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Simple Voting Games and Cartel Damage Proportioning

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Abstract: Individual contributions by infringing firms to the compensation of cartel victims must reflect their "relative responsibility for the harm caused" according to EU legislation. Several studies have argued that the theoretically best way to operationalize this norm is to apply the Shapley value to an equilibrium model of cartel prices. Because calibrating such a model is demanding, legal practitioners prefer workarounds based on market shares. Relative sales, revenues, and profits however fail to reflect causal links between individual behavior and prices. We develop a pragmatic alternative: use simple voting games to describe which cartel configurations can(not) cause significant price increases in an approximate, dichotomous way; then compute the Shapley-Shubik index. Simulations for a variety of market scenarios document that this captures relative responsibility better than market share heuristics can.

Keywords: simple voting games; shapley-shubik index; relative responsibility; cartel damage allocation

JEL Classification: C71; D04; L40; L13; D43

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1. Introduction

Simple voting games are an important subclass of coalitional games with transferable utility (TU games; cf. [1] (Chapter 16)). They partition the set of all possible coalitions among a given set N of players into two categories: a coalition $S \subseteq N$ is either 'winning', which is denoted by assigning worth $v(S) = 1$ to S, or 'losing' ($v(S) = 0$). These games received an entire chapter's attention by von Neumann and Morgenstern [2] (Chapter 10); Taylor and Zwicker [3] devoted a full-length monograph to them. In theoretical analysis, they have played a key role for the axiomatic characterization of solutions ever since Shapley [4] introduced his value on the vector space of TU games via a suitable basis of simple (voting) games. Investigation of their properties–such as the dimensionality of representations of a given characteristic function $v: S \mapsto \{0, 1\}$ by means of voting weight vectors and thresholds (see, e.g., [5–7])–still goes on.

The long list of practical applications of simple games is dominated by voting bodies such as the US Electoral College, the Board of Governors or the Board of Directors of the International Monetary Fund, the EU Council of Ministers, shareholder meetings, etc. The typical concern is the distribution of a priori voting power in the respective institution.[1] The goal of this article is to propose a very different and promising new domain of application.

We will argue that simple games can serve as useful approximations of more complicated TU games in the context of proportioning cartel damages.[2] Let us first provide some background on the pertinent legal environment and economic problem before we develop our proposal.

Around a decade ago, the European Commission passed a *Directive on Antitrust Damages Actions* (2014/104/EU – [12]) in order to reduce legal and procedural hurdles for cartel victims to reclaim antitrust damages. It was motivated by the observation that "... total annual cost for hardcore cartels in the EU can be estimated to range from approximately €25 billion [...] to approximately €69 billion..." and only a small share of accrued damages were later reclaimed (see SWD/2013/203/Final, recitals 65, 67). The

Directive clarified that (i) when a detected cartel is litigated in the EU, all co-infringers are *jointly and severally liable* for the entire harm; and (ii) if a given infringer compensates a victim, other infringers may be required to bear part of the payment.

This means that a customer who has suffered cartel-related price overcharges on its purchases can sue any cartel member for any desired share of its total claim to compensation–independently of whether the pertinent purchases were made from this or another firm. The co-defendant that is sued and convicted to pay compensation to a victim can afterwards force its former cartel partners to contribute "*. . . if it has paid more compensation than its share*" (Directive 2014/104/EU, recital 37). Importantly, the corresponding internal share in the compensation of a victim must reflect the "*. . . relative responsibility for the harm caused by the infringement of competition law*" according to Article 11(5) of the Directive.[3]

With cartel damages and claims to compensation amounting to billions of euros, operationalization of this legal norm and determination of the "relative responsibility for the harm caused" has significant practical relevance. Proposals to invoke the *Shapley value* of appropriately defined TU games were made early on by [13,14]. The underlying reasoning and consequences have been elaborated by [15].

The arguments for using the Shapley value revolve around several properties that a responsibility-based allocation should have: it is almost trivial, for instance, that *all* of the compensation granted to a given victim must somehow be allocated among the cartel members. This translates into *efficiency* of a value in cooperative game theory. A firm's individual share of compensation should not vary with the choice of currency nor unit of account, nor depend on whether the compensation payment included interest or if multiple damages are compensated simultaneously or sequentially (reflected by *linearity*). In order for firms' shares to reflect legal and economic responsibility for harm, the causal connections between an individual firm's cartel membership and the size of a customer's damage are crucial. They ought to be the shares' only determinant. This formally translates into the *marginality* requirement of Young [16]. Moreover, firms whose cartel membership has identical effects on a victim's damage should contribute the same and firms whose membership can be ruled out to have affected the relevant price should have a share of zero (*symmetry* and *null-player* properties).

Well-known results by [4,16] then pin down the Shapley value as the right tool for responsibility-based proportioning: it is the only method that can simultaneously satisfy the indicated properties.[4] Desirability of the properties and the corresponding mathematical conclusion have been endorsed by legal practitioners. For instance, [20] (recital 123) holds that "Shapley values consistently deliver an apportionment according to the relative responsibility for the harm".

The big caveat, however, is that application of the Shapley value necessitates the quantification of counterfactual damages, i.e., estimates of which harm would have obtained had one or several of the cartel members refused to participate in the infringement of competition law. Otherwise, the marginal contributions to harm that determine responsibility are not defined.

So a practitioner wishing to apply the Shapley value to the compensation for a given price overcharge must get hold of a sufficiently rich panel of market data to estimate an economic model that allows to quantify counterfactual harms. Successfully setting up such model is not inconceivable (see, e.g., [21]) and corresponding market simulation techniques are well-established in other areas of competition policy (mainly control of mergers–e.g., [22]). Still it is hard to dispute the assessment by Bornemann [20] (recital 124) that ". . . for almost all real-life cases, such a data panel will be exceedingly difficult or downright impossible to obtain".

So it is not surprising that various ad hoc proportioning heuristics have been suggested. For instance, [23] early on argued that an allocation by ". . . sales of the product during the conspiracy . . . " could be an appropriate benchmark. This has been taken up in Portugal where co-defendants are liable in proportion to average cartel-time sales in the affected market [20] (recital 107). Other ad hoc proposals involve sales during a competitive period preceding or following cartel operations, an allocation by (pre-, post-, or cartel period) revenue shares or profit shares, an equal division by heads, and so on. [15] show that none of these workarounds dominates the others: any given one may uniquely come close to the relative responsibilities identified by the Shapley value in one market scenario but perform very badly in another scenario. The relative accuracy of the heuristics varies in the asymmetry of firm sizes, costs, intensity of product differentiation, etc. Even the respective 'best' among the considered heuristics is in some cases very far off. Our investigation is therefore motivated by two questions: First, how can we pragmatically approximate the marginal contributions to harm when a precise market model is too difficult to construct? Second, how accurate are the resulting inferences on relative responsibilities compared to working with known structural parameters and compared to ad hoc rules of thumb?

After illustrating the damage proportioning problem by a numerical example (Section 2) and setting out some notation (Section 3), we will propose the dichotomous approximation of damages by simple voting games–and thus of marginal contributions by pivotality with respect to causing significant overcharges–as our answer to the first question in Section 4. The performance of the resulting proportionings is then evaluated in Section 5. Dichotomous approximations turn out to compare very well to existing heuristics. Following the concluding remarks in Section 6, we enumerate all conceivable 179 dichotomous damage scenarios with cartels of up to five members and the respective relative responsibilities for easy reference in Appendix A.

2. Illustration

Consider a market with three producers of differentiated goods that compete à la Bertrand, i.e., firms simultaneously set prices p_1, p_2 and p_3 (see, e.g., [24]). Their respective costs are $C_1(q_1) = 30q_1$, $C_2(q_2) = 20q_2$ and $C_3(q_3) = 10q_3$ where $q = (q_1, q_2, q_3)$ denotes the vector of quantities produced by the firms. Let the quantities demanded by customers vary in price vector $p = (p_1, p_2, p_3)$ as follows: $D_1(p) = 100 - 4p_1 + 3p_2 + 0.4p_3$, $D_2(p) = 100 - 4p_2 + 3p_1 + 0.4p_3$ and $D_3(p) = 150 - 3p_3 + 0.4(p_1 + p_2)$. So products 1 and 2 constitute considerably closer substitutes than product 3 but the firm producing product 3 faces a higher demand intercept (i.e., market size) and the lowest unit costs.

The individual maximization of profits yields Bertrand equilibrium prices $p^* = (44.7; 41.0; 35.7)$ rounded to one decimal place. The corresponding equilibrium outputs are $q^* = (58.7; 84.2; 77.1)$ with revenues of $R^* = (2622.5; 3453.7; 2755.1)$ and profits of $\Pi^* = (861.5; 1770.6; 1983.7)$. If the firms form a cartel and maximize total industry profit, prices rise to $p^c = (82.2; 77.2; 47.9)$ while quantities fall to $q^c = (22; 57; 70)$. In the absence of side payments, profits in the cartelized market are $\Pi^c = (1147.6; 3258.4; 2653.7)$ from revenues of $R^c = (1807.6; 4398.4; 3353.7)$.

While profits increase, the illegal coordination by the firms implies an *overcharge damage* of $\Delta p_i = p_i^c - p_i^*$ for each unit of good i sold to the respective buyer. In our scenario, overcharges are $\Delta p = (37.5; 36.1; 12.2)$ per unit and result in product-specific total overcharge damages of $q^c \cdot \Delta p = (824.8; 2059.1; 853.7)$.[5]

Suppose now that a customer k who purchased 10 units from firm 1 at p_1^c, and nothing else, sues for compensation. The customer may take firm 2 to court because the plaintiff is free to choose; perhaps k perceives the best odds for enforcing her claim against the profit champion. If k is then granted compensation for her total overcharges $O^k = 375$, firm 2 must pay out O^k. But it is entitled to reclaim some of this from firms 1 and 3. Table 1 summarizes the implications of several proportioning heuristics ρ that have been discussed by legal practitioners.[6]

Table 1. Proportioning heuristics discussed by legal practitioners.

