcharge: more customers can gain redress in the event of detection but otherwise suffer a big increase of harm.

So on the one hand, Proposition 2 confirms the intuition of Blair and Durrance (2018) and several courts that requiring compensation for a greater share of victims increases cartel deterrence for an *exogenous* cartel size $s < n$: it reduces the maximal overcharge compatible with dynamic stability (possibly to zero as, e.g., for $s = 4$ and $\delta \in (\underline{\delta}_4, \underline{\delta}_4^u)$). But at the same time comparative statics regarding s give rise to situations like Ⓑ: for an *endogenous* cartel size, there may be detrimental structural reactions to the umbrella regime that dominate the beneficial effects pertaining to fixed s.

3.4. Structural Stability

Structural reactions to different redress rules – i.e., changes in cartel size – a priori could go either way: an existing small cartel might lose dynamic stability, become more encompassing and prices double (situation Ⓒ in Figure 1), or an existing all-encompassing cartel loses members and prices are quartered (situation Ⓓ). We therefore follow Escrihuela-Villar (2008, 2009) or Bos and Harrington (2010, 2015) and apply stability analysis à la D'Aspremont et al. (1983) to make more refined predictions.[11]

Define $p_s^{\circledast}(\delta) = 0$ for $s \in \{\underline{s} - 1, n + 1\}$. Then a profitable and dynamically stable cartel of size $\underline{s} \le s \le n$ is called *internally stable* in the default regime if

$$\mathcal{I}_s(\delta) := p_s^{\circledast}(\delta) D_s^*\Big(p_s^{\circledast}(\delta)\Big) - p_{s-1}^{\circledast}(\delta) k \ge 0 \tag{22}$$

i.e., each cartel member's per period profit is at least as high as that from becoming a non-member. It is *externally stable* if

$$\mathcal{E}_s(\delta) := p_s^{\circledast}(\delta) k - p_{s+1}^{\circledast}(\delta) D_{s+1}^*\Big(p_{s+1}^{\circledast}(\delta)\Big) \ge 0, \tag{23}$$

i.e., each non-member's profit weakly exceeds that achievable by becoming a member. Cartels of size $s = \underline{s}$ ($s = n$) are automatically internally (externally) stable. Analogous conditions $\mathcal{I}_s^u(\delta) \ge 0$ and $\mathcal{E}_s^u(\delta) \ge 0$ apply in the umbrella regime. Note that $\mathcal{E}_n^u(\delta) = \mathcal{E}_n(\delta)$ and also that eqs. (22) and (23) imply $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}_s^u(\delta) = -\mathcal{I}_{s+1}^u(\delta)$ for $s < n$. A cartel of s members will be called *structurally stable* in a given regime if it is internally and externally stable.

$\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ (and hence $-\mathcal{E}_{s-1}(\delta)$ and $-\mathcal{E}_{s-1}^u(\delta)$) are piecewise polynomial in δ. Figure 2 illustrates the respective quadratic, linear and constant parts for our example. Supposing δ is big enough such that an industry-wide cartel is dynamically stable, internal stability measure $\mathcal{I}_s(\delta)$ increases on $(\underline{\delta}_5, \underline{\delta}_4)$: the constrained profit maximizer $p_5^{\circledast} = p_s^{\circ}$ rises linearly in δ, causing members' profits to rise quadratically, while an outsider to a cartel of only four firms could not earn positive profit because such cartel is dynamically unstable. The latter however becomes dynamically stable for $\delta \ge \underline{\delta}_4$. Then the linear increase of $p_4^{\circledast} = p_4^{\circ}$ raises outsider profits $p_4^{\circledast} k$ proportionally while negative quantity reactions dampen further increases of $p_5^{\circledast} D_5^*(p_5^{\circledast})$ – in total causing $\mathcal{I}_s(\delta)$ to decrease on $(\underline{\delta}_4, \bar{\delta}_5)$. The initially quadratic decrease becomes linear for $\delta \in (\bar{\delta}_5, \bar{\delta}_4)$ since the encompassing cartel charges the constant unconstrained profit maximizer $p_5^{\circledast} = p_5^*$ for $\delta > \bar{\delta}_5$ while $p_4^{\circledast} k = p_4^{\circ} k$ still increases linearly in δ until $\bar{\delta}_4$. Finally for $\delta \in (\bar{\delta}_4, 1)$, the price faced by outsiders to a cartel of four firms also becomes constant, and so does profit difference $\mathcal{I}_5(\delta)$. Analogous variation in δ applies to internal stability of partial cartels and in the umbrella regime.

[11]Similar results obtain for other conceptions of stability as, e.g., in Diamantoudi (2005).

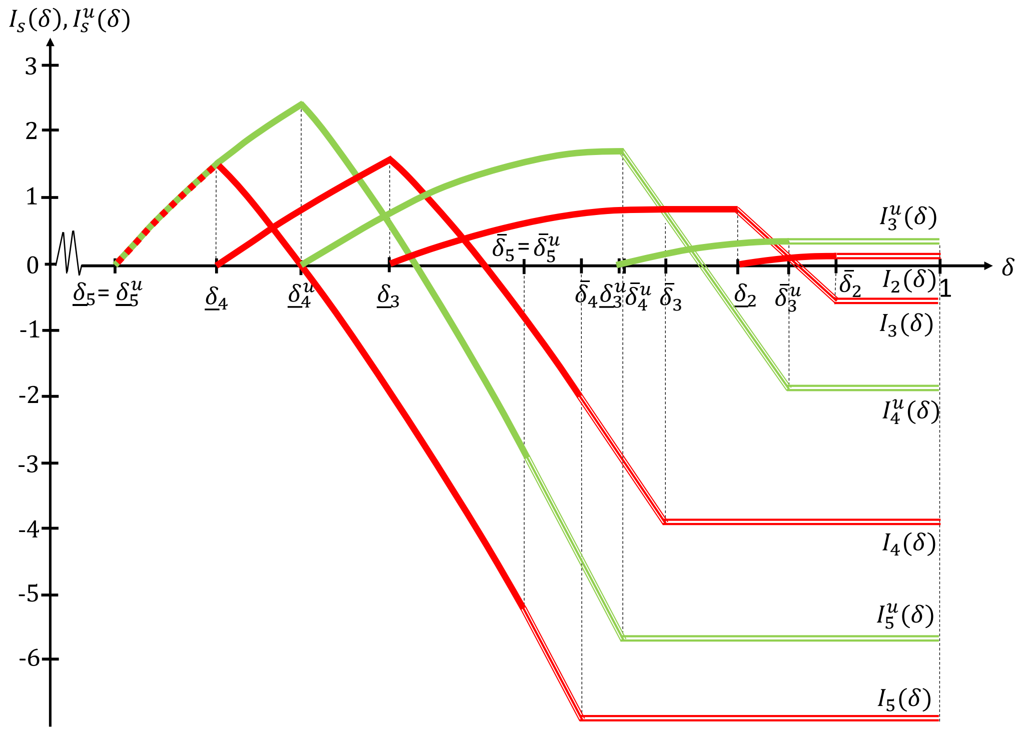


Figure 2: Internal stability measures $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ in default and umbrella regime

Inspection of Figure 2 shows that the range of discount factors where dynamic stability of a cartel entails internal stability is widened by giving legal standing to umbrella victims. Rather tedious case distinctions are necessary regarding which prices $p_s^{\circledast}$, $p_s^{\circledast u}$, $p_{s-1}^{\circledast}$ and $p_{s-1}^{\circledast u}$ in eq. (22) and its analogue for $\mathcal{I}_s^u(\delta)$ correspond to constrained or unconstrained profit maximizers for a given δ, but the phenomenon illustrated by Figure 2 is a general one (the proof is relegated to Appendix A1):

Proposition 3. *Let a profitable cartel of size $s \geq \underline{s}^u$ be dynamically stable in both regimes. If it is internally stable in the default regime, it is also internally stable in the umbrella regime:*

$$\mathcal{I}_s(\delta) > 0 \;\Rightarrow\; \mathcal{I}_s^u(\delta) > 0. \tag{24}$$

The reverse is not true and $\mathcal{I}_s^u(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}_{s-1}^u, 1)$.

