Umbrella Pricing and Cartel Size

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Abstract

It is generally assumed that bigger scale and scope of private antitrust enforcement promotes effective competition. This has motivated several North American and European courts to uphold redress claims not only from clients of a detected cartel but also plaintiffs who were exposed to 'umbrella pricing', i.e. equilibrium price increases by non-colluding competitors. The paper shows that the presumed deterrence effects of obliging infringing firms to compensate aggrieved customers of non-infringing firms can be dominated by adverse cartel size effects. Liability for umbrella damages primarily constrains small partial cartels. It thereby improves the comparative profitability and stability of large ones. More encompassing cartels can form, prices rise, and welfare falls.

Keywords: cartel deterrence; cartel formation; effective competition; umbrella effects; redress payments; cartel size

JEL codes: L40; K21; D43

1. Introduction

Victims of antitrust infringements have a right to act against a detected cartel and to reclaim damages. Forward-looking firms anticipate the applicable redress obligations in the decision to form cartels and to set prices. Private antitrust enforcement can thus affect both the distribution and creation of economic surplus in oligopoly markets.

Legal discussion of private antitrust action is widely presuming that redress on a greater part of total harm is desirable not only on principle and for reasons of justice but because it generates beneficial deterrence effects. This has been acknowledged explicitly in the 2014 decision by the EU Court of Justice that extended legal standing from the customers of cartel members to customers of non-members who were harmed indirectly by an increased industry price level:

> "The right of any individual to claim compensation for such a[n indirect] loss actually strengthens the working of the European Union competition rules, since it discourages agreements or practices, frequently covert, which are liable to restrict or distort competition, thereby making a significant contribution to the maintenance of effective competition in the European Union . . . " (Case C-557/12 *Kone AG v ÖBB-Infrastruktur AG*, ECLI:EU:C:2014:1317, rec. 23).

Victims of 'umbrella effects', signifying detrimental equilibrium reactions by competing firms to cartel activities, are entitled to compensation also in Canada (as affirmed by the Supreme Court in 2019) and in the US depending on the competent court.[1] Leon Higginbotham Jr., former judge at the US Court of Appeals for the Third Circuit, noted in a well-cited case in 1979 (judgement 596 F.2d 573 3rd. Cir.): "Allowing standing [for customers of cartel outsiders] would also encourage enforcement, and thereby deter violation, of the antitrust laws."

Similar views are expressed in scholarly publications. For instance, Blair and Maurer (1982) hold that "[i]t is obvious that the prospect of recovery by purchasers from noncolluding competitors should have a greater deterrent effect than recovery

[1]The US Supreme Court has not yet ruled on the issue. Negative decisions include 600 F.2d 1148 5th Cir. 1979; positive ones 62 F. Supp. 2d 25 1999. See Blair and Durrance (2018), as well as Inderst et al. (2014) and Holler and Schinkel (2017). Laitenberger and Smuda (2015) show that umbrella losses constituted a big part of damages suffered by customers of the European detergent cartel in Germany. Bos et al. (2019) find suggestive evidence for umbrella effects in some but not all considered industries.

limited to direct purchasers, assuming a constant probability of detection". Blair and Durrance (2018) endorse this conclusion.

Our analysis, in contrast, questions the supposed pro-competitive merits of a regime in which customers of non-infringing firms have legal standing. Many former cartels involved a strict subset of suppliers[2] and it can seem very intuitive that redress obligations to the implied victims of umbrella pricing would have made collusion less attractive and encouraged compliance with antitrust law. However, affected cartels might re-optimize their operations rather than stopping them. In particular, the comparative profitability of partial vs. all-encompassing cartels changes if customers of non-infringing firms become entitled to compensation: expected profitability of partial cartels falls but that of industry-wide cartels is unaffected, making industry-wide cartels relatively more attractive.

A legal regime that intends to make a subset of rogue firms comply thus risks prompting previously well-behaved firms to infringe. Larger cartels tend to quote higher prices and reduce welfare more than small ones. Hence intuitively beneficial deterrence can have the adverse side effect that surviving cartels grow in size.

This paper demonstrates that a pro-competitive assessment of extended legal standing is warranted only if a cartel's size can be taken as constant. Then an obligation to compensate umbrella losses reduces the expected profitability of cartels and optimal prices. But insofar as small cartels become unstable while large or all-encompassing ones – with few or no competitors whose customers could claim compensation – remain or newly become stable, standing for indirect cartel victims fosters cartel growth. We demonstrate this so far unacknowledged effect for a non-negligible set of market configurations. For a constant probability of detection (as in Blair and Maurer 1982, Katsoulacos et al. 2015, 2020, etc.) a regime with umbrella compensation can even raise total unatoned overpayments, not just prices and deadweight losses.

We study a dynamic market model based on Bos and Harrington (2010) to show, first, that the deterrence intuition of above quotes holds when the cartel size is fixed

[2] In the recent European truck cartel, for example, only 9 out of 10 trucks were produced by a cartel member. In the data set of Connor (2020), about 60% of the approximately 500 reported cartels to which a market share could be attributed were partial. There are good theoretical reasons to expect partial rather than industry-wide cartels. See, e.g., Bos and Harrington (2010), Gabszewicz et al. (2019), or de Roos and Smirnov (2021) for models featuring capacity constraints, differentiated goods, or imperfectly attentive customers.

exogenously but, second, that total welfare can fall if cartel size is endogenous (Section 2). We assess how umbrella compensation shifts the internal and external stability of cartels in more detail for linear market environments (Section 3). Our observation that greater scope of private enforcement can have negative welfare implications nicely complements results by Bos and Harrington (2015), who showed that greater scale of public antitrust actions may also backfire (viz. higher fines levied on detected cartels). Similar adverse side effects of well-intentioned antitrust measures have been identified by McCutcheon (1997), Andersson and Wengström (2007), Bageri et al. (2013) and Bos et al. (2015).

2. Compensation for Umbrella Losses and Total Welfare

Extended compensation rights can have detrimental side effects if a market exhibits three features: first, it is required that stable partial cartels can emerge in the default regime, i.e. without compensation of umbrella losses; second, the umbrella regime with compensation does not entirely prevent all collusion; third, bigger cartels choose higher prices ceteris paribus. The dynamic stability of partial cartels is then reduced but large cartels gain structural stability.[3]

Namely, expected compensation payments to umbrella victims lower achievable cartel profits. This can render a cartel with s out of n firms downright unprofitable even if it optimally adjusts prices downwards (because expected compensation costs rise in prices). If the cartel remains profitable, it faces tighter constraints on the discount factors that make deviations unattractive. So *for given size s* collusion ends on profitability or dynamic stability grounds, or at the very least cartel prices fall.

However, cartels with few members and low market coverage are disproportionately burdened by compensation payments in the umbrella regime. They must pay compensation to many umbrella victims while a cartel that covers, say, 90% of the market closely resembles the industry-wide cartel, which by definition faces no umbrella victims. Cartels with different market shares are hence affected very differently by the compensation regime.

Structural stability of a cartel comprising s members requires that none of these

[3]We leave aside all moral considerations, fairness and justice concerns, personal criminal sanctions, agency problems and transaction costs of coordination in our investigation.

would prefer to be a non-member of a dynamically stable cartel involving $s-1$ firms, nor would any non-member be better off as member of a dynamically stable cartel of $s+1$ firms. As being or becoming a member loses attractiveness for small s more than for s close or equal to n, cartels of larger size become structurally more stable.

Whether the beneficial dynamic or the detrimental structural effects dominate – either yielding competitive or lower cartel prices for unchanged s, or a cartel of $s+1$ members that possibly charges more than the s original members did – obviously depends on the market at hand. We provide a sufficient condition for the detrimental structural changes caused by umbrella compensation to prevail, so that total welfare falls. The blanket conclusion that compensation of umbrella damages promotes effective competition hence needs to be qualified.[4]

2.1. Formal Setup

We construct a symmetric version of the Bertrand-Edgeworth competition model developed by Bos and Harrington (2010), which can easily accommodate the highlighted market features. Let $n \geq 3$ symmetric firms repeatedly engage in simultaneous price setting for a homogeneous good.[5] Each firm i faces an exogenous capacity constraint $q_i \leq k$ on period production and maximizes the present value of profits for a common discount factor $\delta \in (0, 1)$ with infinite time horizon.

Constant unit costs are denoted by $c \geq 0$. Prices must be integer multiples of a small unit of account $\varepsilon > 0$ and consumers buy at the lowest available price à la Bertrand. Market demand is described by a smooth function $D(p)$ with $D'(p) < 0$ such that $(p-c) \cdot D(p)$ is strictly concave with $D(c) =: a > 0$. Demand is rationed efficiently when a firm's capacity is exhausted and (residual) demand is split equally if several firms post identical prices.

To ensure existence of a static pure-strategy Nash equilibrium, individual capacity k is assumed to be less than monopoly demand but big enough for $n-1$ firms to serve

[4]No qualification is needed for markets that lack some of the listed features: a detrimental increase of cartel size is impossible if the umbrella regime makes *all* collusion unprofitable or dynamically unstable. When no partial cartel is stable in the default regime then there is no scope for umbrella losses to start with, and larger cartels are no worry if their prices are non-increasing in size.

[5]Blair and Durrance (2018) hold that "[t]he economic argument in favor of antitrust standing for customers of the nonconspirators is most compelling when the product is homogeneous". Product differentiation at least makes it legally more demanding to establish causal links between supracompetitive prices for non-cartel customers and the infringement.

the market at cost, i.e.,

$$\frac{a}{n-1} < k < q^m \tag{A1}$$

for given k and n, where $q^m > 0$ denotes the monopoly output associated with c and $D(p)$. This implies two symmetric static equilibria with approximately zero profits: $p = c$ and $p = c + \varepsilon \approx c$. We therefore suppose cartel prices $p \geq c + 2\varepsilon$.