Dividing Compensation ...	**Firms' Shares of** $O^k \propto \Delta p_1$
...equally per head (ρ^0)	$(33.3\%; 33.3\%; 33.3\%)$
...by cartel revenue (ρ^1)	$(18.9\%; 46.0\%; 35.1\%)$
...by cartel sales (ρ^2)	$(14.8\%; 38.3\%; 47.0\%)$
...by competitive revenue (ρ^3)	$(29.7\%; 39.1\%; 31.2\%)$
...by competitive sales (ρ^4)	$(26.7\%; 38.3\%; 35.1\%)$
...by cartel profits (ρ^5)	$(16.3\%; 46.2\%; 37.6\%)$
...by competitive profits (ρ^6)	$(18.7\%; 38.4\%; 43.0\%)$
...by cartel benefits (ρ^7)	$(22.0\%; 55.6\%; 22.4\%)$

The table illustrates two shortcomings of ad hoc proportioning. First, a firm's share of the compensation can vary by up to 20 percentage points when switching from one heuristic to another. There is no (or at least no obvious) normative basis for claiming that one division of k's compensation, e.g., by cartel revenues, reflects relative responsibilities better than another allocation, e.g., equally per head. Differences in the firms' shares across the heuristics seem entirely arbitrary.

Second, all rules assign firm 1 a share of at most 33.3%, i.e., one or both of its competitors are attributed weakly more blame for the increase of p_1 than is firm 1. That other firms bear greater economic responsibility for a given firm's price increase is not inconceivable: as an extreme case, think of a non-cartel member who merely best-responds to the higher industry price level after others secretly formed a (partial) cartel. Intuitively, however, behavior of a product's vendor has the strongest effect on its price. So without further justification, a less than equal share for firm 1 looks rather small.

By contrast, the allocation based on the Shapley shares $\rho^* = (46.0\%; 44.7\%; 9.3\%)$ that we will derive in Section 3 takes the relevant economic fundamentals into account. It uses equilibrium analysis based on full knowledge of the cost and demand structure. Such analysis acknowledges that the observed price increase of good 1 was driven mainly by elimination of competition between firms 1 and 2 given the substantial cross-price effects $\partial D_1/\partial p_2 = \partial D_2/\partial p_1 = 3$ of p_1 and p_2 (which numerically dominate $\partial D_1/\partial p_3 = \partial D_3/\partial p_1 = 0.4$). Significant harm would already have accrued to customer k if only firms 1 and 2 had coordinated their strategies. By comparison, firm 3's cartel participation–despite the market size and cost advantages which imply the highest share under heuristics ρ^2 and ρ^6–contributed little to k's damage.

The listed ad hoc heuristics cannot pick this up because they look at a set of numbers–various measures of market share–that correlate only very loosely with the critical economic fundamentals. At the same time, a casual inspection of the market is likely to reveal that–without an infringement–competition is closest between firms 1 and 2. This is what creates scope for using the Shapley value of an appropriate simple voting game $\tilde{v}$ instead of the heuristics. Such game can serve as a dichotomous approximation of damages for different cartel arrangements. Its Shapley value–also known as $\tilde{v}$'s Shapley-Shubik index–is a good proxy of the Shapley value of the TU game v derived from full-fledged equilibrium analysis, and hence of responsibilities for harm.

In above example, competitive revenue shares $\rho^3 = (29.7\%; 39.1\%; 31.2\%)$ come closest to reflecting responsibility in the sense of having smallest $\|\cdot\|_1$-distance to ρ^* among the heuristics. The respective distance–a mathematical measure of deviations from Shapley-based relative responsibilities–is still 43.8%. We show how easily it can be improved after introducing our formal setup.

3. Basic Notation and Setup

A finite TU game is defined by a set N of n players and a characteristic function $v\colon 2^N \to \mathbb{R}$ that assigns a worth $v(S)$ to each subset $S \subseteq N$ of players, also called coalition S. A TU game (N, v) is referred to as a (monotonic) *simple voting game* or just *simple game* if $v(S) \in \{0,1\}$ for all $S \subseteq N$, $v(\varnothing) = 0$, $v(N) = 1$, and $S \subset T \subseteq N \Rightarrow v(S) \leq v(T)$. In a simple game (N, v), coalitions S with $v(S) = 1$ (0) are called *winning* (*losing*) and the game can be described most concisely by the list $\mathcal{M}(v) = \{T \subseteq N\colon v(T) = 1 \text{ and } S \subset T \Rightarrow v(S) = 0\}$ of its *minimal winning coalitions* (MWC).

In our context, the set N refers to the members of a detected cartel. $v(N)$ represents the financial harm associated with a particular purchase that now needs to be compensated by the co-defendants and split according to their relative responsibilities. Our default interpretation will be that $v(N)$ equals the overcharge damage on a single unit of some good j. It thus corresponds to the difference $\Delta p_j = p_j^c - p_j^*$ between the cartel price p_j^c and the competitive price p_j^* that would have obtained in the absence of an infringement.[7] A victim's total compensation under this interpretation would typically be a fixed multiple of $v(N)$ that accounts for the number of units purchased and that may additionally reflect accrued interest or super-compensation requirements (such as trebling of damages).

We also write $v^j(N)$ instead of $v(N)$ if we want to highlight that overcharges on the specific good j are considered: namely, the division of damages for n different goods involves the consideration of n different TU games. For a strict subset $S \subset N$ of firms, $v^j(S)$ then represents an estimate of the (per unit) price overcharge Δp_j that would have arisen if only the players in S had illegally coordinated their actions to maximize joint profit, with players in $N \setminus S$ maximizing profits in competitive fashion.[8]

Counterfactuals play a crucial role in both the quantification of an actual harm $v(N)$, as is explicitly acknowledged in Directive 2014/104/EU, recital 46, and any causality-based ascription of responsibility. In an ideal world, $v(S)$ can be determined for all $S \subseteq N$ from a 'correct' model of the market. In reality, even sophisticated estimates obtained, e.g., by calibrating a structural model of price or quantity competition to cartel and pre or post-cartel observables, are bound to imply a big error margin for prices pertaining to unobserved cartel configurations. That rather coarse and even dichotomous assessments of $v(S)$, however, can yield good and significantly better ascriptions of relative responsibility than ad hoc measures of market share is the key message of our investigation. Obviously $v(S) = v(\varnothing) = 0$ if $|S| = 1$: joint profit maximization then means competitive behavior by S's single member.

Various game-theoretic solution concepts might be invoked in order to share $v(N)$ among the players. But as shown by Shapley [4] and Young [16], only the *Shapley value* φ, which is defined by

$$\varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \left[v(S) - v(S \setminus \{i\})\right] \quad \text{where } s = |S| \tag{1}$$

for $i \in N$, satisfies all the desirable properties mentioned in Section 1. We refer to [15] for their formal representation and a detailed discussion of why they ensure that $\rho_i^* := \varphi_i(N, v)/v(N)$ can indeed be interpreted as the proportion of damage $v(N)$ that firm i's infringement of the law is responsible for. For a simple game v, φ is also known as the *Shapley-Shubik index* [25]. A player i's marginal contributions $v(S) - v(S \setminus \{i\}) \in \{0,1\}$ can then be viewed as indicator functions for *swing* or *pivot positions* in winning coalitions, i.e., the option for player i to turn S into a losing coalition by leaving.

Table 2 summarizes actual and counterfactual damages to customer k for each of the 10 units of good 1 purchased in the market scenario from Section 2. The indicated marginal contributions $v^1(S) - v^1(S \setminus \{i\})$ for different $S \subseteq N$ and $i \in N$ clarify that cooperation by firm 1 and 2 is the main driver of overcharges on product 1.[9] Firm 3's participation has an effect on p_1, too–but mainly when 1 and 2 are already collaborating. Aggregating the marginal contributions according to Equation (1) yields $\varphi(N, v^1) = (17.2; 16.8; 3.5)$. So, as our earlier economic intuition had it, firm 3's responsibility for k's damage is small; responsibilities of firms 1 and 2 are similar with a slightly bigger share for the vendor, firm 1. Normalization $\rho^{1*} := \varphi(N, v^1)/v^1(N) = (46.0\%; 44.7\%; 9.3\%)$ gives the proportions referred to earlier.[10]

In Table 3, we repeat the analysis for a customer k' who bought units of product 3. Rather small harm for k' is caused if only two firms coordinate their strategies, i.e., significant overcharges arise only if all three firms form a cartel. The corresponding Shapely shares are $\rho^{3*} = (35.5\%; 37.2\%; 27.3\%)$.

Table 2. Marginal contributions to Δp_1.

S	$v^1(S)$	$v^1(S) - v^1(S \setminus \{1\})$	$v^1(S) - v^1(S \setminus \{2\})$	$v^1(S) - v^1(S \setminus \{3\})$
$\emptyset, \{1\}, \{2\}, \{3\}$	0	0	0	0
$\{1,2\}$	28.20	28.20	28.20	0
$\{1,3\}$	1.67	1.67	0	1.67
$\{2,3\}$	0.73	0	0.73	0.73
$\{1,2,3\}$	37.49	36.76	35.82	9.29

Table 3. Marginal contributions to Δp_3.

S	$v^3(S)$	$v^3(S) - v^3(S \setminus \{1\})$	$v^3(S) - v^3(S \setminus \{2\})$	$v^3(S) - v^3(S \setminus \{3\})$
$\emptyset, \{1\}, \{2\}, \{3\}$	0	0	0	0
$\{1,2\}$	3.67	3.67	3.67	0
$\{1,3\}$	1.25	1.25	0	1.25
$\{2,3\}$	1.68	0	1.68	1.68
$\{1,2,3\}$	12.20	10.52	10.95	8.53

4. Dichotomous Approximation

The numbers in Tables 2 and 3 and relative responsibilities ρ^{j*} can be calculated only with the benefit of knowing the underlying market model. This involves precise parameter values that reflect customers' demand behavior and firms' costs. Either may be very hard or impossible to obtain in practice.