Corollary 1. *Every profitable and dynamically stable cartel of size $s \geq \underline{s}^u$ that is externally stable in the umbrella regime is externally stable in the default regime, i.e.,*

$$\mathcal{E}_s^u(\delta) > 0 \;\Rightarrow\; \mathcal{E}_s(\delta) > 0. \tag{25}$$

The reverse is not true and $\mathcal{E}_s(\delta) \geq \mathcal{E}^u_s(\delta)$ *for* $\delta \in (\underline{\delta}^u_s, 1)$.

Proposition 3 and Corollary 1 (implied by $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}^u_s(\delta) = -\mathcal{I}^u_{s+1}(\delta)$) rule out situations like Ⓓ in which a large or even all-encompassing cartel loses members in reaction to extended redress obligations. Whenever the cartel remains profitable and dynamically stable (cf. sections 3.2 and 3.3), its internal stability increases: freeriding on smaller cartels becomes less attractive as these cartels must lower prices the most to maintain their dynamic stability in the umbrella regime. The structural challenge is rather decreasing external stability: non-members – if there are any – may newly want to join in. However, this can only create larger cartels similar to Ⓒ, not smaller ones as in Ⓓ, and never afflicts an all-encompassing cartel.

The possibility that multiple cartel sizes are structurally stable in both regimes and that smaller cartels might somehow become more 'focal' with umbrella compensation can – consistent with findings for static market environments (e.g., Donsimoni et al. 1986; Shaffer 1995) – be ruled out, at least for generic configurations where $\mathcal{I}_s(\delta), \mathcal{I}^u_s(\delta), \mathcal{E}_s(\delta), \mathcal{E}^u_s(\delta) \neq 0$.

Proposition 4. *Structural stability conditions in the default and umbrella regime determine (generically) unique cartel sizes* $s^{*u}(\delta) \geq s^*(\delta)$ *for any given* $\delta > \underline{\delta}_s, \underline{\delta}^u_s$.

Proof. Leaving aside the null set of non-generic discount factors δ where $\mathcal{I}_s(\delta)$ or $\mathcal{I}^u_s(\delta)$ have zeros, external stability of cartels with s members is equivalent to internal instability of cartels with $s+1$ members. So internal (in)stability with s members rules out (implies) external stability with $s-1$ members. For any given $\delta > \underline{\delta}_{\underline{s}}$, the largest $s \in \{\underline{s}, \ldots, n\}$ such that $\mathcal{I}_s(\delta) > 0$ uniquely combines internal stability of size s with external stability (= internal instability of size $s+1$). Hence $s^*(\delta) := \max\left\{s \in \{\underline{s}, \ldots, n\} : \mathcal{I}_s(\delta) > 0\right\}$ is the unique structurally stable cartel size in the default regime. Analogously $s^{*u}(\delta) := \max\left\{s \in \{\underline{s}^u, \ldots, n\} : \mathcal{I}^u_s(\delta) > 0\right\}$ applies to the umbrella regime. Then $s^{*u}(\delta) \geq s^*(\delta)$ follows directly from Proposition 3. □

Sizes of the unique stable cartels and associated prices are illustrated for our example configuration and varying δ in Figure 3. Three intervals of discount factors, with endpoints reflecting zeros of $\mathcal{I}_{s-1}(\delta)$, $\mathcal{I}_s(\delta)$, $\mathcal{I}^u_{s-1}(\delta)$ or $\mathcal{I}^u_s(\delta)$, are highlighted. For discount factors outside of these, a switch from the default to the umbrella redress regime has no effect on the size of structurally stable cartels. A given partial cartel

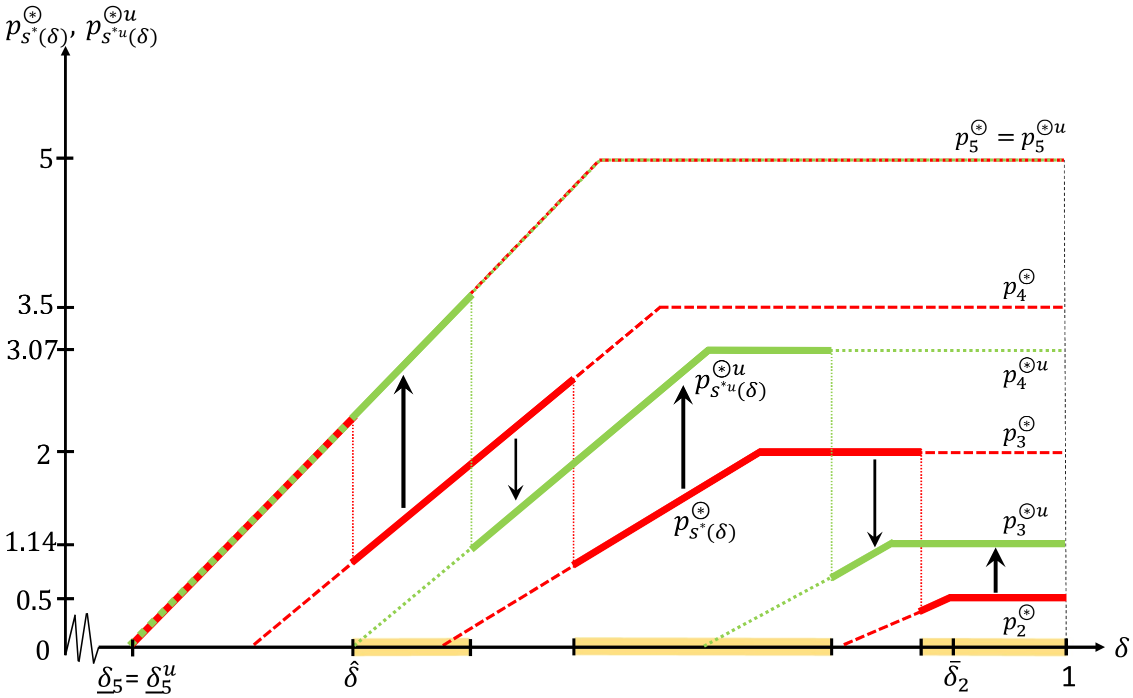


Figure 3: Optimal cartel prices for structurally stable cartel size $s^*(\delta)$ and $s^{*u}(\delta)$ in default and umbrella regime (δ s.t. $p^{\circledast u}_{s^{*u}(\delta)} > p^{\circledast}_{s^*(\delta)}$ highlighted)

must lower its price in order to sustain dynamic stability, so that moving to the umbrella regime lowers damages and raises welfare. However, for discount factors inside the highlighted intervals, a switch to the umbrella regime causes the unique structurally stable cartel size to increase. Disregarding potential transaction costs of such adaptation, the original cartel will become more encompassing. Its prices increase and welfare falls.