As Bos and Harrington (2010, 2015), we allow at most one cartel to operate. Its $2 \leq s \leq n$ members are assumed to use stationary strategies that do not condition on past behavior of non-cartel members but permanently revert to the static zero-profit equilibrium after a deviation (the harshest possible punishment). So non-members will at any point in time maximize their static period profits and undercut the anticipated uniform cartel price p by ε (cf. Bos and Harrington 2010). This leaves a residual demand of $D_s^R(p) = \max\left\{D(p) - (n-s)k, 0\right\}$ for the cartel.

In any period t of cartel operations, the infringement is detected with probability $\alpha \in (0, 1)$. We take α to be fixed, i.e., it depends neither on the legal regime as such, nor on a cartel's size or price choices (Katsoulacos et al. 2015, 2020).[6] Whether below findings extend to variable detection rates would obviously depend on the specific functional relation between α and membership, cartel markups, and redress rules.

In case of detection, each active cartel member must pay a fine of $\tau > 0$ times its period t profit and $\beta > 0$ times eligible overcharges.[7] This gives rise to an individual expected profit for a cartel member of

$$\pi_s(p) = (p-c) \cdot \underbrace{(1 - \alpha(\beta + \tau))D_s^R(p)/s}_{:= D_s^*(p)} = (p-c)D_s^*(p) \tag{1}$$

in the default regime and

$$\pi_s^u(p) = (p-c) \cdot \underbrace{\Big((1 - \alpha\tau)D_s^R(p) - \alpha\beta D(p)\Big)/s}_{:= D_s^{*u}(p)} = (p-c)D_s^{*u}(p) \tag{2}$$

[6] To have some ballpark figure: Bryant and Eckard (1991) estimated the annual probability of getting indicted by federal authorities in the US at between 13% and 17%. Combe et al. (2008) obtained comparable results for a European sample. Ormosi (2014) corroborates these estimates.

[7] We consider only damages for overcharges, since lost profits are rarely recovered in legal practice (see, e.g., Weber 2021, Laborde 2021). Basso and Ross (2010) discuss the extent to which overcharge damages are a good proxy for total harm.

in the umbrella regime, where

$$D_s^*(p) = \frac{1 - \alpha(\beta + \tau)}{s}\Big[D(p) - (n - s)k\Big] \tag{3}$$

and

$$D_s^{*u}(p) = \frac{1 - \alpha(\beta + \tau)}{s}D(p) - \frac{1 - \alpha\tau}{s}(n - s)k \tag{4}$$

describe the *net demand* of a cartel member after subtracting units that cover expected fines and redress payments. Private antitrust enforcement with parameter β is equivalent to purely public action with multiplier $\tau' = \beta + \tau$ in the default regime but not the umbrella case. We assume

$$1 - \alpha(\beta + \tau) > 0 \tag{A2}$$

to ensure that an all-encompassing cartel that chooses $p = c + 2\varepsilon$ would face positive net demand $nD_n^*(p) = nD_n^{*u}(p) \approx \big(1 - \alpha(\beta + \tau)\big)D(c)$ and thus be profitable.

Like Katsoulacos et al. (2015, 2020) we let a detected cartel resume its activities in period $t + 1$, provided the applicable profitability and stability conditions are met.[8] However, cartel activities are ended for good if a member deviates in any way from the agreed price in period t and all firms revert to $p \approx c$ from $t + 1$ on. The best deviation of a cartel member is then either to match the non-members' price $p - \varepsilon$ or to undercut them with $p - 2\varepsilon$, depending on outsiders' joint capacity. In both cases the dominant effect is to raise the respective firm's net demand from $D_s^*(p)$ or $D_s^{*u}(p)$ to k, and we equate the one-off deviation profit with approximately $(p - c)k$.[9]

2.2. Profitability and Dynamic Stability

A cartel with s members is dynamically stable if the present value of profits that accrue from serving net demand $D_s^*(p)$ or $D_s^{*u}(p)$ in the default or umbrella regime, respectively, at cartel price $p \geq c + 2\varepsilon$ is at least as great as realizing the one-off deviation profit of approximately $(p - c)k$ and then reverting to competition. For any given p, this is

[8] One might also suppose that a detected cartel breaks down forever with probability $\gamma \in (0, 1]$ or re-forms in $t+1$ with probability $1-\gamma$. This would scale up the critical discount factors identified below by $\frac{1}{1-\alpha\gamma}$.

[9] This assumes that only active cartel members are fined and liable for redress in case of detection (in line with, e.g., Motta and Polo 2003 or Katsoulacos et al. 2015, 2020). For example, a deviator may apply for leniency and be excluded from fines and compensation payments (see Aubert et al. 2006).

equivalent to the requirement that firms' discount factor $\delta \in (0,1)$ is not smaller than the critical discount factor equal to

$$\delta_s(p) = 1 - \frac{D_s^*(p)}{k} \;\text{ or }\; \delta_s^u(p^u) = 1 - \frac{D_s^{*u}(p^u)}{k}. \tag{5}$$

$(p-c)\cdot D_s^*(p)$ and $(p-c)\cdot D_s^{*u}(p)$ inherit the strict concavity of monopoly profits and so respective unconstrained cartel profit maximizers, denoted p_s^* and p_s^{*u}, are uniquely determined by

$$(p-c)\cdot D'(p) + D(p) = (n-s)k \tag{6}$$

in the default and

$$(p-c)\cdot D'(p) + D(p) = (n-s)k/\mu \tag{7}$$

in the umbrella regime. The constant $\mu := {}^{(1-\alpha(\beta+\tau))}/_{(1-\alpha\tau)} < 1$ will be referred to as the *umbrella coefficient*. It is smaller the greater the compensation multiplier β (e.g. triple vs. single damages), the detection probability α, and the fine multiplier τ.

First focus on the choice of p in the default regime. Adoption of the unconstrained profit maximizer p_s^* determined by (6) can be sustained only if $\delta \geq \bar{\delta}_s$ with

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1-\alpha(\beta+\tau))D(p_s^*) - (1-\alpha(\beta+\tau))(n-s)k}{sk}. \tag{8}$$

If otherwise $\delta < \bar{\delta}_s$, cartel members maximize $sD_s^*(p)(p-c)$ subject to the constraint $\delta_s(p) = \delta$. A cartel cannot be dynamically stable if $\delta < \underline{\delta}_s$ where

$$\underline{\delta}_s := \delta_s(c+2\varepsilon) \approx \delta_s(c) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{sk}. \tag{9}$$

To ensure that $\underline{\delta}_s < 1$, it is necessary that the cartel of size s is profitable, i.e., that it can choose a price $p \geq c+2\epsilon$ with $D_s^*(c+2\epsilon) > 0$. Given that $D_s^*(c+2\epsilon) > 0 \Leftrightarrow D_s^R(c+2\epsilon) > 0$ and noting $D_s^R(c+2\epsilon) \approx D_s^R(c) = a-(n-s)k$, the size of the smallest profitable cartel is $\underline{s} := \lceil n - \frac{a}{k} \rceil$.[10] Condition (A1) implies $2 \leq \underline{s} \leq n-1$ and $\underline{\delta}_{n-1} < \bar{\delta}_{n-1} < 1$. Existence of some dynamically stable partial cartel is thus ensured if $\delta \approx 1$.

For $\delta \in (\underline{\delta}_s, \bar{\delta}_s)$ a cartel of $s \geq \underline{s}$ members can stabilize some prices $c+2\varepsilon \leq p_s < p_s^*$.

[10] $\lceil x \rceil$ denotes the smallest integer not smaller than x.

The corresponding constrained profit maximizer $p_s^\circ(\delta)$ is characterized by

$$\delta = \delta_s(p_s^\circ) = 1 - \frac{D_s^*(p_s^\circ)}{k} \qquad \Leftrightarrow \qquad p_s^\circ(\delta) = D_s^{*-1}\Big((1-\delta)k\Big) \tag{10}$$

and increases continuously to p_s^* as $\delta \to \bar{\delta}_s$. In summary, a cartel of s members is dynamically stable in the default regime iff $\delta \in (\underline{\delta}_s, 1)$ and the respective price is

$$p_s^{\circledast}(\delta) = \begin{cases} p_s^\circ(\delta) & \text{if } \delta \in (\underline{\delta}_s, \bar{\delta}_s), \\ p_s^* & \text{if } \delta \in (\bar{\delta}_s, 1). \end{cases} \tag{11}$$

Umbrella victims suffer a price overcharge of approximately $p_s^{\circledast}(\delta) - c$.