The key premise in this paper is that it is comparatively easy (sometimes very easy) to provide at least a broad picture of substitutabilities and the product-specific competitive landscape in a cartelized market. This is all that is needed in order to come up with a rough, binary assessment of whether a given price would be affected significantly or only modestly if a strict subgroup S of firms N coordinated their action while others compete.

In the above example where customer k bought product 1, for instance, one can likely say without a full-blown quantification of demand and costs that firms 1 and 2 compete in relatively good substitutes, while firm 3 is comparatively isolated (regionally or by its product positioning). This would allow to conclude that collusion by firm 1 and firm 2 would cause significant increases of p_1 and p_2, whereas collusion by firm 3 and either firm 1 or firm 2 is bound to imply only small changes of these prices.

It is then natural to approximate the correct characteristic function v^j for $j = 1$, describing real-valued counterfactual overcharges $v^j(S)$, by a binary function $\tilde{v}^j$. The latter represents dichotomous assessments of whether a given sub-cartel S could be expected to cause a significant or a comparatively insignificant price increase for the good in question.

When products in the concerned market are substitutes, adding members to any partial cartel S cannot result in lower overcharges. So $S \subset T \subseteq N \Rightarrow \tilde{v}^j(S) \leq \tilde{v}^j(T)$. Moreover, if the set of infringing firms S is empty (or singleton), there is no price change; while $S = N$ gives rise to the full harm that was, by definition, significant enough for a court to grant compensation to a litigant. If we then represent no or comparatively small expected price increases by 0 and significant ones by 1, the binary approximation $\tilde{v}^j$ satisfies all defining characteristics of a simple voting game.

For our example, such dichotomous approximation would replace $v^1(S) = 1.67$ and 0.73 for sub-cartels $\{1,3\}$ and $\{2,3\}$ by $\tilde{v}^1(S) = 0$, and any detailed quantification of high overcharges (28.20 and 37.49) caused by coalitions $\{1,2\}$ and $\{1,2,3\}$ with $\tilde{v}^1(S) = 1$. That is

$$\tilde{v}^1(S) = \begin{cases} 1 & \text{if } \{1,2\} \subseteq S, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

Equivalently, we could reflect the broad assessment that a (sub-)cartel S among firms 1, 2 and 3 will significantly harm customer k, i.e. raise price p_1, if and only if $\{1,2\} \subseteq S$ by specifying

$$\mathcal{M}(\tilde{v}^1) = \{\{1,2\}\};$$

that is, $\{1,2\}$ is the unique element of the set of $\tilde{v}^1$'s minimum winning coalitions.

Computing the Shapley-Shubik index for $\tilde{v}^1$ yields shares of $\rho^{1\,D} := \varphi(N,\tilde{v}^1) = (50\%; 50\%; 0)$. These are rather close to $\rho^{1\,*} = (46.0\%; 44.7\%; 9.3\%)$ obtained from the Shapley value of (N, v^1). The respective $\|\cdot\|_1$-distance is 18.6%, i.e., less than half as much as for all market share heuristics considered in Table 1.

A similar logic applies to overcharges on product 3. For Δp_3 to be anywhere near the observed level, all three firms need to stop competing. The natural binary approximation of $v^3(S)$ therefore is the simple game with $\tilde{v}^3(N) = 1$ and $\tilde{v}^3(S) = 0$ for $S \subset N$. The respective Shapley-Shubik index $\varphi(N,\tilde{v}^3) = (33.3\%, 33.3\%, 33.3\%)$ is very close to the Shapley shares evaluated for v^3 (with $\|\cdot\|_1$-distance of 12.1%).

The comparatively minimal role of firm 3 and symmetry of firms 1 and 2 in $\tilde{v}^1$ that follow from a qualitative assessment of the market might arguably motivate a 50:50 division between firms 1 and 2 also without drawing on any cooperative game theory. What is the appropriate ascription of responsibilities is, however, less obvious when, for instance, a cartel involved four firms, say, A, B, C, D, and a qualitative economic assessment shows that any collaboration by firm A with at least one other firm induces significant damage. The corresponding set of MWC would then be

$$\mathcal{M}(\tilde{v}^j) = \{\text{AB, AC, AD}\}$$

where we abbreviate coalition $\{\text{A, B}\}$ by AB, etc.

Here firm A's participation is essential for the considered harm; non-participation by up to two other cartelists would not noticeably change things. Most people's intuition then probably is that the singular importance of A–reflected by a veto position in simple voting game $\tilde{v}^j$–entails greater responsibility for compensating victims. But how much greater? Operationalizing responsibility in a systematic way yields the answer. Presuming one deems the properties discussed in Section 1 desirable, the allocation should be proportional to Shapley-Shubik index

$$\rho^{j\,D} = \varphi(N,\tilde{v}^j) = (75\%; 8.\bar{3}\%; 8.\bar{3}\%; 8.\bar{3}\%).$$

Similar dichotomous approximations may help to divide damages also when a relatively large number of firms is involved. Firm A may, e.g., have more generally been the central member in a hub-and-spoke type cartel with $n-1$ peripheral firms.[11] It might then be obvious without accurate quantitative estimates that significant price increases for a customer at hand accrue if and only if firm A and at least one more firm cooperate, i.e., $\tilde{v}^j(S) = 1$ if $\text{A} \in S$ and $\tilde{v}^j(S) = 0$ if $\text{A} \notin S$. This yields[12]

$$\varphi_{\text{A}}(N,\tilde{v}^j) = \frac{n-1}{n} \text{ and } \varphi_i(N,\tilde{v}^j) = \frac{1}{n(n-1)} \text{ for } i \neq \text{A}. \tag{3}$$

$\varphi_{\text{A}}(N,\tilde{v}^j) = (n-1)/n$ confirms the intuition that the 'hub' firm's responsibility approaches 100% when it conspired with many exchangeable 'spokes'. $(n-1)/n$ also constitutes a tight upper bound for anyone's share of responsibility in a dichotomous assessment of cartel damages because the respective simple game $\tilde{v}^j$ involves as many swing or pivot positions for a single player as are compatible with $\tilde{v}^j(S) = 0$ for $|S| < 2$. Given that a null player i (characterized by $v^j(S \cup \{i\}) = v^j(S)$ for all $S \subset N$) has a

Shapley value of zero, responsibility shares in any simple game approximation thus satisfy $\varphi_i(N, \tilde{v}^j) \in [0, (n-1)/n]$.

These bounds are clearly too broad to do entirely without an assessment of how a firm's marginal contributions in the market translate into binary pivot positions, i.e., 'winning coalitions' for effecting significant price increases that could be swung by refusing to co-infringe the law. For small cartels, however, it is not difficult to enumerate *all* dichotomous damage scenarios which can arise and thus facilitate assessments. The scenarios correspond to structurally distinct simple games $(N, \tilde{v}^j)$ with n players such that $\tilde{v}^j(S) = 1$ implies $|S| \geq 2$. Exactly 19 of them exist for $n \leq 4$, up to a relabeling of players. They are listed in Table 4 with the corresponding Shapley shares.[13]

Table 4. Shapley allocations for all dichotomous damage scenarios with $n \leq 4$ firms.

	$\mathcal{M}(\tilde{v}^j)$	$\varphi(N, \tilde{v}^j)$		$\mathcal{M}(\tilde{v}^j)$	$\varphi(N, \tilde{v}^j)$
1.	AB	$(50\%; 50\%; 0\%; 0\%)$	11.	AB, ACD, BCD	$(33.\bar{3}\%; 33.\bar{3}\%; 16.\bar{6}\%; 16.\bar{6}\%)$
2.	AB, AC	$(66.\bar{6}\%; 16.\bar{6}\%; 16.\bar{6}\%; 0\%)$	12.	AB, AC, AD, BC, BD	$(33.\bar{3}\%; 33.\bar{3}\%; 16.\bar{6}\%; 16.\bar{6}\%)$
3.	AB, AC, BC	$(33.\bar{3}\%; 33.\bar{3}\%; 33.\bar{3}\%; 0\%)$	13.	AB, BC, CD	$(16.\bar{6}\%; 33.\bar{3}\%; 33.\bar{3}\%; 16.\bar{6}\%)$
4.	ABC	$(33.\bar{3}\%; 33.\bar{3}\%; 33.\bar{3}\%; 0\%)$	14.	AB, AC, AD, BC	$(41.\bar{6}\%; 25.0\%; 25.0\%; 8.\bar{3}\%)$
5.	ABC, ABD	$(41.\bar{6}\%; 41.\bar{6}\%; 8.\bar{3}\%; 8.\bar{3}\%)$	15.	ABC, ABD, ACD, BCD	$(25\%; 25\%; 25\%; 25\%)$
6.	ABCD	$(25\%; 25\%; 25\%; 25\%)$	16.	AB, AC, AD, BCD	$(50\%; 16.\bar{6}\%; 16.\bar{6}\%; 16.\bar{6}\%)$
7.	AB, AC, BCD	$(41.\bar{6}\%; 25.0\%; 25.0\%; 8.\bar{3}\%)$	17.	AB, AC, AD, BC, BD, CD	$(25\%; 25\%; 25\%; 25\%)$
8.	AB, AC, AD	$(75.0\%; 8.\bar{3}\%; 8.\bar{3}\%; 8.\bar{3}\%)$	18.	AC, AD, BC, BD	$(25\%; 25\%; 25\%; 25\%)$
9.	AB, CD	$(25\%; 25\%; 25\%; 25\%)$	19.	ABC, ABD, ACD	$(50\%; 16.\bar{6}\%; 16.\bar{6}\%; 16.\bar{6}\%)$
10.	AB, ACD	$(58.\bar{3}\%; 25\%; 8.\bar{3}\%; 8.\bar{3}\%)$		continued for $n = 5$ in Appendix A	

For instance, scenario 1. in Table 4 approximates situations in which only cooperation by firms A and B is critical for overcharges on the good in question; then A and B share responsibility for harm 50:50. We saw that this is a reasonable approximation for the example in Section 2. Scenario 2. corresponds to the big player-small player or hub-and-spoke situation in Equation (3) with $n = 3$. Here, firm D is–with the caveat that we deal with a binary approximation of the real market–a null player and need not contribute to the litigant's compensation. In scenario 3., cooperation by any two firms from $\{A, B, C\}$ causes damage; while that of all three is necessary and sufficient for damage in scenario 4.; etc.[14]

The number of distinct scenarios involving n firms is related to the *Dedekind numbers* in discrete mathematics, which grow at a doubly exponential rate. A list of all dichotomous damage scenarios with $n = 5$ non-null players already involves 160 entries. They are collected in Appendix A.[15] The enumeration may be useful as a reference to quickly obtain ballpark assessments of responsibility, e.g., in negotiations of settlements (and perhaps more generally in context of the 'inverse power index problem'; see [30]).