The strict inequalities that uniquely define $s^*(\delta)$ and $s^{*u}(\delta)$ for almost all $\delta \in (\underline{\delta}_5^u, 1)$ obviously hold in open neighborhoods of the considered parameters a, b, k, α, β and τ. Intersecting price paths as in Figure 3 where $p^{\circledast u}_{s^{*u}(\delta)}(\delta) > p^{\circledast}_{s^*(\delta)}(\delta)$ for multiple intervals of discount factors hence represent a non-negligible subset of market environments.

We can moreover derive a simple sufficient condition for cartels becoming more encompassing in the umbrella regime, i.e. $s^{*u}(\delta) > s^*(\delta)$. For this, consider large discount factors such that the dynamic stability constraints have slack for all profitable cartels. The maximal size of internally stable cartels is then closely connected to the minimal size of profitable cartels (see Proposition 1):

Lemma 2. *Given $\delta \geq \bar{\delta}_{s-1}$, a cartel of size s is internally stable in the default regime only if*

$$s \leq n - \frac{a}{k} + \underbrace{\frac{k - k(n - \frac{a}{k}) + \sqrt{k^2 + (a - kn)(a - k(n + 2\alpha(\beta + \tau)))}}{(1 + \alpha(\beta + \tau))k}}_{:= \Delta} \tag{26}$$

with $\Delta \in (1, 2)$.

The lemma is proven in Appendix A2. Integer part $\lfloor n - \frac{a}{k} + \Delta \rfloor$ of eq. (26)'s RHS describes the largest internally stable cartels in the default regime when $\delta \geq \bar{\delta}_{\underline{s}}$. Because $\Delta \in (1, 2)$ this must either coincide with size $\underline{s} = \lceil n - \frac{a}{k} \rceil$ of the smallest internally stable cartel characterized by Proposition 1, or is one firm larger. Hence the unique structurally stable cartel size s^* in the default regime satisfies $\underline{s} \leq s^* \leq \underline{s} + 1$. (One can show $\underline{s}^u \leq s^{*u} \leq \underline{s}^u + 1$ analogously.)

Because $\underline{s} = \lceil n - \frac{a}{k} \rceil \leq n - 2$ by Proposition 1, a sufficient condition for $s^{*u} > s^*$ is therefore $\underline{s}^u = \lceil n - \mu \cdot \frac{a}{k} \rceil = n$. The latter is equivalent to $n - \mu \cdot \frac{a}{k} > n - 1$ or $\mu = \frac{1-\alpha(\beta+\tau)}{1-\alpha\tau} < \frac{k}{a}$, where $\frac{1}{n-1} < \frac{k}{a}$ by (A1). Moreover, we know $p_n^{*u} = p_n^* > p_{n-1}^* > p_{n-2}^*$ from Proposition 2. This together implies

Proposition 5. *If δ is sufficiently close to 1 ($\delta \geq \bar{\delta}_{\lceil n - \frac{a}{k} \rceil}$) and $\mu \leq \frac{1}{n-1}$, extending legal standing to umbrella victims increases the unique profitable, dynamically and structurally stable cartel size to $s^{*u} > s^*$, prices rise to (approximately) $p_{s^{*u}}^{*u} > p_{s^*}^*$, and welfare is lowered.*

4. Concluding Remarks

Proposition 5 provides a relatively coarse sufficient condition for extended legal standing to reduce welfare (as is obvious from noting $\mu = \frac{7}{9} > \frac{1}{4} = \frac{1}{n-1}$ in our earlier example). The proposition however shows that is easy to construct other examples in which the umbrella regime gives rise to larger cartels with higher prices: for given n, a, b, and k that satisfy (A1) one can simply combine enforcement parameters α, β, and τ such that $0 < 1 - \alpha(\beta + \tau) \leq \frac{1-\alpha\tau}{n-1}$ with a discount factor close to one.

An obligation to compensate victims of umbrella pricing can clearly have adverse cartel size and welfare effects also for smaller discount factors, but it need not. In our illustrating example, this depends on whether δ lies inside the intervals that are highlighted in Figure 3 or outside. Welfare increases from extending the scope for redress are possible but, alas, limited to two comparatively small intervals (since no umbrella losses accrue for $\delta \in (\underline{\delta}_5, \hat{\delta})$ with $\mathcal{I}_5(\hat{\delta}) = 0$ nor $\delta < \underline{\delta}_5$).[12]

In the light of the stated propositions it would seem quite optimistic to uphold the presumption of extended standing "making a significant contribution to the maintenance of effective competition" (CJEU C-557/12 2014, 23) or "hav[ing] greater deterrent effect than recovery limited to direct purchasers" (Blair and Maurer 1982). We do not wish to doubt here that good arguments for entitling *all* victims of an antitrust infringement to redress, no matter whether they were harmed directly or indirectly, can be put forward.[13] But in contrast with first intuition, these should rather be reasons of distributional justice or legal principle – not deterrence and effective competition.

[12]From an a priori perspective, the magnitude of possible price drops seems unlikely to dominate the smallish range of δ s.t. $p^{*u}_{s^{*u}} < p^*_{s^*}$. For instance, if we assume δ to be distributed uniformly on $(0, 1)$ like Katsoulacos et al. (2015), the expected price $\mathbf{E}p^{*u}_{s^{*u}} \approx 1.01$ in the umbrella regime is 63% higher than $\mathbf{E}p^*_{s^*} \approx 0.62$ in the default regime.

[13]See Argenton et al. (2020, p. 269), however, for caveats related to the open question: "[W]here to stop the causal chain set in motion by the initial liability-generating behaviour?" Upstream firms that supplied (non-)cartel members, producers of complement goods and their suppliers, etc. may all have suffered indirect harm, too. Going beyond a partial equilibrium framework one might even pinpoint ripple effects on wages or bond rates.

Appendix

A1. Proof of Proposition 3

That $\mathcal{I}^u_s(\delta) > 0$ does not imply $\mathcal{I}_s(\delta) > 0$ is obvious from our example. It is also obvious that implication (24) is true for $\delta \in (\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$ because $\mathcal{I}^u_s(\underline{\delta}^u_s) = 0$ and $\mathcal{I}^u_s(\delta)$ strictly increases on $(\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$. (Recall that a cartel of s firms is dynamically stable in both regimes if $\delta \in (\underline{\delta}^u_s, 1)$.)