Analoguously, we can derive

$$p_s^{\circledast u}(\delta) = \begin{cases} p_s^{\circ u}(\delta) & \text{if } \delta \in (\underline{\delta}_s^u, \bar{\delta}_s^u), \\ p_s^{*u} & \text{if } \delta \in (\bar{\delta}_s^u, 1), \end{cases} \tag{12}$$

in the umbrella regime with

$$\delta = \delta_s(p_s^{\circ u}) \qquad \Leftrightarrow \qquad p_s^{\circ u}(\delta) = D_s^{*u-1}\Big((1-\delta)k\Big) \tag{13}$$

as the constrained price and

$$\bar{\delta}_s^u := \delta_s^u(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))D(p_s^{*u}) - (1-\alpha\tau)(n-s)k}{sk}, \tag{14}$$

$$\underline{\delta}_s^u := \delta_s^u(c+2\varepsilon) \approx \delta_s^u(c) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{sk} \tag{15}$$

as the relevant critical discount factors. It is noteworthy that the size $\underline{s}^u := \lceil n - \mu \cdot \frac{a}{k} \rceil$ of the smallest cartel that satisfies dynamic stability for $\delta \approx 1$ in the umbrella regime is weakly greater than size $\underline{s}$ in the default regime because $\mu < 1$. Relatedly one can show:

Proposition 1. *For a cartel of size $s < n$, extending legal standing to umbrella victims*

(i) increases the minimal discount factor that is needed to sustain the agreement, i.e., $\underline{\delta}_s^u > \underline{\delta}_s$;

(ii) decreases prices to (approximately) $p_s^{\circledast u}(\delta) < p_s^{\circledast}(\delta)$.

(iii) Thresholds $\underline{\delta}_s^u$ and $\underline{\delta}_s$ are decreasing and prices $p_s^{\circledast}(\delta)$ and $p_s^{\circledast u}(\delta)$ are increasing in the cartel's size $s \in \{2, \ldots, n\}$.

Proof. Part (*i*) follows directly from (9), (15) and $\mu^{-1} > 1$. Part (*ii*) is implied by two observations. First, the LHS of equations (6) and (7) coincide and decrease in p, while the RHS of eq. (7) is greater than that of eq. (6) given $\mu < 1$. Hence $p^*_s > p^{*u}_s$. Second, because $D^{*u}_s(p) < D^*_s(p)$ for all relevant p, any given sales q go with lower prices $D^{*u-1}_s(q) < D^{*-1}_s(q)$ in the umbrella regime. So setting $q = (1-\delta)k$ implies $p^{\circ}_s(\delta) > p^{\circ u}_s(\delta)$ and we can, overall, conclude $p^{\circledast u}_s(\delta) < p^{\circledast}_s(\delta)$.[11]

Moreover, for $2 < s \leq n$ we have

$$\underline{\delta}_s - \underline{\delta}_{s-1} \approx \frac{(1-\alpha(\beta+\tau))(a-(n-s+1)k)}{(s-1)k} - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{sk} \tag{16}$$

$$= \underbrace{(a-nk)}_{<0} \cdot \underbrace{\frac{1-\alpha(\beta+\tau)}{s(s-1)k}}_{>0} < 0 \tag{17}$$

by equation (9) and assumptions (A1) and (A2). $\underline{\delta}^u_s - \underline{\delta}^u_{s-1} < 0$ in part (iii) follows similarly. That p^*_s (p^{*u}_s) is increasing in s follows since the RHS of eq. (6) (eq. (7)) is decreasing in s, whereas the LHS is independant of s. $p^{\circ}_s(\delta)$ ($p^{\circ u}_s(\delta)$) is increasing in s since $D^*_s(p)$ ($D^{*u}_s(p)$) is increasing in s given (A2). □

At an intuitive level, the reduction of prices observed in part (ii) follows from the cartel's expected compensation payments being lower, the lower its overcharge. The critical discount factor in part (i) rises because a defecting member's deviation profit for $p = c + 2\varepsilon$ is not affected by compensation of umbrella victims while expected profits from collusion fall. These lowered profits also gave rise to $\underline{s}^u \geq \underline{s}$, i.e., the smallest cartel that is profitable is (weakly) larger in the umbrella regime. Turning to part (iii), a cartel's influence on margins and the implied prices increase in s because greater market coverage makes undercutting by capacity-constrained outsiders less of a concern. Greater market coverage also explains the wider range of discount factors that render larger cartels dynamically stable, i.e. $\underline{\delta}_s < \underline{\delta}_{s-1}$ and $\underline{\delta}^u_s < \underline{\delta}^u_{s-1}$: a cartel first becomes dynamically stable when it can charge $p = c + 2\varepsilon$ and discourage defections with fixed deviation profit εk. This discouragement gets easier the greater the respective firm-specific collusion profit, which for $p = c + 2\varepsilon$ is proportional to per-firm demand and increases in s.

[11] If $\bar{\delta}_s < \bar{\delta}^u_s$ and $\delta \in (\bar{\delta}_s, \bar{\delta}^u_s)$, then $p^{\circledast u}_s(\delta) = p^{\circ u}_s(\delta) < p^{*u}_s < p^*_s = p^{\circledast}_s(\delta)$. If $\bar{\delta}^u_s < \bar{\delta}_s$ and $\delta \in (\bar{\delta}^u_s, \bar{\delta}_s)$, then $p^{\circledast u}_s(\delta) = p^{*u}_s = p^{\circ u}_s(\bar{\delta}^u_s) < p^{\circ}_s(\bar{\delta}^u_s) < p^{\circ}_s(\delta) = p^{\circledast}_s(\delta)$.

2.3. *Structural Stability*

Provided that partial cartels remain dynamically stable in the umbrella regime, their continued operation is conditional on firms' incentives for being a member or non-member in the long run. We follow Escrihuela-Villar (2008, 2009) or Bos and Harrington (2010, 2015) and apply structural stability analysis à la d'Aspremont et al. (1983). Similar results obtain for other conceptions of stability as, e.g., in Diamantoudi (2005) at least when demand is linear.

Define $p_s^{\circledast}(\delta) = c$ for $s \in \{\underline{s} - 1, n + 1\}$. Then a dynamically stable cartel of size $\underline{s} \leq s \leq n$ is called *internally stable* in the default regime if

$$\mathcal{I}_s(\delta) := (p_s^{\circledast}(\delta) - c)D_s^*\Big(p_s^{\circledast}(\delta)\Big) - (p_{s-1}^{\circledast}(\delta) - c)k \geq 0 \tag{18}$$

i.e., each cartel member's per period profit is at least as high as that from becoming a non-member. It is *externally stable* if

$$\mathcal{E}_s(\delta) := (p_s^{\circledast}(\delta) - c)k - (p_{s+1}^{\circledast}(\delta) - c)D_{s+1}^*\Big(p_{s+1}^{\circledast}(\delta)\Big) \geq 0, \tag{19}$$

i.e., each non-member's profit weakly exceeds that achievable by becoming a member. Cartels of size $s = \underline{s}$ ($s = n$) are automatically internally (externally) stable, i.e. $\mathcal{I}_{\underline{s}}(\delta) > 0$ ($\mathcal{E}_n(\delta) > 0$). Analogous conditions $\mathcal{I}_s^u(\delta) \geq 0$ and $\mathcal{E}_s^u(\delta) \geq 0$ apply in the umbrella regime. Note that $\mathcal{E}_n^u(\delta) = \mathcal{E}_n(\delta)$ and also that eqs. (18) and (19) imply $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}_s^u(\delta) = -\mathcal{I}_{s+1}^u(\delta)$ for $s < n$. The latter plays a key role in the analysis.

A dynamically stable cartel of s members will be considered *structurally stable* or simply *stable* in a given regime if it is internally and externally stable. If a unique size s is compatible with structural stability, a corresponding cartel will be assumed to form. In case multiple sizes are compatible with structural stability, a stable cartel of maximal size is presumed to operate: the latter is not just focal in that it maximizes total profits subject to stability but it is of the generically unique size that makes coordinated switches from one to another stable cartel individually unprofitable.[12]

[12]To elaborate on this, let $\tilde{s}$ be the maximal size s such that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$. Then $\mathcal{E}_{\tilde{s}-1}(\delta) = -\mathcal{I}_{\tilde{s}}(\delta) < 0$ renders it impossible for an $\tilde{s} - 1$-sized cartel to be stable. But it is conceivable (only) with non-linear demand that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$ for some $\hat{s}$ with $\underline{s} \leq \hat{s} \leq \tilde{s} - 2$: no single member finds it profitable to leave a cartel of $\hat{s}$ members, nor does a single non-member find it profitable to join the cartel. However, in view of $p_{\hat{s}}^{\circledast}(\delta) < p_{\tilde{s}-1}^{\circledast}(\delta)$ and fixed capacities k, profits of an outsider to an $\hat{s}$-sized cartel are smaller than those of an outsider to an $\tilde{s} - 1$-sized cartel. The latter are smaller than those of an insider to an $\tilde{s}$-sized cartel by $\mathcal{I}_{\tilde{s}}(\delta) > 0$. So it would be profitable for $\tilde{s} - \hat{s}$ outsiders of an $\hat{s}$-sized cartel

From these premises we can conclude:

Proposition 2. *Let firms be sufficiently patient so that some (partial) cartel is dynamically stable in the umbrella (default) regime. Then extending legal standing to customers of non-infringing firms lowers welfare if* $\mathcal{I}_n(\delta) < 0$ *and* $\mathcal{I}_n^u(\delta) > 0$.

Proof. If any cartel is dynamically stable in a given regime, so is the all-encompassing one (recalling that $\underline{\delta}_s^u$ and $\underline{\delta}_s$ decrease in s). With $\mathcal{E}_n(\delta) > 0$ holding automatically, $\mathcal{I}_n^u(\delta) > 0$ implies that the industry-wide cartel is structurally stable in the umbrella regime. It is also the largest stable cartel – so if cartels of smaller size should also be stable it is unique in not providing incentives to form a different stable cartel. Recalling that an industry-wide cartel faces no umbrella victims, the market price equals $p_n^{\circledast u}(\delta) = p_n^{\circledast}(\delta)$ in the umbrella regime.