In sum, the crucial advantages of binary simple game approximations are that (i) they require just a big-or-small classification of counterfactual damages, not a full-blown market simulation; and (ii) the corresponding Shapley-Shubik index still reflects marginal contributions and hence "relative responsibility for the harm caused". Some approximation error, translating into a distance of $\|\rho^{1D} - \rho^{1*}\|_1 = 18.6\%$ in the example, is generically unavoidable. Proportioning harm by some market share benchmark is also very easy and may per chance have smaller error in other settings. Albeit it is impossible to know when, because market shares are essentially independent of whether a firm's (non-)participation in the cartel affects a particular price or not. In contrast, the Shapley-Shubik index of a dichotomous approximation is always based on causal links and increases in any additional pivot position that a firm could conceivably have used to reduce harm.

5. Comparisons to Other Heuristics in Linear Market Environments

To obtain a better picture of unavoidable approximation errors and also to compare how the Shapley-Shubik index of dichotomous damage scenarios performs in comparison to ad hoc heuristics let us conduct some in silico experiments. We consider situations where the underlying market model involves linear cost and demand for a wide range of parameter configurations. As in Section 2, specific parameter values determine equilibrium prices and sales, revenues, etc. that the listed heuristics draw on. They also deliver equilibrium predictions for prices in partial cartels $S \subset N$. These define the characteristic function and Shapley shares for any given price overcharge Δp_j, $j \in N$. We will compare these to the Shapley-Shubik index of dichotomous approximations as well as heuristic apportionment based on market shares.

We write v^j, $\tilde{v}^j$, $\rho^{j\,*} = \varphi(N, v^j)/v^j(N) = (\rho_1^{j\,*}, \ldots, \rho_n^{j\,*})$, and $\rho^{j\,D} = \varphi(N, \tilde{v}^j)/\tilde{v}^j(N) = \varphi(N, \tilde{v}^j)$ when we proportion damages for product $j \in N$. The analysis focuses on an economic assessment of the mismatch between product-specific Shapley contributions $\rho^{j\,*}$ and aggregate market shares or $\rho^{j\,D}$, rather than $\|\cdot\|_1$-distances. Such an assessment weights each per-unit deviation with corresponding sales and distinguishes whether compensation of *one* litigant with purchases of a single good is shared, or whether we divide compensation at the market level assuming *all* victims in the market have successfully litigated the cartel. The latter case allows purchase-specific deviations from responsibility-based contributions to cancel out across products and customers. This gives market share heuristics a relatively good shot at matching responsibility shares at least in aggregate terms.

A broad assessment of substitutability and the product-specific competitive landscape may well result in different dichotomous approximations $\tilde{v}^j$ of a product j-specific characteristic function v^j when conducted by different experts. We somehow need to decide which simple game $\tilde{v}^j$ serves as our benchmark in what follows. For computational reasons, we choose a $\tilde{v}^j$ that constitutes a 'best' dichotomous approximation of v^j in terms of minimizing $\|\rho^{j\,D} - \rho^{j\,*}\|_1$ for a given product-specific overcharge Δp_j. As a robustness check we have also conducted computations for simple games that yield a second or third-best approximation (in this metric). The respective figures corroborate that choosing $\tilde{v}^j$ by minimal $\|\rho^{j\,D} - \rho^{j\,*}\|_1$-distance does *not* imply minimal mismatch in aggregate terms. In particular, bigger but opposing mistakes in approximating the respective individual v^j by 'second or third-best' simple games may cancel. The total payment implied by $\varphi(N, v^j)$ across all $j \in N$ is then sometimes matched better than using the 'best' simple games. We therefore deem our comparison between simple game approximations with minimal product-level distance $\|\rho^{j\,D} - \rho^{j\,*}\|_1$ and market share heuristics as reasonably fair. Still we acknowledge that obtaining good dichotomous approximations of v^j will not always be as straightforward as in Section 2's example.

5.1. Linear Market Model

Let firms simultaneously set prices à la Bertrand. Best-responding non-cartel members $j \notin S$ maximize own profits whereas cartel members $i \in S \subseteq N$ simultaneously maximize joint collusive profits.

Each firm $i \in N = \{1, \ldots, n\}$ with $n \geq 3$ produces a single good.[16] Products are differentiated substitutes. Firm-specific unit costs γ_i are constant with $\gamma_i \geq 0$. A firm's demand function is assumed to be linearly decreasing in its own price p_i and increasing in p_j for $j \neq i$:

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \quad \text{for } a_i > \gamma_i,\ d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i. \tag{4}$$

Private antitrust enforcement would be superfluous if firm i could not profitably sell positive quantities. Hence, we assume $D_i(\gamma) > 0$. To ensure existence and uniqueness of a Nash equilibrium we will additionally assume that the well-known dominant diagonal conditions are satisfied (see Section 6.2 in [24] and Corollary 4.6 in [31]), i.e.,

$$\alpha_i := d_i / \sum_{j \neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N. \tag{5}$$

Parameter α_i measures the degree of differentiation between the product produced by firm i and products of the remaining market participants $j \neq i \in N$. For $\alpha_i \approx 1$, they are close substitutes and cartel formation causes big overcharge damages. With increasing values of α_i, substitutability and also the damages caused by a cartel decrease.

We let $p^S = (p_1^S, \ldots, p_n^S)$ denote the Nash equilibrium when firms in $S \subseteq N$ coordinate their strategies and all remaining firms in $N \setminus S$ act competitively. See [15] for an explicit derivation of p^S assuming that firms are symmetric and Chapter 8 in [32] for an elegant procedure to determine p^S in general linear market environments. $\Delta p = p^c - p^* = p^N - p^{\varnothing}$ then is the vector of unit price overcharges for which compensation can be claimed. If a customer who bought 10 units of product 1 acts against former cartel members, overcharge $10\Delta p_1$ will be proportioned between the n co-defendants. The characteristic function v^j that describes per unit damage for good j is defined by $v^j(S) = p_j^S - p_j^*$.

5.2. Symmetric Firms

When firms are symmetric, that is, when $a_i = a$, $d_i = d$, $b_{ij} = b$ and $\gamma_i = \gamma$, Shapley shares ρ^{j*} depend only on differentiation parameter α, the number of co-defendants n, and firm identity j. For the firm $j \in N$ that sold the product for which a given customer claims compensation–firm 1 for the above-mentioned litigant–the Shapley share ρ_j^{j*} that pertains to its own sales lies in $(1/n, 1/2)$ whereas all remaining firms $i \neq j$ have to contribute equally with $\rho_i^{j*} \in (0.5/(n-1), 1/n)$ [15] (Corollary 1).[17]

The precise Shapley shares for firm $j = 1$ and remaining firms $i \neq 1$ are illustrated in panel (a) of Figure 1 for a cartel of $n = 4$ firms and parameters $a = 30$, $\gamma = 2$, $d = 3$, $b = d/(3\alpha)$ as the degree of differentiation α is varied. Because firms are symmetric at the market level, all heuristics that draw on the symmetric sales, revenue or profit shares naturally coincide with an equal per head allocation of the compensation for sales of good 1 (indicated as $\bar{\rho} = 25\%$ in Figure 1). But own-price effects of cartel participation are larger than cross-price effects. The greater the product differentiation, the more the responsibility of firm 1 for harm to its customers therefore exceeds that of firms 2, 3 and 4. Panel (a) of Figure 1 shows how this difference increases in α.