It remains to show $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on the interval $(\underline{\delta}^u_{s-1}, 1)$ where sub-cartels of size $s-1 \geq \underline{s}^u$ would be profitable and dynamically stable. Depending on market parameters and considered cartel size s, this interval can be split into quadratic, linear and constant parts of $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ differently from Figure 2. Namely, several pairwise comparisons of the critical discount factors that determine if $p^{\circledast}_s(\delta) = p^{\circ}_s(\delta)$ or p^*_s and if $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ or p^*_{s-1} in eq. (22), and its analogue, can go either way. Figure 4 shows the Hasse diagram of the corresponding partially ordered set (cf. Proposition 2). This gives rise to seven possible cases. Before turning to each, we establish some properties that hold whenever one or two pairwise comparisons, e.g., between $\underline{\delta}_{s-1}$ and $\bar{\delta}_s$, go in a particular way.

Claim 1. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} < \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is strictly concave and decreasing on $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left((\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ and linearly decreasing on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ $\left((\bar{\delta}^u_s, \bar{\delta}^u_{s-1})\right)$. $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ falls faster in the latter intervals than in the former.

Proof. For $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ dynamic stability constraints are binding for cartel sizes s and $s-1$. Substituting $p^{\circledast}_s(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast u}_s(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ and $p^{\circledast u}_{s-1}(\delta) = p^{\circ u}_{s-1}(\delta)$ into $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ and abbreviating $e := 1 - \alpha(\beta+\tau) \in (0,1)$, one obtains

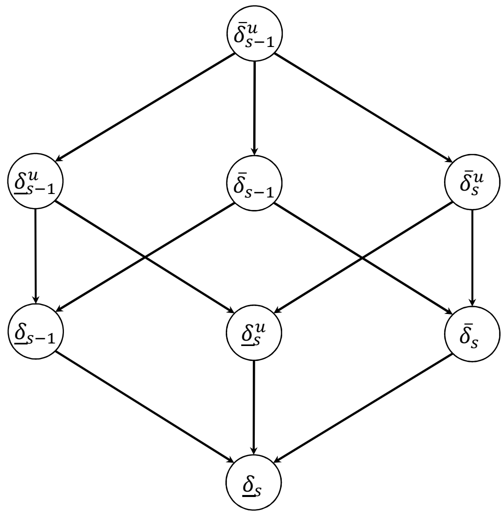
$$\mathcal{I}_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta + k\delta(n+1-s\delta) - k\alpha(1+n\delta-s\delta)(\beta+\tau)\Big] \tag{27}$$

and

$$\mathcal{I}^u_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta - k\alpha\tau + k\delta\Big(n+1-s\delta-(n-s)\alpha\tau\Big)\Big] \tag{28}$$

after some algebra. Corresponding derivatives with respect to δ are

$$\frac{\partial\mathcal{I}_s(\delta)}{\partial\delta} = \frac{k}{b}\Big[-a + k(n-s) - k\Big(s(2\delta-1)-1\Big)e^{-1}\Big] \tag{29}$$



Case	Ordering
1	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s-1}$
2	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
3	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s} < \bar{\delta}^u_{s-1}$
4	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
5	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s} < \bar{\delta}^u_{s-1}$
6	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s} < \bar{\delta}^u_{s} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
7	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s} < \bar{\delta}^u_{s-1}$

Figure 4: Hasse diagram for critical discount factors ($x \to y$ indicating $x > y$) and compatible orderings that partition $(\underline{\delta}^u_{s-1}, 1)$

and

$$\frac{\partial \mathcal{I}^u_s(\delta)}{\partial \delta} = \frac{k}{b}\Big[-a + k\Big(1 + n - 2s\delta - (n-s)\alpha\tau\Big)e^{-1}\Big] \tag{30}$$

with

$$\frac{\partial^2 \mathcal{I}_s(\delta)}{\partial \delta^2} = \frac{\partial^2 \mathcal{I}^u_s(\delta)}{\partial \delta^2} = \frac{-2k^2 s}{be} < 0. \tag{31}$$

So $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ are strictly concave in δ.

Substituting $\delta = \underline{\delta}_{s-1}$ from eq. (13) into (29) yields

$$\frac{\partial \mathcal{I}_s(\underline{\delta}_{s-1})}{\partial \delta} = \frac{k}{be}\Bigg(\frac{e(s+1)(a-kn)}{s-1} - k(s\alpha(\beta+\tau) - 1)\Bigg) < 0. \tag{32}$$

Inequality (32) is satisfied iff

$$\frac{(s+1)(a-kn)}{s-1} < \frac{k(s\alpha(\beta+\tau)-1)}{1-\alpha(\beta+\tau)}. \tag{33}$$

Making the LHS as large as possible, that is, substituting $a = k(n-1)$ yields

$$\begin{aligned}
&-\frac{s+1}{s-1} < \frac{s\alpha(\beta+\tau)-1}{1-\alpha(\beta+\tau)} \\
\Leftrightarrow\ & -(s+1)(1-\alpha(\beta+\tau)) < (s-1)(s\alpha(\beta+\tau)-1) \\
\Leftrightarrow\ & 0 < 2-\alpha(\beta+\tau)+s^2\alpha(\beta+\tau)-2s\alpha(\beta+\tau) \\
\Leftrightarrow\ & 0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha(\beta+\tau)(s-2)}_{>0}.
\end{aligned} \tag{34}$$

Similarly substituting $\delta = \underline{\delta}^u_{s-1}$ from eq. (17) into (30) gives

$$\frac{\partial \mathcal{I}^u_s(\underline{\delta}^u_{s-1})}{\partial \delta} = \frac{-k}{b(s-1)e}(-a(1+s)e - k(n(1+s)(-1+\alpha\tau)+(s-1)(1-s\alpha\tau))) < 0 \tag{35}$$

which is satisfied iff

$$0 < -a(1+s)e - k(n(1+s)(-1+\alpha\tau)+(s-1)(1-s\alpha\tau)). \tag{36}$$

Making the RHS as small as possible by substituting $a = k(n-1)$ yields

$$\begin{aligned}
&0 < -(n-1)(1+s)e - n(1+s)(-1+\alpha\tau) - (s-1)(1-s\alpha\tau) \\
\Leftrightarrow\ & 0 < -n(1-\alpha\beta-\alpha\tau) - ns(1-\alpha\beta-\alpha\tau) + (1-\alpha\beta-\alpha\tau) + s(1-\alpha\beta-\alpha\tau) + n - n\alpha\tau + ns \\
& - ns\alpha\tau - s + s^2\alpha\tau + 1 - s\alpha\tau \\
\Leftrightarrow\ & 0 < 2 - \alpha\beta - \alpha\tau + s^2\alpha\tau - 2s\alpha\tau + n\alpha\beta + ns\alpha\beta - s\alpha\beta \\
\Leftrightarrow\ & 0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha\tau(s-2)}_{\geq 0} + \underbrace{s\alpha\beta(n-1)}_{>0}.
\end{aligned} \tag{37}$$

So both derivatives $\frac{\partial \mathcal{I}_s(\delta)}{\partial \delta}$ and $\frac{\partial \mathcal{I}^u_s(\delta)}{\partial \delta}$ are negative at the respective left endpoints of $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$. By (31) they are falling. Hence the slopes of $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ must be negative for all $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$, respectively.