In the default regime, by contrast, the industry-wide cartel is unstable if $\mathcal{I}_n(\delta) < 0$. Because a cartel of size $s = \underline{s}$ with $2 \le \underline{s} \le n-1$ is automatically internally stable, i.e., $\mathcal{I}_{\underline{s}}^u(\delta) > 0$, there must exist a largest size $\hat{s}$ with $\underline{s} \le \hat{s} < n$ such that $\mathcal{I}_{\hat{s}}^u(\delta) > 0$ and $\mathcal{I}_{\hat{s}+1}^u(\delta) = -\mathcal{E}_{\hat{s}}^u(\delta) < 0$. When this forms (or some smaller stable cartel in defiance of joint entry incentives), the corresponding market price $p_{\hat{s}}^{\circledast}(\delta)$ is strictly below $p_n^{\circledast}(\delta)$ by Proposition 1. For well-behaved cost and demand functions, as in our model, this implies greater total surplus in the default regime than in the umbrella regime. □

The conditions on internal stability in Proposition 2 are not overly difficult to satisfy. It is sufficient that there are enough firms with a capacity somewhere between monopoly output q^m and fraction μ of competitive output $a = D(c)$:

Proposition 3. *For any parameters* c *and* $\mu a < k < q^m$ *there exists* $\hat{n} > 2$ *such that* $\mathcal{I}_n(\delta) < 0$, $\mathcal{I}_n^u(\delta) > 0$ *and welfare is lowered for all* $n \ge \hat{n}$ *and* $\delta \in (\hat{\delta}_n, 1)$ *where* $\hat{\delta}_n := \max\{\underline{\delta}_{n-1}, \underline{\delta}_n^u\} < 1$.

Proof. $\underline{\delta}_n^u < 1$ follows directly from eq. (14); $\underline{\delta}_{n-1} < 1$ is implied by eq. (9) and $k < q^m < a$. So interval $(\hat{\delta}_n, 1)$ is non-empty. An industry-wide cartel can sustain price $p_n^{\circledast u}(\delta) \ge 2\varepsilon$ in the umbrella regime for any $\delta \in (\hat{\delta}_n, 1)$ and, similarly, a cartel of $n-1$ members can sustain the choice of $p_{n-1}^{\circledast}(\delta) \ge 2\varepsilon$ in the default regime. The patience requirements in Proposition 2 are thus satisfied.

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to simultaneously become cartel members. Hence an $\hat{s}$-sized cartel fails to be stable against coordinated entry of multiple outsiders, while both individual and joint exits by members of the $\tilde{s}$-sized cartel would be unprofitable.

The price $p_n^{\circledast}(\delta)$ set by an industry-wide cartel in the default regime is bounded by p_n^*. The latter price is characterized by $(p-c)D'(p) = -D(p)$ and constant in n. So total profits of an all-encompassing cartel have a fixed upper bound and associated per-member profits of $(p_n^{\circledast} - c)D(p_n^{\circledast})/n$ vanish as $n \to \infty$.[13] A cartel of only $n-1$ members also makes positive profits given that $\delta > \hat{\delta}_n$. This implies a supracompetitive profit for the single non-member of at least εk, which does *not* vanish as $n \to \infty$. Therefore, some $\tilde{n} > 2$ exists such that for all $n \geq \hat{n}$ permanently undercutting a cartel of size $n-1$ is more profitable than being member of a cartel of size n, i.e. $\mathcal{I}_n(\delta) < 0$.

For the umbrella regime note that $\mu a < k < q^m$ implies $\underline{s}^u = \lceil n - \mu \cdot \frac{a}{k} \rceil = n$ for all $n \geq 3$. So any partial cartel is unprofitable and a fortiori unstable. In contrast, an industry-wide cartel earns positive profit and can be sustained given $\delta \in (\hat{\delta}_n, 1)$. Therefore $\mathcal{I}_n^u(\delta) > 0$ for all $n \geq 3$. Proposition 2 then yields the claim. □

The proof of Proposition 3 draws on the trivial observation that per-firm rents are small when the monopoly output must be divided by many. This causes at least a few firms to prefer staying outside of a partial cartel in the default regime. However, benefits to being non-member of a partial cartel only exist if the latter is profitable and dynamically stable. When non-members have sufficiently many customers who require compensation in the umbrella regime – reflected by a non-member's capacity k exceeding fraction μ of competitive demand – this fails to be the case. Then permanent freeriding on a partial cartel is no option and collection even of small rents in an industry-wide cartel is the best a firm can do. In other words, the industry-wide cartel, which was unstable by default, is stabilized by partial cartels' expected costs of umbrella compensation.

The sufficient condition in Proposition 3 pertains to cases with potentially many patient firms of which most but not all collude in the default regime.[14] Umbrella compensation can have harmful overall effects also for cartels involving few firms and relatively small discount factors. Demonstrating this however requires closed form solutions and concrete parameter values. We pursue this route in the next section assuming linear demand. The latter also makes it possible to show that umbrella

[13] The model's key assumption (A1) on capacities is implied by $\mu a < k < q^m$ for all $n \geq 2$. The only caveat to considering large n is that minimal discount factors also become large because $\hat{\delta}_n \to 1$.

[14] The minimum number $\hat{n}$ and what 'many' means in practice will clearly depend on demand, technology and enforcement parameters. The data set compiled by Connor (2020) involves an average number of 10.2 firms (a median of 4.0 firms), with an average (median) market coverage of 87% (92%).

compensation strengthens the internal stability of all profitable cartels – implying that weakly greater cartels are formed. It moreover ensures that for any $\delta > \delta_n$ ($\delta > \delta_n^u$) there generically exists a unique size s (s^u) such that $\mathcal{I}_s(\delta) > 0$ and $\mathcal{E}_s(\delta) > 0$ ($\mathcal{I}^u_{s^u}(\delta) > 0$ and $\mathcal{E}^u_{s^u}(\delta) > 0$).

3. Linear Market Environments

Suppose now that demand is linear with $D(p) = a - bp$ and set marginal costs to zero.[15] Assumption (A1) simplifies to

$$\frac{a}{n-1} < k < \frac{a}{2} \tag{A1$'$}$$

and now requires $n \geq 4$.

The profit maximizing cartel price in the default regime evaluates to

$$p_s^{\circledast}(\delta) = \begin{cases} p_s^{\circ}(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b} & \text{if } \delta \in (\underline{\delta}_s, \bar{\delta}_s), \\ p_s^{*} = \frac{1}{2b}\Big(a - (n-s)k\Big) & \text{if } \delta \in (\bar{\delta}_s, 1), \end{cases} \tag{20}$$

with

$$\bar{\delta}_s := \delta_s(p_s^*) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{2sk}, \tag{21}$$

and

$$\underline{\delta}_s := \delta_s(2\varepsilon) \approx \delta_s(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k)}{sk}. \tag{22}$$

The profit maximizing cartel price in the umbrella regime is

$$p_s^{\circledast u}(\delta) = \begin{cases} p_s^{\circ u}(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b} & \text{if } \delta \in (\underline{\delta}_s^u, \bar{\delta}_s^u), \\ p_s^{*u} = \frac{1}{2b}\Big(a - (n-s)k\cdot\mu^{-1}\Big) & \text{if } \delta \in (\bar{\delta}_s^u, 1), \end{cases} \tag{23}$$

with

$$\bar{\delta}_s^u := \delta_s^u(p_s^{*u}) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{2sk}, \tag{24}$$

and

$$\underline{\delta}_s^u := \delta_s^u(2\varepsilon) \approx \delta_s^u(0) = 1 - \frac{(1-\alpha(\beta+\tau))(a-(n-s)k\cdot\mu^{-1})}{sk}. \tag{25}$$

[15] For $c > 0$, reinterpret p as the price markup: consider prices $\tilde{p} = c + p$ and demand $\tilde{D}(\tilde{p}) = \tilde{a} - b\tilde{p} = a - bp$ with $a = \tilde{a} + bc$. Then maximizing $(\tilde{p} - c)\tilde{D}(\tilde{p})$ is equivalent to maximizing $p(a - bp)$.

This implies $\bar{\delta}_s < \bar{\delta}_s^u$ for $s < n$ (whereas $\bar{\delta}_s \geq \bar{\delta}_s^u$ is possible for non-linear demand).

Linearity of demand makes $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ (and hence $\mathcal{E}_{s-1}(\delta)$ and $\mathcal{E}_{s-1}^u(\delta)$) piecewise polynomial in δ. Figure 1 illustrates the specific quadratic, linear and constant parts when $n = 5$, $a = 10$, $b = 1$, $k = 3$, $\alpha = 1/5$, $\beta = 1$, and $\tau = 1/2$, which represents a case that is neither covered by Proposition 2 nor Proposition 3 (as $\mathcal{I}_n^u(\delta) < 0$ and $\mu a > k$). Supposing δ is big enough to ensure that at least the industry-wide cartel is dynamically stable, internal stability $\mathcal{I}_5(\delta)$ increases on $(\underline{\delta}_5, \underline{\delta}_4)$: the constrained profit maximizer $p_5^{\circledast} = p_s^{\circ}$ rises linearly in δ, causing members' profits to rise quadratically, while an outsider to a cartel of only four firms could not earn positive profit because such cartel is dynamically unstable. The latter however becomes dynamically stable for $\delta \geq \underline{\delta}_4$. Then the linear increase of $p_4^{\circledast} = p_4^{\circ}$ raises outsider profits $p_4^{\circledast} k$ proportionally while negative quantity reactions dampen further increases of $p_5^{\circledast} D_5^*(p_5^{\circledast})$ – in total causing $\mathcal{I}_5(\delta)$ to decrease on $(\underline{\delta}_4, \bar{\delta}_5)$. The quadratic decrease becomes linear for $\delta \in (\bar{\delta}_5, \bar{\delta}_4)$ since the encompassing cartel charges the constant unconstrained profit maximizer $p_5^{\circledast} = p_5^*$ for $\delta > \bar{\delta}_5$ while $p_4^{\circledast} k = p_4^{\circ} k$ still increases linearly in δ until $\bar{\delta}_4$. Finally for $\delta \in (\bar{\delta}_4, 1)$, the price faced by outsiders to a cartel of four firms also becomes constant, and so does profit difference $\mathcal{I}_5(\delta)$. Analogous variation in δ applies to internal stability of partial cartels and in the umbrella regime.