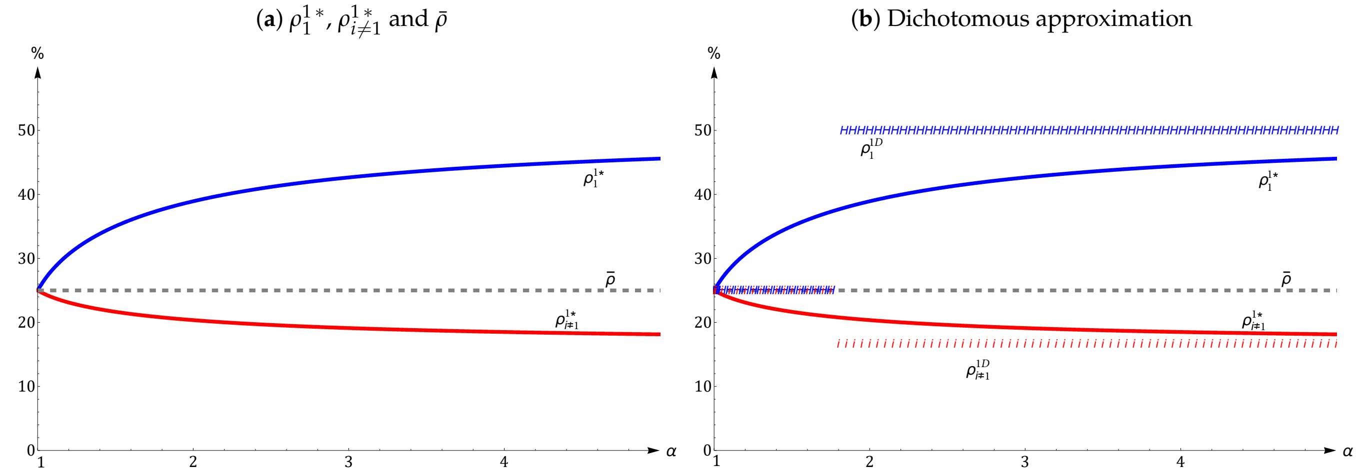

Figure 1. (**a**) Shapley shares ρ^{1*} and equal ad hoc shares $\bar{\rho}$ vs. (**b**) shares ρ^{1D} implied by dichotomous approximation when proportioning compensation for purchases of good 1 in a linear Bertrand model with symmetric firms.

For $\alpha \approx 1$ and thus firms producing very close substitutes, a qualitative assessment of the market is likely to predict that a significant price increase of any good j (incl. p_1) would be unsustainable if one or more firms decide to act competitively. The corresponding simple game with $\mathcal{M}(\tilde{v}^j) = \{1234\}$ has a Shapley-Shubik index of $\rho^{jD} = (25\%, 25\%, 25\%, 25\%)$ (cf. scenario 6, Table 4). If, by contrast, products are only mediocre substitutes then a qualitative assessment plausibly diagnoses that (i) already partial cartels involving three firms can cause significant price increases, (ii) participation of firm 1 is critical for the equilibrium level of p_1 to be close to p_1^*, and (iii) firms 2, 3, and 4 play entirely exchangeable roles for Δp_1. This makes scenario 19. in Table 4 with $\mathcal{M}(\tilde{v}^1) = \{123, 124, 134\}$ the focal candidate for a dichotomous simple game approximation of overcharges on product 1. This game $\tilde{v}^1$ yields good approximations $\rho^{1\,D} = (50\%, 16.\bar{6}\%, 16.\bar{6}\%, 16.\bar{6}\%)$ of the respective Shapley value $\rho^{1\,*}$, which varies continuously in α. It is the best approximation in terms of $\|\cdot\|_1$-distance if $\alpha > 1.8$, as illustrated in panel (b) of Figure 1. The figure also visualizes that $\rho^{1\,D}$ is everywhere at least as close to $\rho^{1\,*}$ as are equal shares $\bar{\rho}$ prescribed by heuristics that look at the market rather than the product level.[18]

We can give market share heuristics a much better shot if we assume that all (or equal measures of) customers act against all former cartel members. Then the aggregate overcharge damage of

$$O := \sum_{i \in N} q_i^c \cdot \Delta p_i \tag{6}$$

ends up being shared, with q_i^c denoting the cartel quantity sold by firm i.

If damages connected to each good j are allocated according to Shapley shares $\rho^{j\,*}$, then aggregate Shapley payments of

$$\Phi_i := \sum_{j \in N} q_j^c \cdot \varphi_i(N, v^j) = \sum_{j \in N} q_j^c \cdot \Delta p_j \cdot \rho_i^{j\,*} \tag{7}$$

must be made by a given firm i for its own sales and those of others. Product-level differences between the vendor of a good and its competitors cancel under symmetry. So $\Phi_i = O/n$.

Similarly let H_i^ρ denote i's aggregate payments when some heuristic shares ρ are applied instead of $\rho^{1\,*}, \ldots, \rho^{n\,*}$. With symmetric firms, all heuristics ρ that go with an equal division $\bar{\rho}$ imply $H_i^\rho = O/n = \Phi_i$. Hence the practical proposals discussed in Section 2 work well if all victims seek compensation: product-level deviations from relative responsibility that arise because firms are non-exchangeable for individual customers (cf. Figure 1) vanish in the aggregate provided firms are symmetric at the market level.

5.3. Asymmetric Firms

For asymmetric market configurations, however, it is likely that firm i contributes more in total than its responsibility share ($H_i^\rho > \Phi_i$) or too little ($H_i^\rho < \Phi_i$) for any given heuristic ρ. We will then evaluate the *aggregate mis-allocation* of damages $\sum_{i \in N} |\Phi_i - H_i^\rho|$ for competing proportioning methods, namely for dichotomous product-level approximations and market share heuristics. To make (mis-)allocations of compensation payments comparable across different asymmetric scenarios, we normalize the mis-allocation by the respective total overcharge damage O. In particular, Figure 2 shows percentage values

$$M^\rho := \sum_{i \in N} \mid \Phi_i - H_i^\rho \mid / O \tag{8}$$

that allow unit-free statements on how well different methods are mimicking shares based on relative responsibility for the harm caused. The figure takes up Section 5.2's parameters $a = 30$, $\gamma = 2$, $d = 3$, $n = 4$, $b = d/(3\alpha)$ as baseline and reports results for one of four kinds of asymmetry at a time and for the six most prominent entries in Table 1.

In the two top panels (a) and (b) of Figure 2, firms differ in their market size or saturation quantity a_i. The dichotomous approximation method ρ^D strictly outperforms ad hoc heuristics for $\alpha > 1.4$. With increasing values of α, only an allocation based on competitive sales ρ^4 stays close to the Shapley-based allocation $\rho^*(N, v)$. However, ρ^D does even better.

The discontinuous jumps that can occur when using dichotomous simple game approximations (ρ^D) is particularly visible in panel (c), where firms 1 and 2 are twice as efficient as 3 and 4. For $\alpha = 1.55$ to the left of the first jump, aggregate overcharge damages are roughly

$O = 321$ and using product-specific 'best' dichotomous approximations $\tilde{v}^j$ gives rise to equal shares $\rho_i^{j\,D} \equiv 25\%$ for each firm i in each firm j's product. This yields $H_i^{\rho^D} = 80.3$ for $i \in N$. Now look at one of the two inefficient firms, say firm 3. Its product-specific Shapley shares $\rho_3^{1\,*}, \rho_3^{2\,*}, \rho_3^{3\,*}, \rho_3^{4\,*}$ turn out to be 21%, 21%, 35.4%, and 20.6% (yielding total Shapley payments $\Phi_3 = 77.2$). So $\rho_3^{3\,D} = 25\%$ significantly underestimates firm 3's responsibility for the overcharges on its own sales but $\rho_3^{1\,D} = \rho_3^{2\,D} = \rho_3^{4\,D} = 25\%$ overestimate firm 3's responsibility for other firms' overcharges. These over and underestimations mostly cancel, implying $H_3^{\rho^D} - \Phi_3 = 80.3 - 77.2 = 3.1$ and $M^{\rho^D} = 4 \cdot 3.1/321 = 3.9\%$.[19]

Now move from $\alpha = 1.55$ to $\alpha = 1.6$ on the right of the jump, with $O = 286$. There $\rho_3^{1\,D}$, $\rho_3^{2\,D}, \rho_3^{3\,D}, \rho_3^{4\,D}$ in the respective 'best' product-specific binary assessments abruptly change to $16.\bar{6}\%, 16.\bar{6}\%, 25\%$, and 25% with $H_3^{\rho^D} = 58.1$. The corresponding Shapley shares $\rho_3^{j\,*}$, by contrast, vary continuously. They are 20.8%, 20.8%, 35.9%, and 20.4% for $\alpha = 1.6$ (yielding $\Phi_3 = 68.7$). The simple voting games then underestimate firm 3's actual Shapley share not only for own sales but also when customers of firms 1 and 2 sue. With less cancellations, the aggregate deviation between firm 3's (and analogously firm 4's) payments using characteristic functions v^j vs. simple games $\tilde{v}^j$ increases to $\Phi_3 - H_3^{\rho^D} = 10.6$, giving rise to $M^{\rho^D} = 4 \cdot 10.6/286 = 14.8\%$.