For $\delta \in (\bar{\delta}_s, \bar{\delta}_{s-1})$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Profits of s cartel members consequently become constant, too, and $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have slope

$$-k\frac{\partial p^{\circ}_{s-1}(\delta)}{\partial \delta} = -k\frac{\partial p^{\circ u}_{s-1}(\delta)}{\partial \delta} = -\frac{(s-1)k^2}{eb}. \tag{38}$$

This is less than the slopes identified for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ in eq. (29) $\left(\text{eq. (30)}\right)$ where cartel profits still increase in δ. This proves Claim 1. ■

Claim 2. If $\underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is linearly decreasing for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_{s-1})\right)$.

Proof. For $\delta > \underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\delta > \underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Collusive profits of s members then are constant in δ whereas cartel prices and profits of an outsider are linearly increasing in δ for a cartel of size $s-1$. Hence $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have the slope already identified in eq. (38). ■

Claim 3. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$ then $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$.

Proof. $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ are then given by eqs. (27) and (28) above. Hence

$$\begin{aligned}\mathcal{I}^u_s(\delta) - \mathcal{I}_s(\delta) &= \frac{k}{be}\Big[-k\alpha\tau - k\delta(n-s)\alpha\tau + k\alpha(1+n\delta-s\delta)(\beta+\tau)\Big] \\ &= \frac{k^2\alpha\beta}{be}\Big(1+(n-s)\delta\Big) > 0.\end{aligned} \tag{39}$$

■

Claim 4. $\Delta\bar{\delta}^u_s := \bar{\delta}^u_{s-1} - \bar{\delta}^u_s > \bar{\delta}_{s-1} - \bar{\delta}_s := \Delta\bar{\delta}_s$.

Proof. Applying eq. (12) and simplifying yields

$$\Delta\bar{\delta}_s = \frac{-e(a-(n-s)k)2(s-1)k + 2ske(a-(n-(s-1))k)}{4sk^2(s-1)} = -\frac{(a-kn)e}{2k(s-1)s} \tag{40}$$

and similarly eq. (16) gives

$$\Delta\bar{\delta}^u_s = -\frac{ae-kn(1-\alpha\tau)}{2k(s-1)s}. \tag{41}$$

So

$$\Delta\bar{\delta}^u_s - \Delta\bar{\delta}_s = \frac{-ae+kn(1-\alpha\tau)+ae-kn(1-\alpha\beta-\alpha\tau)}{2k(s-1)s} = \frac{n\alpha\beta}{2(s-1)s} > 0. \tag{42}$$

■

Claim 5. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}^u_{s-1}, 1)$.

Proof. $\delta > \bar{\delta}^u_{s-1}$ implies $\delta > \bar{\delta}_{s-1}$ and $\delta > \bar{\delta}^u_s \geq \bar{\delta}_s$. Hence, all relevant dynamically stable cartels can choose the respective unconstrained profit maximizers $p^{\circledast}_{s-1}(\delta) = p^*_{s-1} =$

$\frac{1}{2b}\Big(a-(n-(s-1))k\Big)$, $p^{\circledast u}_{s-1}(\delta)=p^{*u}_{s-1}=\frac{1}{2b}\Big(a-(n-(s-1))k\cdot\mu^{-1}\Big)$, $p^{\circledast u}_{s}(\delta)=p^{*u}_{s}$ and $p^{\circledast}_{s}(\delta)=p^{*}_{s}$. Again abbreviating $e:=1-\alpha(\beta+\tau)\in(0,1)$ and using $\mu^{-1}=1+\frac{\alpha\beta}{e}$, $\mathcal{I}_s(\delta)<\mathcal{I}^u_s(\delta)$ holds iff

$$
\begin{aligned}
& \frac{e\Big(a-(n-s)k\Big)^2}{4bs}-\frac{a-(n-s+1)k}{2b}k<\frac{e\Big(a-(n-s)k\mu^{-1}\Big)^2}{4bs}-\frac{a-(n-s+1)k\mu^{-1}}{2b}k\\
\Leftrightarrow\quad & e\Big(a-(n-s)k\Big)^2+2ks\Big((n-s+1)k-a\Big)<\\
& \qquad e\Big(a-(n-s)k-\frac{\alpha\beta}{e}(n-s)k\Big)^2+2ks\Big((n-s+1)k\mu^{-1}-a\Big)\\
\Leftrightarrow\quad & 2k^2s(n-s+1)<-2e\Big(a-k(n-s)\Big)\frac{\alpha\beta}{e}k(n-s)+e\Big(\frac{\alpha\beta}{e}k(n-s)\Big)^2+2k^2s(n-s+1)\mu^{-1}\\
\Leftrightarrow\quad & 0<2\alpha\beta\Big(k(n-s)-a\Big)k(n-s)+\frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2-2k^2s(n-s+1)(1-\mu^{-1})\\
\Leftrightarrow\quad & 0<2\alpha\beta\Big(k(n-s)-a\Big)k(n-s)+\frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2+2k^2s(n-s+1)\frac{\alpha\beta}{e}\\
\Leftrightarrow\quad & 0<2\Big(k(n-s)-a\Big)k(n-s)e+\alpha\beta\Big(k(n-s)\Big)^2+2k^2s(n-s+1). \qquad (43)
\end{aligned}
$$

The RHS of (43) is decreasing in a. So it suffices to observe it is positive for the maximal value $a=k(n-1)$ that satisfies (A1). In particular,

$$
\begin{aligned}
& 0<2k\Big(k(n-s)-k(n-1)\Big)(n-s)e+\alpha\beta\Big(k(n-s)\Big)^2+2k^2s(n-s+1)\\
\Leftrightarrow\quad & 0<k^2\Big[2(-s+1)(n-s)e+\alpha\beta(n-s)^2+2s(n-s+1)\Big]\\
\Leftrightarrow\quad & 0<2(1-s)(n-s)e+\alpha\beta(n-s)^2+2s(n-s+1) \qquad (44)\\
\Leftrightarrow\quad & 0<2(n-s)e-2s(n-s)\Big(1-\alpha(\beta+\tau)\Big)+\alpha\beta(n-s)^2+2s+2s(n-s)\\
\Leftrightarrow\quad & 0<\underbrace{2(n-s)e+2s(n-s)\alpha(\beta+\tau)+\alpha\beta(n-s)^2}_{\geq 0}+\underbrace{2s}_{>0}.
\end{aligned}
$$

■

We are now ready to verify $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ for $\delta\in(\underline{\delta}^u_{s-1},1)$ in the seven cases identified in Figure 4:

Case 1 with $\ldots<\underline{\delta}^u_{s-1}<\bar{\delta}^u_{s-1}$. Then $\delta\in(\underline{\delta}^u_{s-1},1)$ implies $\delta>\bar{\delta}_{s-1},\bar{\delta}_s$. So $p^{\circledast}_{s-1}(\delta)=p^{*}_{s-1}$ and $p^{\circledast}_{s}(\delta)=p^{*}_{s}$ in the default regime, which renders $\mathcal{I}_s(\delta)$ constant for $\delta\in(\underline{\delta}^u_{s-1},1)$. $\mathcal{I}^u_s(\delta)$ linearly decreases from $\mathcal{I}^u_s(\underline{\delta}^u_{s-1})$ to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1})$ according to claim 2 and then stays constant. By claim 5, $\mathcal{I}^u_s(\bar{\delta}^u_{s-1})>\mathcal{I}_s(\bar{\delta}^u_{s-1})$. Hence $\mathcal{I}^u_s(\underline{\delta}^u_{s-1})>\mathcal{I}_s(\underline{\delta}^u_{s-1})$ to avoid a contradiction. So $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ holds for all $\delta\in(\underline{\delta}^u_{s-1},1)$.