Inspection of Figure 1 shows that the range of discount factors where dynamic stability of a cartel entails internal stability is widened by giving legal standing to umbrella victims. This holds irrespective of our specific parameter choices:

Proposition 4.

(i) Every dynamically stable cartel of size $s \geq \underline{s}^u$ that is internally stable in the default regime is also internally stable in the umbrella regime:

$$\mathcal{I}_s(\delta) > 0 \;\Rightarrow\; \mathcal{I}_s^u(\delta) > 0. \tag{26}$$

The reverse is not true and $\mathcal{I}_s^u(\delta) > \mathcal{I}_s(\delta)$ holds for $\delta \in (\underline{\delta}_{s-1}^u, 1)$.

(ii) Every dynamically stable cartel of size $s \geq \underline{s}^u$ that is externally stable in the umbrella regime is also externally stable in the default regime:

$$\mathcal{E}_s^u(\delta) > 0 \;\Rightarrow\; \mathcal{E}_s(\delta) > 0. \tag{27}$$

The reverse is not true and $\mathcal{E}_s(\delta) \geq \mathcal{E}_s^u(\delta)$ holds for $\delta \in (\underline{\delta}_s^u, 1)$ with strict inequality for $s < n$.

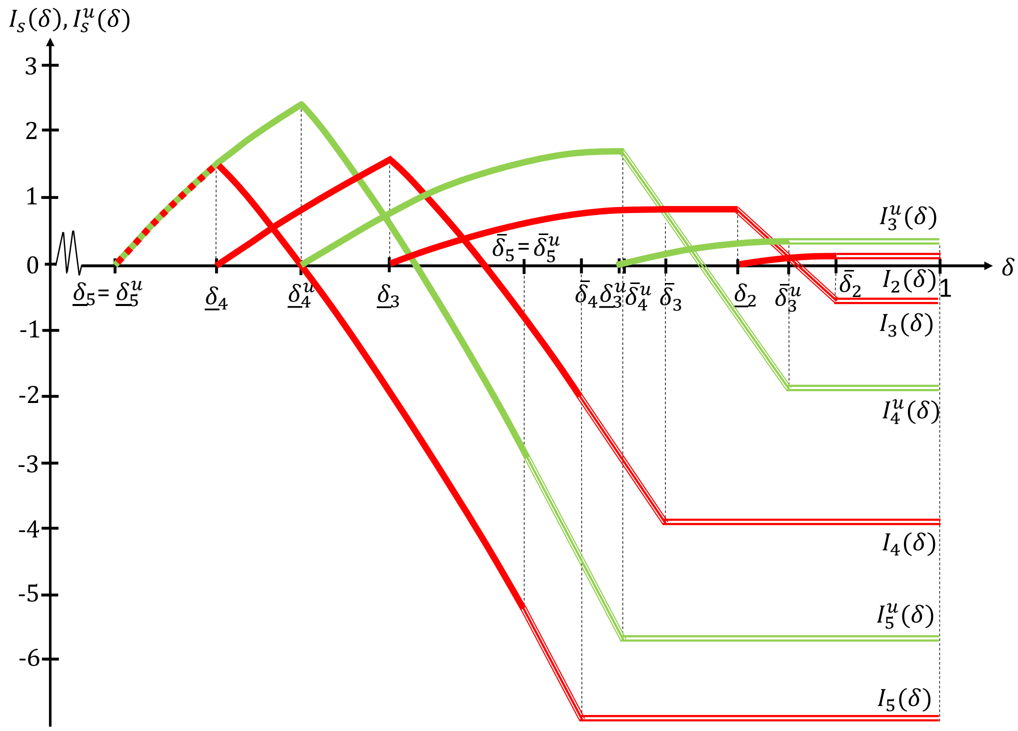


Figure 1: Internal stability measures $\mathcal{I}_s(\delta)$ and $\mathcal{I}_s^u(\delta)$ in default and umbrella regime

Proof of Proposition 4 involves tedious case distinctions between constrained vs. unconstrained cartel prices and is provided in the Appendix. The observations rule out that a large or even all-encompassing cartel loses members because umbrella compensation is mandated. Whenever a cartel remains dynamically stable, its internal stability increases: freeriding on smaller cartels becomes less attractive as these cartels must lower prices the most to remain dynamically stable in the umbrella regime. The structural challenge is instead that non-members have incentives to join in. So operating cartels never shrink as legal standing is extended and rather tend to grow.

Consistent with findings for static market environments in, e.g., Donsimoni et al. (1986) or Shaffer (1995), the possibility that multiple cartel sizes are structurally stable in either regime can for linear demand be ruled out generically:

Proposition 5. *For any $\delta > \underline{\delta}_s$ ($\delta > \underline{\delta}_s^u$) there exists a generically unique stable cartel size $s^*(\delta)$ ($s^{*u}(\delta)$) in the default regime (umbrella regime) with $s^{*u}(\delta) \geq s^*(\delta)$.*

Proof. Leaving aside the null set of non-generic discount factors δ where $\mathcal{I}_s(\delta)$ or $\mathcal{I}_s^u(\delta)$ have zeros, internal (in)stability with s members rules out (implies) external stability with $s-1$ members. For any $\delta > \underline{\delta}_{\underline{s}}$, there is a largest $s \in \{\underline{s}, \ldots, n\}$ such that

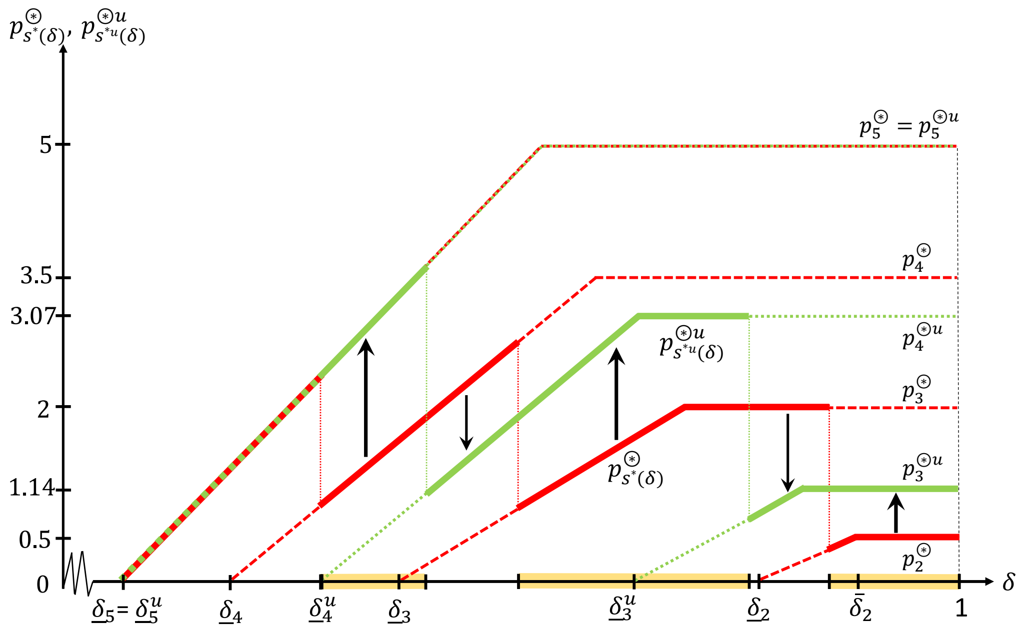


Figure 2: Optimal cartel prices for structurally stable cartel size $s^*(\delta)$ and $s^{*u}(\delta)$ in default and umbrella regime (δ s.t. $p^{\circledast u}_{s^{*u}(\delta)} > p^{\circledast}_{s^*(\delta)}$ highlighted)

$\mathcal{I}_s(\delta) > 0$. This s uniquely combines internal stability of size s with external stability (= internal instability of size $s + 1$). So the unique stable cartel size in the default regime is $s^*(\delta) := \max\left\{s \in \{\underline{s}, \ldots, n\} : \mathcal{I}_s(\delta) > 0\right\}$ and, analogously, that in the umbrella regime is $s^{*u}(\delta) := \max\left\{s \in \{\underline{s}^u, \ldots, n\} : \mathcal{I}^u_s(\delta) > 0\right\}$. $s^{*u}(\delta) \geq s^*(\delta)$ follows directly from Proposition 4. □

When inequality $s^{*u}(\delta) \geq s^*(\delta)$ is strict, i.e., detrimental structural stability effects of the umbrella regime induce a larger cartel, this is partly compensated by the beneficial dynamic stability effect captured by Proposition 1. Namely, profit maximizing prices are smaller in the umbrella than the default regime conditional on size (Prop. 1(ii)), even though a bigger cartel charges higher prices conditional on the compensation regime. It is conceivable that the cartel grows in the umbrella regime but still charges less than in the default situation. However, this requires the cartel in the umbrella regime to remain a partial one: if $s^{*u}(\delta) = n > s^*(\delta)$ then $p^{\circledast u}_n(\delta) = p^{\circledast}_n(\delta) > p^{\circledast}_{s^*(\delta)}(\delta)$ implies that prices rise, as exploited in the proof of Prop. 3.