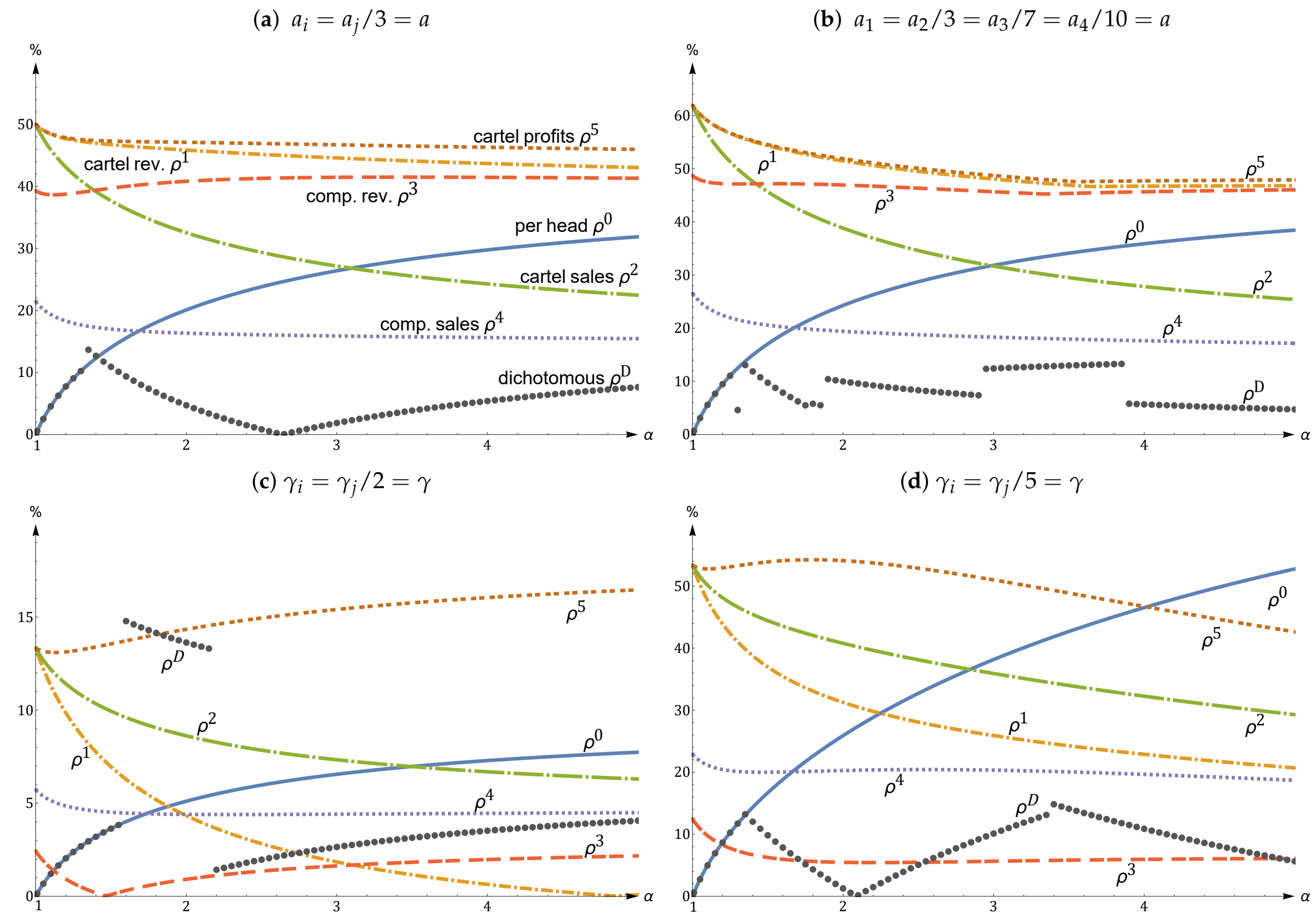

Figure 2. *Cont.*

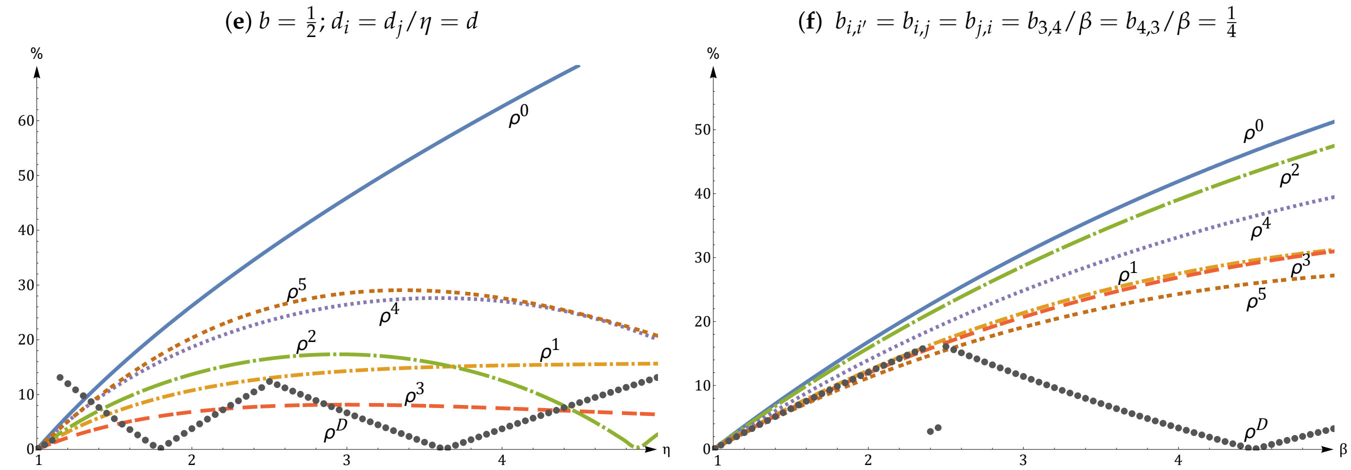

Figure 2. Normalized mis-allocation M^{ρ} implied by market share heuristics and using dichotomous simple game approximations in a linear Bertrand model with asymmetric firms, letting $i \in \{1,2\}$ and $j \in \{3,4\}$ and respectively deviating from symmetric benchmark parameters $a = 30$, $\gamma = 2$, $d = 3$, $n = 4$, $b = d/(3\alpha)$ as indicated in the respective panels (**a**–**f**). The black dotted line reflects the use of dichotomous simple game approximations.

The comparison of mis-allocations using dichotomous approximations and market share heuristics in Figure 2 shows that ρ^D may sometimes perform worse than all ad hoc heuristics (for $1.6 \leq \alpha \leq 1.9$ in panel (c)). It should be noted, though, that this happens rarely and that the corresponding deviations in panel (c) are only around 15%. Any of the market share heuristics is for many of the configurations in Figure 2 much further off. Note also that a measure of market shares that performs very well for one kind of asymmetry– namely, sales during competitive conduct ρ^4 for market size asymmetry in panels (a,b); competitive revenues ρ^3 for cost asymmetry in panels (c,d) or asymmetric own-price effects in panel (e); cartel profits ρ^5 for asymmetric cross-price effects in panel (f) – turns out to perform very badly for some other kind.[20] In contrast to this, allocations based on dichotomous simple game approximations always deduce responsibility from causal links between cartel participation and prices and also keep M^{ρ^D} robustly below 20% in all considered scenarios.

The latter bound increases to a still moderate 25% if approximations of the actual product-specific characteristic function v^j involve simple voting games $\tilde{v}^j$ whose Shapley-Shubik index generates the second or third-smallest $\|\cdot\|_1$-distance to ρ^{j*} rather than to minimize $\|\varphi(N,\tilde{v}^j) - \varphi(N,v^j)/v^j(N)\|_1$. So simple game approximations quite robustly deliver good estimates of the full information shares that reflect relative responsibility for the harm caused. They perform still better compared to market shares or equal per head divisions when only individual overcharges–not aggregate damages–are concerned (cf. in Figure 1).

6. Concluding Remarks

The above analysis has hopefully demonstrated the potential of using simple voting games as first approximations of harder to define TU games. Finding the right approximation $\tilde{v}^j$ of an underlying but unknown characteristic function v^j for cartel overcharges on a given product j will require some expertise and involve a certain degree of freedom. The proposed method still seems much simpler to implement than estimation of a full structural market model and is a lot less ad hoc than applying any of various equally (un)convincing market share heuristics to the problem at hand.

Specific circumstance may call for finer approximations of the underlying characteristic function, as suggested by the multi-level approximation in endnote 12. It could therefore be promising to consider (j,k) *simple games* (see [33]) instead of standard simple voting games in future research. These allow to translate $j \geq 2$ intensities of individual cartel participation into $k \geq 2$ levels of harm.

The mathematical properties of the Shapley value and–closely related–the Shapley-Shubik index in our view make these solutions for cooperative games uniquely suitable for the task of identifying responsibility for a real-valued or binarily approximated financial damage. Similar reasoning may, however, extend in other domains of application to the Banzhaf value and the Penrose-Banzhaf index ([34,35]), as well as other values for TU games and their simple game equivalents. For example, weighted versions of the Shapley value and Shapley-Shubik index [36] could be invoked to model exemptions that EU legislation provides for whistle-blowers or increased responsibilities that may for a given cost and demand structure derive from organizational leadership of a cartel.

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Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. All Dichotomous Damage Scenarios with $n = 5$ Firms