Case 2 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. For $\delta \in (\bar{\delta}^u_{s-1}, 1)$ we have $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ according to claim 5. Claim 2 ensures that $\mathcal{I}^u_s(\delta)$ falls linearly on $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_{s-1})$ and in particular on sub-interval $(\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$ where $\mathcal{I}_s(\delta)$ is constant. Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$ in order not to contradict $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$. For $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$, both $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ decrease with identical slope (invoking claim 1 or 2 depending on $\bar{\delta}_s \lesseqgtr \underline{\delta}_{s-1}$). Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ must also hold for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$.

Case 3 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 1.

Case 4 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. That $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$ can be concluded from claims 2 and 5 just as for case 1. $\mathcal{I}_s(\delta)$ decreases linearly with slope $-\frac{(s-1)k^2}{eb}$ on $(\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ for $\bar{\delta}_s < \underline{\delta}_{s-1}$ and on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ for $\bar{\delta}_s > \underline{\delta}_{s-1}$, and so does $\mathcal{I}^u_s(\delta)$ on $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ (claim 2). Hence, considering $\delta \geq \underline{\delta}^u_{s-1}$, $\mathcal{I}_s(\delta)$ assumes a maximum of $\mathcal{I}_s(\underline{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1}) + \frac{(s-1)k^2}{eb} \cdot (\bar{\delta}_{s-1} - \underline{\delta}^u_{s-1})$ at $\delta = \underline{\delta}^u_{s-1}$. $\mathcal{I}^u_s(\delta)$ exceeds that value at $\bar{\delta}^u_s > \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\}$ and assumes even higher values on $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$ because it is decreasing on this interval to $\mathcal{I}_s(\underline{\delta}^u_{s-1})$.[14] Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 5 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 4.

Case 6 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. Given $\underline{\delta}_{s-1} < \underline{\delta}^u_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$, claim 3 yields $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$. Then $\mathcal{I}_s(\delta)$ falls linearly on $(\bar{\delta}_s, \bar{\delta}^u_s)$ while $\mathcal{I}^u_s(\delta)$ decreases in a slower strictly concave fashion for $\delta \in (\bar{\delta}_s, \bar{\delta}^u_s)$ (claim 1). So $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on $(\bar{\delta}_s, \bar{\delta}^u_s)$. For $\delta \in (\bar{\delta}^u_s, \bar{\delta}_{s-1})$, both functions fall linearly with same slope (claim 1), extending $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ at $\delta = \bar{\delta}^u_s$ to interval $(\bar{\delta}^u_s, \bar{\delta}_{s-1})$. $\mathcal{I}_s(\delta)$ turns constant for $\delta \in (\bar{\delta}_{s-1}, 1)$ while $\mathcal{I}^u_s(\delta)$ continues its decrease – but only to a value of $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1})$ (claim 5). Then $\mathcal{I}^u_s(\delta)$ turns constant too, implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 7 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$. For $\delta \in (\bar{\delta}_{s-1}, 1)$, $\mathcal{I}_s(\delta)$ is constant. By contrast, $\mathcal{I}^u_s(\delta)$ is constant to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$ on $(\bar{\delta}^u_{s-1}, 1)$ (claim 5) and, focusing on $(\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$, decreasing to this value from $\mathcal{I}^u_s(\bar{\delta}_{s-1})$ – implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ can be concluded in direct analogy to case 6. □

[14] That $\mathcal{I}^u_s(\bar{\delta}^u_s) > \mathcal{I}_s(\underline{\delta}^u_{s-1})$ is most easily seen by looking at $\mathcal{I}^u_s$'s behavior from the right, i.e., moving down from $\delta = 1$ to $\delta = \underline{\delta}^u_{s-1}$: it switches from constant to increasing with slope $\left|-\frac{(s-1)k^2}{eb}\right|$ already at $\bar{\delta}^u_{s-1} > \bar{\delta}_{s-1}$ and continues this increase over an interval $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ that is wider than the corresponding interval $(\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ of $\mathcal{I}_s$'s increase for $\delta \geq \underline{\delta}^u_{s-1}$ because $\bar{\delta}_{s-1} - \bar{\delta}_s < \bar{\delta}^u_{s-1} - \bar{\delta}^u_s$ (claim 4) and $\bar{\delta}_s \leq \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\} < \underline{\delta}^u_{s-1}$ imply $\bar{\delta}_{s-1} - \bar{\delta}^u_{s-1} < \underline{\delta}^u_{s-1} - \bar{\delta}^u_s$.

A2. Proof of Lemma 2

For $\delta \geq \bar{\delta}_{s-1}$ and abbreviating $e := 1 - \alpha(\beta + \tau) \in (0,1)$, $\mathcal{I}_s(\delta) \geq 0$ is equivalent to

$$\begin{aligned}
&\frac{e[a-(n-s)k]^2}{4bs} - \frac{a-\big(n-(s-1)\big)k}{2b} \cdot k \geq 0 \\
\Leftrightarrow\quad & e[a^2 - 2akn + 2aks + n^2k^2 - 2nk^2s + k^2s^2] - [2aks - 2k^2ns + 2k^2s^2 - 2k^2s] \geq 0 \qquad (45)\\
\Leftrightarrow\quad & k^2(e-2)\cdot s^2 + 2\Big[(e-1)k[a-nk] + k^2\Big]\cdot s + e[a^2 - 2akn + k^2n^2] \geq 0.
\end{aligned}$$

This implies

$$\begin{aligned}
s &\leq \frac{(e-1)(\frac{a}{k}-n)+1}{2-e} + \sqrt{\left(\frac{(e-1)(\frac{a}{k}-n)+1}{2-e}\right)^2 - \frac{e(a-kn)^2}{(e-2)k^2}} \\
&= n - \frac{a}{k} + \frac{k - k(n-\frac{a}{k}) + \sqrt{\big((e-1)(a-kn)+k\big)^2 - e(e-2)(a-kn)^2}}{(2-e)k} \qquad (46)\\
&= n - \frac{a}{k} + \frac{k - k(n-\frac{a}{k}) + \sqrt{k^2 + (a-kn)\big(a-kn+2k(e-1)\big)}}{(2-e)k}
\end{aligned}$$

as claimed. Moreover, $\Delta > 1$ means

$$\sqrt{k^2 + (a-kn)\big(a-kn+2k(e-1)\big)} > (2-e)k + k(n-1) - a, \qquad (47)$$

which, noting $k(n+1) - a + (2-e)k > 0$, is equivalent to

$$\begin{aligned}
& k^2 + (a-kn)\big(a-kn+2k(e-1)\big) > \\
& \quad (2-e)^2k^2 + k^2(n-1)^2 + a^2 + 2(n-1)(2-e)k^2 - 2ak(n-1) - 2a(2-e)k \\
\Leftrightarrow\quad & k^2 + a^2 - 2akn + 2ak(e-1) + k^2n^2 - 2k^2n(e-1) > \\
& \quad k^2[4-4e+e^2] + k^2[n^2-2n+1] + a^2 + k^2[4n-2en-4+2e] - 2akn - 2ak + 2aek \\
\Leftrightarrow\quad & k - 2an + 2a(e-1) + kn^2 - 2kn(e-1) > k[n^2+2n+1-2e+e^2-2en] - 2an - 2a + 2ae \\
\Leftrightarrow\quad & 0 > ke(e-2) \qquad (48)
\end{aligned}$$