Figure 2 plots prices in both regimes as a function of δ for the parameters $n = 5$, $a = 10$, $b = 1$, $k = 3$, $\alpha = 1/5$, $\beta = 1$, and $\tau = 1/2$ considered in Figure 1. Endpoints of the three highlighted intervals reflect zeros of $\mathcal{I}_{s-1}(\delta)$, $\mathcal{I}_s(\delta)$, $\mathcal{I}^u_{s-1}(\delta)$ or $\mathcal{I}^u_s(\delta)$ (where the left endpoint of the first interval given by $\mathcal{I}_n(\delta) = 0$ is close to $\underline{\delta}_4^u$). $p_{s+1}^{\circledast u}(\delta) > p_s^{\circledast}(\delta)$ holds for all $2 \leq s < n$. So welfare in the umbrella regime is lower than in the default regime for all δ such that $s^{*u}(\delta) > s^*(\delta)$. Prices fall instead and welfare rises if $s^{*u}(\delta) = s^*(\delta)$, i.e., if cartel size remains constant.

For any discount factor inside the highlighted intervals, a regime change pushes up the required size for stability. Presuming that related transaction costs are second-order, the cartel grows by one member, its prices increase to $p_{s+1}^{\circledast u}(\delta) > p_s^{\circledast}(\delta)$, and welfare falls. For discount factors $\delta > \underline{\delta}_4^u$ outside of the highlighted intervals, a switch to the umbrella regime leaves the unique stable cartel size constant. The original cartel then continues its operations and lowers prices just enough to maintain dynamic stability.

The parametric example shows that price drops and welfare increases from extended legal standing are possible. Alas, they are restricted to two comparatively small intervals of δ. If we assumed that δ is a priori distributed uniformly on $(0, 1)$, like Katsoulacos et al. (2015), the expected price $\mathbf{E}p_{s^{*u}}^{*u} = 1.01$ in the umbrella regime would be approximately 63% higher than $\mathbf{E}p_{s^*}^{*} = 0.62$ in the default regime.[16]

It is also noteworthy that unatoned overpayments can be higher in the umbrella regime. Consider, e.g., discount factors $\delta \geq \bar{\delta}_2$ in our example. Standing for umbrella victims raises the size of the stable cartel from $s = 2$ to $s' = 3$ and respective (umbrella) prices paid by customers increase to (just below) $p_3^{*u} = 1.14$ compared to $p_2^* = 0.50$. The expected uncompensated damage for customers in the default scenario comprises an overcharge of (approximately) 0.50 for all $3k$ units purchased from outsiders, plus $D_2^R(0.50) = 0.50$ units from the cartel that remain uncompensated with probability $1 - \alpha = 4/5$ – which makes $0.50 \cdot (9 + 4/5 \cdot 0.50) = 4.70$ in total. In contrast, an overcharge of 1.14 on 8.86 units accrues in the umbrella scenario, yielding an expected uncompensated overcharge damage of $1.14 \cdot 4/5 \cdot 8.86 = 8.08$. It may seem paradoxical but total *uncompensated* overpayments increase by almost 72% in a legal regime where a strict superset of customers has the right to be compensated.

4. Concluding Remarks

The key message of above analysis is that the size of cartels should be expected to vary in the legal compensation regime, just as expected cartel profitability and the time

[16] Numbers are rounded to two decimal places.

preference required to sustain collusion do. The effect of extending legal standing to umbrella victims on the latter is beneficial, as one would expect. However, there is an effect on cartel size that goes into the opposite direction: large cartels that were hitherto unstable can become stable. They could not form when freeriding on a cartel with comparatively small market coverage was more lucrative than membership of a larger cartel in the default regime, but that ceases to be an issue if collusion by small cartels is turned sufficiently less profitable or dynamically too demanding.

As illustrated in Section 3, it is possible that the unique stable cartel size remains the same in both legal regimes, or that prices fall even though cartel size increases. There are many configurations, however, in which this is not the case. Then cartels grow because of the umbrella regime, charge higher prices and may, assuming an unchanged detection probability, induce greater uncompensated damages in expectation – despite a greater set of victims having standing in court.[17]

In light of these observations it would seem quite optimistic to uphold the presumption of extended standing "making a significant contribution to the maintenance of effective competition" (CJEU C-557/12 2014, 23) or "hav[ing] greater deterrent effect than recovery limited to direct purchasers" (Blair and Maurer 1982). To be clear: we do not wish to doubt that good arguments for entitling *all* victims of an antitrust infringement to redress, no matter whether they were harmed directly or indirectly, can be put forward. The point is that, in contrast with first intuition, these should primarily be reasons of distributional justice or legal principle – not deterrence and effective competition.

[17]Let us point to Argenton et al. (2020, p. 269) for the related question: "[W]here to stop the causal chain set in motion by the initial liability-generating behaviour?" Upstream firms that supplied (non-)cartel members, producers of complement goods and their suppliers, etc. may all have suffered indirect harm, too. Going beyond a partial equilibrium framework one might even pinpoint ripple effects on wages or bond rates.

Appendix: Proof of Proposition 4

First consider part (i). That $\mathcal{I}^u_s(\delta) > 0$ does not imply $\mathcal{I}_s(\delta) > 0$ is obvious from our example. It is also obvious that implication (26) is true for $\delta \in (\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$ because $\mathcal{I}^u_s(\underline{\delta}^u_s) = 0$ and $\mathcal{I}^u_s(\delta)$ strictly increases on $(\underline{\delta}^u_s, \underline{\delta}^u_{s-1})$. (Recall that a cartel of s firms is dynamically stable in both regimes if $\delta \in (\underline{\delta}^u_s, 1)$.)

It remains to show $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on the interval $(\underline{\delta}^u_{s-1}, 1)$ where sub-cartels of size $s - 1 \geq \underline{s}^u$ would be profitable and dynamically stable. Depending on market parameters and considered cartel size s, this interval can be split into quadratic, linear and constant parts of $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ differently from Figure 1. Namely, several pairwise comparisons of the critical discount factors that determine if $p^{\circledast}_s(\delta) = p^{\circ}_s(\delta)$ or p^*_s and if $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ or p^*_{s-1} in eq. (18), and its analogue, can go either way. Figure 3 shows the Hasse diagram of the corresponding partially ordered set. This gives rise to seven possible cases. Before turning to each, we establish some properties that hold whenever one or two pairwise comparisons, e.g., between $\underline{\delta}_{s-1}$ and $\bar{\delta}_s$, go in a particular way.

Claim 1. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} < \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is strictly concave and decreasing on $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left((\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ and linearly decreasing on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ $\left((\bar{\delta}^u_s, \bar{\delta}^u_{s-1})\right)$. $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ falls faster in the latter intervals than in the former.

Proof. For $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ dynamic stability constraints are binding for cartel sizes s and $s-1$. Substituting $p^{\circledast}_s(\delta) = \frac{a-(n-s)k}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast u}_s(\delta) = \frac{a-(n-s)k\cdot\mu^{-1}}{b} - \frac{(1-\delta)sk}{(1-\alpha(\beta+\tau))b}$, $p^{\circledast}_{s-1}(\delta) = p^{\circ}_{s-1}(\delta)$ and $p^{\circledast u}_{s-1}(\delta) = p^{\circ u}_{s-1}(\delta)$ into $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ and abbreviating $e := 1 - \alpha(\beta + \tau) \in (0, 1)$, one obtains

$$\mathcal{I}_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta + k\delta(n+1-s\delta) - k\alpha(1+n\delta-s\delta)(\beta+\tau)\Big] \tag{28}$$

and

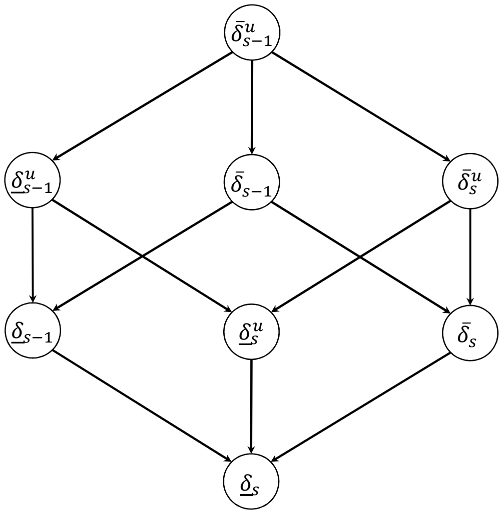
$$\mathcal{I}^u_s(\delta) = \frac{k}{be}\Big[a\alpha\delta(\beta+\tau) - a\delta - k\alpha\tau + k\delta\Big(n+1-s\delta-(n-s)\alpha\tau\Big)\Big] \tag{29}$$

after some algebra. Corresponding derivatives with respect to δ are

$$\frac{\partial\mathcal{I}_s(\delta)}{\partial\delta} = \frac{k}{b}\Big[-a + k(n-s) - k\Big(s(2\delta-1)-1\Big)e^{-1}\Big] \tag{30}$$

and

$$\frac{\partial\mathcal{I}^u_s(\delta)}{\partial\delta} = \frac{k}{b}\Big[-a + k\Big(1+n-2s\delta-(n-s)\alpha\tau\Big)e^{-1}\Big] \tag{31}$$



Case	Ordering
1	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_{s-1}$
2	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
3	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$
4	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
5	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$
6	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$
7	$\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$

Figure 3: Hasse diagram for critical discount factors ($x \to y$ indicating $x > y$) and compatible orderings that partition $(\underline{\delta}^u_{s-1}, 1)$

with

$$\frac{\partial^2 \mathcal{I}_s(\delta)}{\partial \delta^2} = \frac{\partial^2 \mathcal{I}^u_s(\delta)}{\partial \delta^2} = \frac{-2k^2 s}{be} < 0. \tag{32}$$

So $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ are strictly concave in δ.