	$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$		$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$
1.–19.	see Table 4 on p. 8		71.	AB, AC, ADE, BCDE	(30, 10, 10, 5, 5)
20.	AB, AC, AD, AE	(48, 3, 3, 3, 3)	72.	AB, AC, ADE, BDE, CDE	(24, 9, 9, 9, 9)
21.	AB, AC, AD, AE, BC	(28, 13, 13, 3, 3)	73.	AB, AC, BC, ADE	(22, 17, 17, 2, 2)
22.	AB, AC, AD, AE, BC, BD	(23, 18, 8, 8, 3)	74.	AB, AC, BC, ADE, BDE	(19, 19, 14, 4, 4)
23.	AB, AC, AD, AE, BC, BD, BE	(21, 21, 6, 6, 6)	75.	AB, AC, BC, ADE, BDE, CDE	(16, 16, 16, 6, 6)
24.	AB, AC, AD, AE, BC, BD, BE, CD	(16, 16, 11, 11, 6)	76.	AB, AC, BC, DE	(14, 14, 14, 9, 9)
25.	AB, AC, AD, AE, BC, BD, BE, CD, CE	(14, 14, 14, 9, 9)	77.	AB, AC, BCD, BCE	(22, 17, 17, 2, 2)
26.	AB,AC,AD,AE,BC,BD,BE,CD,CE,DE	(12, 12, 12, 12, 12)	78.	AB, AC, BCD, BCE, BDE	(19, 19, 14, 4, 4)
27.	AB, AC, AD, AE, BC, BD, BE, CDE	(18, 18, 8, 8, 8)	79.	AB, AC, BCD, BCE, BDE, CDE	(16, 16, 16, 6, 6)
28.	AB, AC, AD, AE, BC, BD, CD	(18, 13, 13, 13, 3)	80.	AB, AC, BCD, BDE	(22, 17, 12, 7, 2)
29.	AB, AC, AD, AE, BC, BD, CE	(18, 13, 13, 8, 8)	81.	AB, AC, BCD, BDE, CDE	(19, 14, 14, 9, 4)
30.	AB, AC, AD, AE, BC, BD, CE, DE	(16, 11, 11, 11, 11)	82.	AB, AC, BCDE	(28, 13, 13, 3, 3)
31.	AB, AC, AD, AE, BC, BD, CDE	(20, 15, 10, 10, 5)	83.	AB, AC, BD, ADE	(22, 17, 7, 12, 2)
32.	AB, AC, AD, AE, BC, BDE	(25, 15, 10, 5, 5)	84.	AB, AC, BD, ADE, BCE	(19, 19, 9, 9, 4)
33.	AB, AC, AD, AE, BC, BDE, CDE	(22, 12, 12, 7, 7)	85.	AB, AC, BD, ADE, BCE, CDE	(16, 16, 11, 11, 6)
34.	AB, AC, AD, AE, BC, DE	(20, 10, 10, 10, 10)	86.	AB, AC, BD, ADE, CDE	(19, 14, 9, 14, 4)
35.	AB, AC, AD, AE, BCD	(33, 8, 8, 8, 3)	87.	AB, AC, BD, CD, ADE	(17, 12, 12, 17, 2)
36.	AB, AC, AD, AE, BCD, BCE	(30, 10, 10, 5, 5)	88.	AB, AC, BD, CD, ADE, BCE	(14, 14, 14, 14, 4)
37.	AB, AC, AD, AE, BCD, BCE, BDE	(27, 12, 7, 7, 7)	89.	AB, AC, BD, CDE	(17, 17, 12, 12, 2)
38.	AB,AC,AD,AE,BCD,BCE,BDE,CDE	(24, 9, 9, 9, 9)	90.	AB, AC, BD, CE	(12, 17, 17, 7, 7)
39.	AB, AC, AD, AE, BCDE	(36, 6, 6, 6, 6)	91.	AB, AC, BD, CE, ADE	(14, 14, 14, 9, 9)
40.	AB, AC, AD, BC, BD, CDE	(17, 17, 12, 12, 2)	92.	AB, AC, BD, CE, DE	(12, 12, 12, 12, 12)
41.	AB, AC, AD, BC, BD, CE	(15, 15, 15, 10, 5)	93.	AB, AC, BDE	(25, 15, 10, 5, 5)
42.	AB, AC, AD, BC, BD, CE, DE	(13, 13, 13, 13, 8)	94.	AB, AC, BDE, CDE	(22, 12, 12, 7, 7)
43.	AB, AC, AD, BC, BDE	(22, 17, 12, 7, 2)	95.	AB, AC, DE	(22, 7, 7, 12, 12)
44.	AB, AC, AD, BC, BDE, CDE	(19, 14, 14, 9, 4)	96.	AB, AC, DE, BCD	(19, 9, 9, 14, 9)
45.	AB, AC, AD, BC, BE	(20, 20, 10, 5, 5)	97.	AB, AC, DE, BCD, BCE	(16, 11, 11, 11, 11)
46.	AB, AC, AD, BC, BE, CDE	(17, 17, 12, 7, 7)	98.	AB, ACD, ACE	(37, 12, 7, 2, 2)
47.	AB, AC, AD, BC, BE, DE	(15, 15, 10, 10, 10)	99.	AB, ACD, ACE, ADE	(39, 9, 4, 4, 4)
48.	AB, AC, AD, BC, DE	(17, 12, 12, 12, 7)	100.	AB, ACD, ACE, ADE, BCD	(24, 14, 9, 9, 4)
49.	AB, AC, AD, BCD, BCE	(27, 12, 12, 7, 2)	101.	AB, ACD, ACE, ADE, BCD, BCE	(21, 16, 11, 6, 6)
50.	AB, AC, AD, BCD, BCE, BDE	(24, 14, 9, 9, 4)	102.	AB, ACD, ACE, ADE, BCD, BCE, BDE	(18, 18, 8, 8, 8)

	$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$		$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$
51.	AB, AC, AD, BCD, BCE, BDE, CDE	(21, 11, 11, 11, 6)	103.	AB,ACD,ACE,ADE,BCD,BCE,BDE,CDE	(15, 15, 10, 10, 10)
52.	AB, AC, AD, BCDE	(33, 8, 8, 8, 3)	104.	AB, ACD, ACE, ADE, BCD, BCE, CDE	(18, 13, 13, 8, 8)
53.	AB, AC, AD, BCE	(30, 10, 10, 5, 5)	105.	AB, ACD, ACE, ADE, BCD, CDE	(21, 11, 11, 11, 6)
54.	AB, AC, AD, BCE, BDE	(27, 12, 7, 7, 7)	106.	AB, ACD, ACE, ADE, BCDE	(27, 12, 7, 7, 7)
55.	AB, AC, AD, BCE, BDE, CDE	(24, 9, 9, 9, 9)	107.	AB, ACD, ACE, ADE, CDE	(24, 9, 9, 9, 9)
56.	AB, AC, AD, BE	(25, 15, 5, 5, 10)	108.	AB, ACD, ACE, BCD	(22, 17, 12, 7, 2)
57.	AB, AC, AD, BE, BCD	(22, 17, 7, 7, 7)	109.	AB, ACD, ACE, BCD, BCE	(19, 19, 14, 4, 4)
58.	AB, AC, AD, BE, BCD, CDE	(19, 14, 9, 9, 9)	110.	AB, ACD, ACE, BCD, BCE, CDE	(16, 16, 16, 6, 6)
59.	AB, AC, AD, BE, CDE	(22, 12, 7, 7, 12)	111.	AB, ACD, ACE, BCD, BDE	(19, 19, 9, 9, 4)
60.	AB, AC, AD, BE, CE	(20, 10, 10, 5, 15)	112.	AB, ACD, ACE, BCD, BDE, CDE	(16, 16, 11, 11, 6)
61.	AB, AC, AD, BE, CE, BCD	(17, 12, 12, 7, 12)	113.	AB, ACD, ACE, BCD, CDE	(19, 14, 14, 9, 4)
62.	AB, AC, AD, BE, CE, DE	(18, 8, 8, 8, 18)	114.	AB, ACD, ACE, BCDE	(25, 15, 10, 5, 5)
63.	AB, AC, AD, BE, CE, DE, BCD	(15, 10, 10, 10, 15)	115.	AB, ACD, ACE, BDE	(22, 17, 7, 7, 7)
64.	AB, AC, ADE	(42, 7, 7, 2, 2)	116.	AB, ACD, ACE, BDE, CDE	(19, 14, 9, 9, 9)
65.	AB, AC, ADE, BCD	(27, 12, 12, 7, 2)	117.	AB, ACD, ACE, CDE	(22, 12, 12, 7, 7)
66.	AB, AC, ADE, BCD, BCE	(24, 14, 14, 4, 4)	118.	AB, ACD, BCD, CDE	(17, 17, 12, 12, 2)
67.	AB, AC, ADE, BCD, BCE, BDE	(21, 16, 11, 6, 6)	119.	AB, ACD, BCDE	(23, 18, 8, 8, 3)
68.	AB, AC, ADE, BCD, BCE, BDE, CDE	(18, 13, 13, 8, 8)	120.	AB, ACD, BCE	(20, 20, 10, 5, 5)
69.	AB, AC, ADE, BCD, BDE	(24, 14, 9, 9, 4)	121.	AB, ACD, BCE, CDE	(17, 17, 12, 7, 7)
70.	AB, AC, ADE, BCD, BDE, CDE	(21, 11, 11, 11, 6)	122.	AB, ACD, CDE	(20, 15, 10, 10, 5)

	$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$
123.	AB, ACDE	(33, 18, 3, 3, 3)
124.	AB, AC, ADE, BDE	(27, 12, 7, 7, 7)
125.	AB, ACDE, BCDE	(21, 21, 6, 6, 6)
126.	AB, CD, ACE	(17, 12, 17, 12, 2)
127.	AB, CD, ACE, ADE	(19, 9, 14, 14, 4)
128.	AB, CD, ACE, ADE, BCE	(16, 11, 16, 11, 6)
129.	AB, CD, ACE, ADE, BCE, BDE	(13, 13, 13, 13, 8)
130.	AB, CE, ACE, BDE	(14, 14, 14, 14, 4)
131.	AB, CDE	(18, 18, 8, 8, 8)
132.	ABC, ABD, ABE	(27, 27, 2, 2, 2)
133.	ABC, ABD, ABE, ACD	(32, 12, 7, 7, 2)
134.	ABC, ABD, ABE, ACD, ACE	(34, 9, 9, 4, 4)
135.	ABC, ABD, ABE, ACD, ACE, ADE	(36, 6, 6, 6, 6)
136.	ABC, ABD, ABE, ACD, ACE, ADE, BCD	(21, 11, 11, 11, 6)
137.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE	(18, 13, 13, 8, 8)
138.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE	(15, 15, 10, 10, 10)
139.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE	(12, 12, 12, 12, 12)
140.	ABC, ABD, ABE, ACD, ACE, ADE, BCDE	(24, 9, 9, 9, 9)
141.	ABC, ABD, ABE, ACD, ACE, BCD	(19, 14, 14, 9, 4)
142.	ABC, ABD, ABE, ACD, ACE, BCD, BCE	(16, 16, 16, 6, 6)
143.	ABC, ABD, ABE, ACD, ACE, BCD, BDE	(16, 16, 11, 11, 6)
144.	ABC, ABD, ABE, ACD, ACE, BCD, BDE, CDE	(13, 13, 13, 13, 8)
145.	ABC, ABD, ABE, ACD, ACE, BCDE	(22, 12, 12, 7, 7)
146.	ABC, ABD, ABE, ACD, ACE, BDE	(19, 14, 9, 9, 9)
147.	ABC, ABD, ABE, ACD, ACE, BDE, CDE	(16, 11, 11, 11, 11)
148.	ABC, ABD, ABE, ACD, BCD	(17, 17, 12, 12, 2)
149.	ABC, ABD, ABE, ACD, BCD, CDE	(14, 14, 14, 14, 4)
150.	ABC, ABD, ABE, ACD, BCDE	(20, 15, 10, 10, 5)
151.	ABC, ABD, ABE, ACD, BCE	(17, 17, 12, 7, 7)
152.	ABC, ABD, ABE, ACD, BCE, CDE	(14, 14, 14, 9, 9)
153.	ABC, ABD, ABE, ACD, CDE	(17, 12, 12, 12, 7)
154.	ABC, ABD, ABE, ACDE	(30, 15, 5, 5, 5)
155.	ABC, ABD, ABE, ACDE, BCDE	(18, 18, 8, 8, 8)
156.	ABC, ABD, BCE	(30, 10, 10, 5, 5)
157.	ABC, ABD, ABE, CDE	(15, 15, 10, 10, 10)
158.	ABC, ABD, ACD, BCE	(15, 15, 15, 10, 5)