and guaranteed by $e \in (0,1)$. However, assuming $\Delta \geq 2$ is equivalent to

$$\sqrt{k^2 + (a-kn)\big(a-kn+2k(e-1)\big)} \geq k(3+n-2e) - a \qquad (49)$$

and implies

$$
\begin{aligned}
& k^2 + a^2 - 2akn + 2ak(e-1) + k^2n^2 - 2k^2n(e-1) \geq k^2(3+n-2e)^2 - 2ak(3+n-2e) + a^2 \\
\Leftrightarrow\quad & k - 2an + 2a(e-1) + kn^2 - 2kn(e-1) \geq k(3+n-2e)^2 - 2a(3+n-2e) \\
\Leftrightarrow\quad & k\Big(1 + n^2 - 2n(e-1) - (3+n-2e)^2\Big) \geq 2an - 2a(e-1) - 2a(3+n-2e) \\
\Leftrightarrow\quad & k\Big(1 + n^2 - 2n(e-1) - (9 + n^2 + 4e^2 + 6n - 12e - 4en)\Big) \geq 2a(e-2) \\
\Leftrightarrow\quad & k\Big(n + 2(1-e)\Big)(e-2) \geq 2a(e-2) \\
\Leftrightarrow\quad & k \leq \frac{a}{n + 2(1-e)} \qquad (50)
\end{aligned}
$$

in contradiction to (A1). □

A3. Non-Linear Demand

The closed-form expressions that can be derived for linear demand make the comparison of default vs. umbrella regime particularly transparent. Linear demand is however not essential for our key message: extending compensation to 'umbrella losses' can have unintended cartel size effects and raise prices. For instance, a quadratic demand function like $D(p) = 10 - p^2$ changes the specific numbers reported in Table 1 but leaves the unique structurally stable cartel sizes $s^* = 2 < s^{*u} = 3$ unchanged (respective cartels satisfying dynamic stability for $\delta \approx 1$). It also entails $p^{*u}_{s^{*u}} > p^*_{s^*}$.

Of course, evidence of a phenomenon such as increased cartel size and higher prices for some elements of a given set of market environments a fortiori constitutes evidence also when considering supersets of them (i.e., allowing non-linear demand, asymmetric capacities, etc.). It is still worth noting that several of the more specific findings in the main text extend rather straighforwardly to variations of our baseline setup. For illustration consider potentially positive unit costs $c \geq 0$ and a general smooth demand function $D(p)$ with $D'(p) < 0$ such that $(p - c) \cdot D(p)$ is strictly concave. Let $D(c + 2\varepsilon) \approx D(c) =: a > 0$ satisfy

$$
\frac{a}{n-1} \leq k < q^m \qquad \text{(A1')}
$$

for given k and n, where $q^m > 0$ is the monopoly output induced by c and $D(p)$. (A2) ensures that cartel profit functions

$$\pi_s(p) = (p-c) \cdot \underbrace{(1-\alpha(\beta+\tau))D_s^R(p)/s}_{:= D_s^*(p)} = (p-c)D_s^*(p) \tag{51}$$

and

$$\pi_s^u(p) = (p-c) \cdot \underbrace{\Big((1-\alpha\tau)D_s^R(p) - \alpha\beta D(p)\Big)/s}_{:= D_s^{*u}(p)} = (p-c)D_s^{*u}(p) \tag{52}$$

with $D_s^R(p) = D(p) - (n-s)k$ are also strictly concave and $p = c$ is the competitive benchmark. Profitability requirements $D_s^*(c+2\varepsilon) \approx D_s^*(c) > 0$ and $D_s^{*u}(c+2\varepsilon) \approx D_s^{*u}(c) > 0$ for partial cartels of s members are unchanged. So Lemma 1 and Proposition 1 still apply without any essential modification.

Dynamic stability of a profitable cartel requires

$$\delta \geq \delta_s(p) = 1 - \frac{D_s^*(p)}{k} \quad \text{and} \quad \delta \geq \delta_s^u(p^u) = 1 - \frac{D_s^{*u}(p^u)}{k} \tag{53}$$

and unconstrained profit maximizers p_s^* and p_s^{*u} must satisfy

$$(p-c) \cdot D'(p) + D(p) = (n-s)k \tag{54}$$

in the default and

$$(p-c) \cdot D'(p) + D(p) = (n-s)k/\mu \tag{55}$$

in the umbrella regime. The LHS of eq. (54) and eq. (55) coincide and decrease in p, while the RHS of eq. (55) is greater than that of eq. (54) given $\mu < 1$. Hence $p_s^* > p_s^{*u}$.

Constrained profit maximizers p_s° and $p_s^{\circ u}$ for small δ are determined by

$$\delta = \delta_s(p_s^\circ) \qquad \Leftrightarrow \qquad p_s^\circ(\delta) = D_s^{*-1}\Big((1-\delta)k\Big) \tag{56}$$

and

$$\delta = \delta_s(p_s^{\circ u}) \qquad \Leftrightarrow \qquad p_s^{\circ u}(\delta) = D_s^{*u-1}\Big((1-\delta)k\Big). \tag{57}$$

Because $D_s^{*u}(p) < D_s^*(p)$ for all relevant p, any given sales q go with lower prices $D_s^{*u-1}(q) < D_s^{*-1}(q)$ in the umbrella regime and $q = (1-\delta)k$ implies $p_s^\circ(\delta) > p_s^{\circ u}(\delta)$.

The minimal values of δ that permit a cartel of s members to be dynamically stable are

$$\underline{\delta}_s := \delta_s(c+2\varepsilon) \approx \delta_s(c) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k))}{sk} \tag{58}$$

and

$$\underline{\delta}_s^u := \delta_s^u(c+2\varepsilon) \approx \delta_s^u(c) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{sk} \tag{59}$$

just as for linear demand. $\mu^{-1} > 1$ implies $\underline{\delta}_s^u > \underline{\delta}_s$ exactly as in Proposition 2(i).