Substituting $\delta = \underline{\delta}_{s-1}$ from eq. (9) into (30) yields

$$\frac{\partial \mathcal{I}_s(\underline{\delta}_{s-1})}{\partial \delta} = \frac{k}{be}\left(\frac{e(s+1)(a-kn)}{s-1} - k(s\alpha(\beta+\tau)-1)\right) < 0. \tag{33}$$

Inequality (33) is satisfied iff

$$\frac{(s+1)(a-kn)}{s-1} < \frac{k(s\alpha(\beta+\tau)-1)}{1-\alpha(\beta+\tau)}. \tag{34}$$

Making the LHS as large as possible, that is, substituting $a = k(n-1)$ yields

$$\begin{aligned} & -\frac{s+1}{s-1} < \frac{s\alpha(\beta+\tau)-1}{1-\alpha(\beta+\tau)} \\ \Leftrightarrow\ & -(s+1)(1-\alpha(\beta+\tau)) < (s-1)(s\alpha(\beta+\tau)-1) \\ \Leftrightarrow\ & 0 < 2 - \alpha(\beta+\tau) + s^2\alpha(\beta+\tau) - 2s\alpha(\beta+\tau) \\ \Leftrightarrow\ & 0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha(\beta+\tau)(s-2)}_{>0}. \end{aligned} \tag{35}$$

Similarly substituting $\delta = \underline{\delta}^u_{s-1}$ from eq. (15) into (31) gives

$$\frac{\partial \mathcal{I}^u_s(\underline{\delta}^u_{s-1})}{\partial \delta} = \frac{-k}{b(s-1)e}(-a(1+s)e - k(n(1+s)(-1+\alpha\tau) + (s-1)(1-s\alpha\tau))) < 0 \tag{36}$$

which is satisfied iff

$$0 < -a(1+s)e - k(n(1+s)(-1+\alpha\tau) + (s-1)(1-s\alpha\tau)). \tag{37}$$

Making the RHS as small as possible by substituting $a = k(n-1)$ yields

$$\begin{aligned}
&0 < -(n-1)(1+s)e - n(1+s)(-1+\alpha\tau) - (s-1)(1-s\alpha\tau)\\
\Leftrightarrow\ &0 < -n(1-\alpha\beta-\alpha\tau) - ns(1-\alpha\beta-\alpha\tau) + (1-\alpha\beta-\alpha\tau) + s(1-\alpha\beta-\alpha\tau) + n - n\alpha\tau + ns\\
&- ns\alpha\tau - s + s^2\alpha\tau + 1 - s\alpha\tau\\
\Leftrightarrow\ &0 < 2 - \alpha\beta - \alpha\tau + s^2\alpha\tau - 2s\alpha\tau + n\alpha\beta + ns\alpha\beta - s\alpha\beta\\
\Leftrightarrow\ &0 < \underbrace{1+e}_{>0} + \underbrace{s\alpha\tau(s-2)}_{\geq 0} + \underbrace{s\alpha\beta(n-1)}_{>0}.
\end{aligned} \tag{38}$$

So both derivatives $\frac{\partial \mathcal{I}_s(\delta)}{\partial \delta}$ and $\frac{\partial \mathcal{I}^u_s(\delta)}{\partial \delta}$ are negative at the respective left endpoints of $(\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$. By (32) they are falling. Hence the slopes of $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ must be negative for all $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ and $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$, respectively.

For $\delta \in (\bar{\delta}_s, \bar{\delta}_{s-1})$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Profits of s cartel members consequently become constant, too, and $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have slope

$$-k\frac{\partial p^{\circ}_{s-1}(\delta)}{\partial \delta} = -k\frac{\partial p^{\circ u}_{s-1}(\delta)}{\partial \delta} = -\frac{(s-1)k^2}{eb}. \tag{39}$$

This is less than the slopes identified for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_s)$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)\right)$ in eq. (30) $\left(\text{eq. (31)}\right)$ where cartel profits still increase in δ. This proves Claim 1. ■

Claim 2. If $\underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$ then $\mathcal{I}_s(\delta)$ $\left(\mathcal{I}^u_s(\delta)\right)$ is linearly decreasing for $\delta \in (\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ $\left(\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}^u_{s-1})\right)$.

Proof. For $\delta > \underline{\delta}_{s-1} > \bar{\delta}_s$ $\left(\delta > \underline{\delta}^u_{s-1} > \bar{\delta}^u_s\right)$, $p^{\circledast}_s(\delta)$ and $p^{\circledast u}_s(\delta)$ are constant to p^*_s and p^{*u}_s. Collusive profits of s members then are constant in δ whereas cartel prices and profits of an outsider are linearly increasing in δ for a cartel of size $s-1$. Hence $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ both have the slope already identified in eq. (39). ■

Claim 3. If $\underline{\delta}_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$ then $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$.

Proof. $\mathcal{I}_s(\delta)$ and $\mathcal{I}^u_s(\delta)$ are then given by eqs. (28) and (29) above. Hence

$$\begin{aligned}\mathcal{I}^u_s(\delta) - \mathcal{I}_s(\delta) &= \frac{k}{be}\Big[-k\alpha\tau - k\delta(n-s)\alpha\tau + k\alpha(1+n\delta-s\delta)(\beta+\tau)\Big] \\ &= \frac{k^2\alpha\beta}{be}\Big(1+(n-s)\delta\Big) > 0.\end{aligned} \tag{40}$$

■

Claim 4. $\Delta\bar{\delta}^u_s := \bar{\delta}^u_{s-1} - \bar{\delta}^u_s > \bar{\delta}_{s-1} - \bar{\delta}_s := \Delta\bar{\delta}_s$.

Proof. Applying eq. (8) and simplifying yields

$$\Delta\bar{\delta}_s = \frac{-e(a-(n-s)k)2(s-1)k + 2ske(a-(n-(s-1))k)}{4sk^2(s-1)} = -\frac{(a-kn)e}{2k(s-1)s} \tag{41}$$

and similarly eq. (24) gives

$$\Delta\bar{\delta}^u_s = -\frac{ae - kn(1-\alpha\tau)}{2k(s-1)s}. \tag{42}$$

So

$$\Delta\bar{\delta}^u_s - \Delta\bar{\delta}_s = \frac{-ae + kn(1-\alpha\tau) + ae - kn(1-\alpha\beta-\alpha\tau)}{2k(s-1)s} = \frac{n\alpha\beta}{2(s-1)s} > 0. \tag{43}$$

■

Claim 5. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}^u_{s-1}, 1)$.

Proof. $\delta > \bar{\delta}^u_{s-1}$ implies $\delta > \bar{\delta}_{s-1}$ and $\delta > \bar{\delta}^u_s \geq \bar{\delta}_s$. Hence, all relevant dynamically stable cartels can choose the respective unconstrained profit maximizers $p^{\circledast}_{s-1}(\delta) = p^*_{s-1} = \frac{1}{2b}\Big(a-(n-(s-1))k\Big)$, $p^{\circledast u}_{s-1}(\delta) = p^{*u}_{s-1} = \frac{1}{2b}\Big(a-(n-(s-1))k\cdot\mu^{-1}\Big)$, $p^{\circledast u}_s(\delta) = p^{*u}_s$ and $p^{\circledast}_s(\delta) = p^*_s$. Again abbreviating $e := 1-\alpha(\beta+\tau) \in (0,1)$ and using $\mu^{-1} = 1 + \frac{\alpha\beta}{e}$, $\mathcal{I}_s(\delta) < \mathcal{I}^u_s(\delta)$ holds iff

$$
\begin{aligned}
&\frac{e\Big(a-(n-s)k\Big)^2}{4bs}-\frac{a-(n-s+1)k}{2b}k<\frac{e\Big(a-(n-s)k\mu^{-1}\Big)^2}{4bs}-\frac{a-(n-s+1)k\mu^{-1}}{2b}k\\
\Leftrightarrow\quad & e\Big(a-(n-s)k\Big)^2+2ks\Big((n-s+1)k-a\Big)<\\
&\qquad e\Big(a-(n-s)k-\frac{\alpha\beta}{e}(n-s)k\Big)^2+2ks\Big((n-s+1)k\mu^{-1}-a\Big)\\
\Leftrightarrow\quad & 2k^2s(n-s+1)<-2e\Big(a-k(n-s)\Big)\frac{\alpha\beta}{e}k(n-s)+e\Big(\frac{\alpha\beta}{e}k(n-s)\Big)^2+2k^2s(n-s+1)\mu^{-1}\\
\Leftrightarrow\quad & 0<2\alpha\beta\Big(k(n-s)-a\Big)k(n-s)+\frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2-2k^2s(n-s+1)(1-\mu^{-1})\\
\Leftrightarrow\quad & 0<2\alpha\beta\Big(k(n-s)-a\Big)k(n-s)+\frac{(\alpha\beta)^2}{e}\Big(k(n-s)\Big)^2+2k^2s(n-s+1)\frac{\alpha\beta}{e}\\
\Leftrightarrow\quad & 0<2\Big(k(n-s)-a\Big)k(n-s)e+\alpha\beta\Big(k(n-s)\Big)^2+2k^2s(n-s+1). \qquad (44)
\end{aligned}
$$

The RHS of (44) is decreasing in a. So it suffices to observe it is positive for the maximal value $a=k(n-1)-\epsilon\approx k(n-1)$ that satisfies (A1′). In particular,

$$
\begin{aligned}
&0<2k\Big(k(n-s)-k(n-1)\Big)(n-s)e+\alpha\beta\Big(k(n-s)\Big)^2+2k^2s(n-s+1)\\
\Leftrightarrow\quad & 0<k^2\Big[2(-s+1)(n-s)e+\alpha\beta(n-s)^2+2s(n-s+1)\Big]\\
\Leftrightarrow\quad & 0<2(1-s)(n-s)e+\alpha\beta(n-s)^2+2s(n-s+1) \qquad (45)\\
\Leftrightarrow\quad & 0<2(n-s)e-2s(n-s)\Big(1-\alpha(\beta+\tau)\Big)+\alpha\beta(n-s)^2+2s+2s(n-s)\\
\Leftrightarrow\quad & 0<\underbrace{2(n-s)e+2s(n-s)\alpha(\beta+\tau)+\alpha\beta(n-s)^2}_{\geq 0}+\underbrace{2s}_{>0}.
\end{aligned}
$$

■

We are now ready to verify $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ for $\delta\in(\underline{\delta}^u_{s-1},1)$ in the seven cases identified in Figure 3:

Case 1 with $\ldots<\underline{\delta}^u_{s-1}<\bar{\delta}^u_{s-1}$. Then $\delta\in(\underline{\delta}^u_{s-1},1)$ implies $\delta>\bar{\delta}_{s-1},\bar{\delta}_s$. So $p^{\circledast}_{s-1}(\delta)=p^*_{s-1}$ and $p^{\circledast}_s(\delta)=p^*_s$ in the default regime, which renders $\mathcal{I}_s(\delta)$ constant for $\delta\in(\underline{\delta}^u_{s-1},1)$. $\mathcal{I}^u_s(\delta)$ linearly decreases from $\mathcal{I}^u_s(\underline{\delta}^u_{s-1})$ to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1})$ according to claim 2 and then stays constant. By claim 5, $\mathcal{I}^u_s(\bar{\delta}^u_{s-1})>\mathcal{I}_s(\bar{\delta}^u_{s-1})$. Hence $\mathcal{I}^u_s(\underline{\delta}^u_{s-1})>\mathcal{I}_s(\underline{\delta}^u_{s-1})$ to avoid a contradiction. So $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ holds for all $\delta\in(\underline{\delta}^u_{s-1},1)$.

Case 2 with $\ldots<\underline{\delta}^u_{s-1}<\bar{\delta}_{s-1}<\bar{\delta}^u_{s-1}$. For $\delta\in(\bar{\delta}^u_{s-1},1)$ we have $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ according to claim 5. Claim 2 ensures that $\mathcal{I}^u_s(\delta)$ falls linearly on $(\underline{\delta}^u_{s-1},\bar{\delta}^u_{s-1})$ and in particular on sub-interval $(\bar{\delta}_{s-1},\bar{\delta}^u_{s-1})$ where $\mathcal{I}_s(\delta)$ is constant. Hence $\mathcal{I}^u_s(\delta)>\mathcal{I}_s(\delta)$ for $\delta\in(\bar{\delta}_{s-1},\bar{\delta}^u_{s-1})$

in order not to contradict $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$. For $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$, both $\mathcal{I}^u_s(\delta)$ and $\mathcal{I}_s(\delta)$ decrease with identical slope (invoking claim 1 or 2 depending on $\bar{\delta}_s \lesseqgtr \underline{\delta}_{s-1}$). Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ must also hold for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$.

Case 3 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 1.

Case 4 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. That $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$ can be concluded from claims 2 and 5 just as for case 1. $\mathcal{I}_s(\delta)$ decreases linearly with slope $-\frac{(s-1)k^2}{eb}$ on $(\underline{\delta}_{s-1}, \bar{\delta}_{s-1})$ for $\bar{\delta}_s < \underline{\delta}_{s-1}$ and on $(\bar{\delta}_s, \bar{\delta}_{s-1})$ for $\bar{\delta}_s > \underline{\delta}_{s-1}$, and so does $\mathcal{I}^u_s(\delta)$ on $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ (claim 2). Hence, considering $\delta \geq \underline{\delta}^u_{s-1}$, $\mathcal{I}_s(\delta)$ assumes a maximum of $\mathcal{I}_s(\underline{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1}) + \frac{(s-1)k^2}{eb} \cdot (\bar{\delta}_{s-1} - \underline{\delta}^u_{s-1})$ at $\delta = \underline{\delta}^u_{s-1}$. $\mathcal{I}^u_s(\delta)$ exceeds that value at $\bar{\delta}^u_s > \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\}$ and assumes even higher values on $(\underline{\delta}^u_{s-1}, \bar{\delta}^u_s)$ because it is decreasing on this interval to $\mathcal{I}_s(\underline{\delta}^u_{s-1})$.[18] Hence $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 5 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$ is directly analogous to case 4.

Case 6 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}^u_s < \bar{\delta}_{s-1} < \bar{\delta}^u_{s-1}$. Given $\underline{\delta}_{s-1} < \underline{\delta}^u_{s-1} < \bar{\delta}_s$ and $\underline{\delta}^u_{s-1} < \bar{\delta}^u_s$, claim 3 yields $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_s)$. Then $\mathcal{I}_s(\delta)$ falls linearly on $(\bar{\delta}_s, \bar{\delta}^u_s)$ while $\mathcal{I}^u_s(\delta)$ decreases in a slower strictly concave fashion for $\delta \in (\bar{\delta}_s, \bar{\delta}^u_s)$ (claim 1). So $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ on $(\bar{\delta}_s, \bar{\delta}^u_s)$. For $\delta \in (\bar{\delta}^u_s, \bar{\delta}_{s-1})$, both functions fall linearly with same slope (claim 1), extending $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ at $\delta = \bar{\delta}^u_s$ to interval $(\bar{\delta}^u_s, \bar{\delta}_{s-1})$. $\mathcal{I}_s(\delta)$ turns constant for $\delta \in (\bar{\delta}_{s-1}, 1)$ while $\mathcal{I}^u_s(\delta)$ continues its decrease – but only to a value of $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1}) = \mathcal{I}_s(\bar{\delta}_{s-1})$ (claim 5). Then $\mathcal{I}^u_s(\delta)$ turns constant too, implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for all $\delta \in (\underline{\delta}^u_{s-1}, 1)$.

Case 7 with $\ldots < \underline{\delta}^u_{s-1} < \bar{\delta}_s < \bar{\delta}_{s-1} < \bar{\delta}^u_s < \bar{\delta}^u_{s-1}$. For $\delta \in (\bar{\delta}_{s-1}, 1)$, $\mathcal{I}_s(\delta)$ is constant. By contrast, $\mathcal{I}^u_s(\delta)$ is constant to $\mathcal{I}^u_s(\bar{\delta}^u_{s-1}) > \mathcal{I}_s(\bar{\delta}^u_{s-1})$ on $(\bar{\delta}^u_{s-1}, 1)$ (claim 5) and, focusing on $(\bar{\delta}_{s-1}, \bar{\delta}^u_{s-1})$, decreasing to this value from $\mathcal{I}^u_s(\bar{\delta}_{s-1})$ – implying $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\bar{\delta}_{s-1}, 1)$. $\mathcal{I}^u_s(\delta) > \mathcal{I}_s(\delta)$ for $\delta \in (\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ can be concluded in direct analogy to case 6.

This proves part (i) of the proposition. Part (ii) then follows from recalling $\mathcal{E}_n(\delta) = \mathcal{E}^u_n(\delta)$ and that $\mathcal{E}_s(\delta) = -\mathcal{I}_{s+1}(\delta)$ and $\mathcal{E}^u_s(\delta) = -\mathcal{I}^u_{s+1}(\delta)$ for $s < n$. □

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[18] That $\mathcal{I}^u_s(\bar{\delta}^u_s) > \mathcal{I}_s(\underline{\delta}^u_{s-1})$ is most easily seen by looking at $\mathcal{I}^u_s$'s behavior from the right, i.e., moving down from $\delta = 1$ to $\delta = \underline{\delta}^u_{s-1}$: it switches from constant to increasing with slope $\left|-\frac{(s-1)k^2}{eb}\right|$ already at $\bar{\delta}^u_{s-1} > \bar{\delta}_{s-1}$ and continues this increase over an interval $(\bar{\delta}^u_s, \bar{\delta}^u_{s-1})$ that is wider than the corresponding interval $(\underline{\delta}^u_{s-1}, \bar{\delta}_{s-1})$ of $\mathcal{I}_s$'s increase for $\delta \geq \underline{\delta}^u_{s-1}$ because $\bar{\delta}_{s-1} - \bar{\delta}_s < \bar{\delta}^u_{s-1} - \bar{\delta}^u_s$ (claim 4) and $\bar{\delta}_s \leq \max\left\{\underline{\delta}_{s-1}, \bar{\delta}_s\right\} < \underline{\delta}^u_{s-1}$ imply $\bar{\delta}_{s-1} - \bar{\delta}^u_{s-1} < \underline{\delta}^u_{s-1} - \bar{\delta}^u_s$.

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