The main caveat regarding Proposition 2 is that the critical discount factors that permit unconstrained pricing,

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1-\alpha(\beta+\tau))D(p_s^*) - (1-\alpha(\beta+\tau))(n-s)k}{sk} \tag{60}$$

and

$$\bar{\delta}_s^u := \delta_s^u(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))D(p_s^{*u}) - (1-\alpha\tau)(n-s)k}{sk}, \tag{61}$$

satisfy $\bar{\delta}_s^u > \bar{\delta}_s$ for linear demand but the opposite relation can hold in general: prices in the umbrella regime may turn constant from $p_s^{\circ u}(\delta)$ to p_s^{*u} for a smaller δ than optimal prices in the default regime (e.g., $\bar{\delta}_s^u < \bar{\delta}_s$ for $c = 0$ and $D(p) = 10 - p^2$). Still $p_s^\circ(\delta) > p_s^{\circ u}(\delta)$ and $p_s^* > p_s^{*u}$ imply that observation $p_s^{\circledast u} < p_s^{\circledast}$ in Proposition 2(ii) is robust.[15]

Unfortunately, the (already unpleasant) case distinctions that are needed to prove increased internal stability, $\mathcal{I}_s(\delta) > 0 \Rightarrow \mathcal{I}_s^u(\delta) > 0$, and uniqueness of stable sizes $s^*(\delta) \geq s^{*u}(\delta)$ in Propositions 3 and 4 become intractable for general demand functions. Even a comparison of $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ for δ close to 1 is very unwieldy if prices and ensuing profits are defined only implicitly. An exception is that $\mathcal{I}_n^u(\delta) > \mathcal{I}_n(\delta)$ for all $\delta > \underline{\delta}_{s-1}^u$: this holds because collusive profits for $s = n$ are identical in both regimes while a non-member's approximate profits for $s = n-1$ satisfy $(p_{s-1}^{\circledast u} - c)k < (p_{s-1}^{\circledast} - c)k$.

[15] Assuming $\delta_n = \delta_n^u = 1 - \Big(1-\alpha(\beta+\tau))D(p_n^*)\Big)/(nk) > 1$ yields a contradiction to (A2). So $p_n^{\circledast} = p_n^{\circledast u}$ must switch from increasing function $p_n^\circ(\delta)$ to constant p_n^* for some $\bar{\delta}_n = \bar{\delta}_n^u < 1$, as in Figure 1.

Concerning Proposition 5, Lemma 1 ensures that (a) only the industry-wide cartel is profitable in the umbrella regime (hence $s^{*u} = n$ for δ sufficiently close to 1) while (b) both $s = n - 1$ and $s = n$ are profitable in the default regime when $\mu a < k < q^m$. This inequality can always be satisfied by an appropriate choice of parameters α, β and τ. Then $s^{*u} = n > s^*$ is implied if

$$\mathcal{I}_n(\delta) = \frac{1 - \alpha(\beta + \tau)}{n}(p_n^* - c)D(p_n^*) - (p_{n-1}^* - c)k < 0. \tag{62}$$

So, given (A2), a sufficient condition analogous to Proposition 5 is the existence of p_n and p_{n-1} such that

$$(p_n - c)D(p_n) < (p_{n-1} - c)kn \tag{63}$$

$$(p_n - c)D'(p_n) = -D(p_n) \tag{64}$$

$$(p_{n-1} - c)D'(p_{n-1}) = k - D(p_{n-1}). \tag{65}$$

Unique solutions p_n and p_{n-1} to eqs. (64) and (65) are ensured by the curvature restriction on $(p - c)D(p)$ and k's upper bound in (A1′). These solutions do not depend on n. Moreover, larger n relaxes the lower bound imposed by (A1′). Hence for given c, $D(p)$ and $k < q^m$ we can jointly choose: parameters α, β and τ big enough to ensure profitability of cartel sizes n and $n - 1$ in the default regime, but only n in the umbrella regime; δ big enough to guarantee dynamic stability; and n big enough to satisfy eq. (63). We here exploit that if partial cartels are profitable then sharing a fixed monopoly rent among *all* firms becomes unstable for large n. Hence stable cartel sizes satisfy $s^{*u} = n > n - 1 \geq s^*$, prices rise to (approximately) $p_{s^{*u}}^{*u} > p_{s^*}^*$ and welfare is lowered. In other words: by treating n as a free rather than given parameter one can obtain examples where the umbrella regime induces larger cartels, higher prices, etc. in direct analogy to our linear baseline and Proposition 5.

References

Argenton, C., D. Geradin, and A. Stephan (2020). *EU Cartel Law and Economics*. Oxford: Oxford University Press.

Blair, R. D. and C. P. Durrance (2018). Umbrella damages: Towards a coherent antitrust policy. *Contemporary Economic Policy 36*(2), 241–254.

Blair, R. D. and V. G. Maurer (1982). Umbrella pricing and antitrust standing: An economic analysis. *Utah Law Review 1982*(4), 761–796.

Bos, I. and J. E. Harrington, Jr. (2010). Endogenous cartel formation with heterogeneous firms. *RAND Journal of Economics 41*(1), 92–117.

Bos, I. and J. E. Harrington, Jr. (2015). Competition policy and cartel size. *International Economic Review 56*(1), 133–153.

Bos, I., W. Letterie, and N. Scherl (2019). Industry impact of cartels: Evidence from the stock market. *Journal of Competition Law and Economics 15*(2-3), 358–379.

Bryant, P. G. and E. W. Eckard (1991). Price fixing: The probability of getting caught. *Review of Economics and Statistics 73*(3), 531–536.

Combe, E., C. Monnier, and R. Legal (2008). Cartels: The probability of getting caught in the European Union. Bruges European Economic Research papers 12.

D'Aspremont, C., A. Jacquemin, J. J. Gabszewicz, and J. A. Weymark (1983). On the stability of collusive price leadership. *Canadian Journal of Economics 16*(1), 17–25.

Diamantoudi, E. (2005). Stable cartels revisited. *Economic Theory 26*(4), 907–921.

Donsimoni, M.-P., N. Economides, and H. Polemarchakis (1986). Stable cartels. *International Economic Review 27*(2), 317–327.

Easterbrook, F. H., W. M. Landes, and R. A. Posner (1980). Contribution among antitrust defendants: A legal and economic analysis. *Journal of Law and Economics 23*(2), 331–370.

Escrihuela-Villar, M. (2008). On endogenous cartel size under tacit collusion. *Investigaciones Económicas 32*(3), 325–338.

Escrihuela-Villar, M. (2009). A note on cartel stability and endogenous sequencing with tacit collusion. *Journal of Economics 96*(2), 137–147.

Holler, E. and M. P. Schinkel (2017). Umbrella effects: Correction and extension. *Journal of Competition Law and Economics 13*(1), 185–189.

Inderst, R., F. P. Maier-Rigaud, and U. Schwalbe (2014). Umbrella effects. *Journal of Competition Law and Economics 10*(3), 739–763.

Katsoulacos, Y., E. Motchenkova, and D. Ulph (2015). Penalizing cartels: The case for basing penalties on price overcharge. *International Journal of Industrial Organization 42*, 70–80.

Katsoulacos, Y., E. Motchenkova, and D. Ulph (2020). Combining cartel penalties and private damage actions: The impact on cartel prices. *International Journal of Industrial Organization 73*, 102604.

Laitenberger, U. and F. Smuda (2015). Estimating consumer damages in cartel cases. *Journal of Competition Law and Economics 11*(4), 955–973.

Motta, M. and M. Polo (2003). Leniency programs and cartel prosecution. *International Journal of Industrial Oganization 21*(3), 347–379.

Napel, S. and D. Welter (2020). Relative responsibility for cartel damages. Discussion paper, University of Bayreuth.

Shaffer, S. (1995). Stable cartels with a Cournot fringe. *Southern Economic Journal 61*(3), 744–754.