	$\mathcal{M}(\tilde{v}^j)$	$60 \cdot \varphi(N, \tilde{v}^j)$
159.	ABC, ABD, ACD, BCE, BDE	(12, 17, 12, 12, 7)
160.	ABC, ABD, ACD, BCE, BDE, CDE	(9, 14, 14, 14, 9)
161.	ABC, ABD, ACD, BCDE	(18, 13, 13, 13, 3)
162.	ABC, ABD, ACE, ADE	(32, 7, 7, 7, 7)
163.	ABC, ABD, ACE, ADE, BCDE	(20, 10, 10, 10, 10)
164.	ABC, ABD, ACE, BCDE	(18, 13, 13, 8, 8)
165.	ABC, ABD, ACE, BDE	(15, 15, 10, 10, 10)
166.	ABC, ABD, ACE, BDE, CDE	(12, 12, 12, 12, 12)
167.	ABC, ABD, ACDE	(28, 13, 8, 8, 3)
168.	ABC, ABD, ACDE, BCDE	(16, 16, 11, 11, 6)
169.	ABC, ABD, CDE	(13, 13, 13, 13, 8)
170.	ABC, ABDE	(23, 23, 8, 3, 3)
171.	ABC, ABDE, ACDE	(26, 11, 11, 6, 6)
172.	ABC, ABDE, ACDE, BCDE	(14, 14, 14, 9, 9)
173.	ABC, ADE	(28, 8, 8, 8, 8)
174.	ABC, ADE, BCDE	(16, 11, 11, 11, 11)
175.	ABCD, ABCE	(18, 18, 18, 3, 3)
176.	ABCD, ABCE, ABDE	(21, 21, 6, 6, 6)
177.	ABCD, ABCE, ABDE, ACDE	(24, 9, 9, 9, 9)
178.	ABCD, ABCE, ABDE, ACDE, BCDE	(12, 12, 12, 12, 12)
179.	ABCDE	(12, 12, 12, 12, 12)

Notes

1. Refs [8,9] provide comprehensive introductions. For a concise overview see [10].
2. Another application of simple games and voting power indices *outside* voting contexts has recently been studied by Kovacic and Zoli [11]. They show that the Penrose-Banzhaf index can improve the prediction of violent conflict in ethnically polarized societies.
3. Litigants can also strive to find out-of-court settlements, or settle with some firms and take the remaining ones to court. After a settlement "*…the [remaining] claim of the injured party should be reduced by the settling infringer's share of the harm caused …*" (Directive 2014/104/EU, recital 51).
4. Other values fail to satisfy at least one property. For instance, the Banzhaf value and its restriction to simple voting games, the Penrose-Banzhaf index, do not satisfy efficiency; their normalized variants are efficient but violate linearity. They are hence unsuitable for the purpose at hand. [17–19] invoke similar reasoning for liability shares in successive torts.
5. Additional harm stems from deadweight losses: customers who would have made (additional) purchases, and thus would have enjoyed surplus had prices only been p^*, failed to do so. We are unaware of cases in which compensation for this has successfully been claimed and disregard these losses in what follows.
6. Cartel benefits (ρ^7) reflect normalized relative profit increases of the cartel members. Yet more heuristics are conceivable: for instance, proportioning based on product-specific total overcharge damages would yield shares of (22.1%; 55.1%; 22.8%) which are very similar to heuristic ρ^7.
7. It is easiest to think of each firm $i \in N$ producing a single good but it is possible to let the set of products $M \ni j$ be distinct from the set of cartel members N. This can reflect multi-product firms as well as goods produced by non-cartel members. The latter's price may have increased due to the passing on of cartel margins along a vertical value chain or due to 'umbrella effects' that derive from best response behavior of cartel outsiders.
8. We will assume that once a cartel has formed, other firms become at least implicitly aware of its existence. The cartel outsiders will adapt optimally to the new market environment, as is already anticipated by the cartel members. This and that the latter maximize joint profits are standard assumptions in industrial organization and seem reasonable defaults for the analysis of cartel counterfactuals. However, if there is sufficient evidence that firms pursued alternative objectives in a given cartel case then computations of $v^j(S)$ could be based on these other objectives.
9. For example, in the counterfactual scenario $S = \{1,2\}$, the price of product 1 increases to 72.90. Hence the damage caused by coalition $S = \{1,2\}$ is $v^1(\{1,2\}) = 72.90 - 44.70 = 28.20$. If either member left S then prices would become competitive, i.e., $v^1(S \setminus \{i\}) = 0$ for $i \in \{1,2\}$.
10. The analogous table for Δp_2, i.e., overcharges on product 2, is very similar to Table 2. Respective Shapley shares are $\rho^{2*} = (44.4\%, 45.9\%, 9.7\%)$.
11. To fix ideas, think of a crooked architect A who is remunerated in fixed proportion to contract volumes and can define specifications so as to steer procurement for customers towards any building companies B, C, … that are willing to inflate prices.

12 The easiest way to compute the Shapley value in this scenario is to use equation (5) of [15].–A qualitatively different scenario could be that all player pairs $\{A, i\} \subseteq S$ with $i \in \{B, C, \ldots\}$ cause similar *incremental* damages, independently of each other. The corresponding mapping $\hat{v}^j$ with $\hat{v}^j(S) = s - 1$ if $A \in S$ and $\hat{v}^j(S) = 0$ otherwise, is *not* a simple game. Still, it and the resulting Shapley value with $\varphi_A(N, \hat{v}^j) = \frac{1}{2}\hat{v}^j(N)$ and $\varphi_i(N, \hat{v}^j) = \frac{1}{2(n-1)}\hat{v}^j(N)$ for $i \neq A$ may constitute a straightforward *multi-level* rather than dichotomous approximation of causal links and responsibilities when a structural model is difficult to estimate. Simple game approximations may sometimes be refined easily.

13 The median number of firms in price fixing US cartels is 4 according to analysis by [26], which reflects 329 cases.

14 Consider, e.g., the European plasterboard cartel. When detected, four companies (*BPB PLC, Gebrüder Knauf Westdeutsche Gipswerke KG, Société Lafarge SA* and *Gyproc Benelux NV*) were active in the cartel. They all operated in several countries but their abilities to influence prices differed locally. For instance, the first three firms were large players in Germany and France while *Gyproc* in France held a market share below 5%. According to the European Commission, *"[i]t is clear that the three operators considered it necessary to make Gyproc take part in the exchange as far as the German market was concerned, where that undertaking, which overall was much smaller than the three others, had a significant market share"* (see [27] (recital 268)). To do justice to this case would require much deeper analysis, but this description already hints that in the German market all four firms were necessary to cause significant damage (scenario 6), whereas *Gyproc*'s contribution to harm was rather negligible in France (scenario 4).

15 See [28] for $n \leq 4$ and [29] for $n = 5$. Our list comprises fewer games because $\tilde{v}^j(S) = 1$ requires $|S| \geq 2$ in a cartel context but our Appendix A corrects several typos hidden in Baldan's list. Some games in the list, such as scenario 9, would be considered as *improper* in the context of voting: they involve disjoint winning coalitions. If we think of A and B as two producers and of C and D as their retailers, damage may plausibly arise already if the producers or the retailers cooperate. If there is little scope for additional marginalization by vertical collusion, $\mathcal{M}(\tilde{v}^j) = \{AB, CD\}$ makes good sense.

16 For a duopoly with $n = 2$, cartel participation of either firm is essential for raising prices. Relative responsibilities then are $\rho^{1*} = \rho^{2*} = (50\%, 50\%)$ irrespective of cost or demand asymmetries.

17 $\rho_j^{j*} > 1/n$ and $\rho_i^{j*} < 1/n$, $i \neq j$, can be shown to apply to symmetric firms also for non-linear demand and costs, both under price and quantity competition (cf. Proposition 2 in [15]).

18 For very high degrees of differentiation, a qualitative assessment might diagnose significant scope to increase p_1 already if firm 1 colludes with one, not two other firms, or if all three competitors of 1 collude. The resulting MWC are then $\mathcal{M}(\tilde{v}^1) = \{12, 13, 14, 234\}$ with, again, $\rho^{1D} = (50\%, 16.\bar{6}\%, 16.\bar{6}\%, 16.\bar{6}\%)$. Extreme differentiation could conceivably lead to a bad approximation by $\mathcal{M}(\tilde{v}^1) = \{12, 13, 14\}$ with $\rho^{1D} = (75\%, 8.\bar{3}\%, 8.\bar{3}\%, 8.\bar{3}\%)$. This would be incompatible, however, with linear costs and demand for symmetric firms since these imply $25\% < \rho_1^{1*} < 50\%$.

19 Inefficient firms 3 and 4 each pay 3.1 too much; so each efficient firm, 1 and 2, pays 3.1 too little.

20 Although the baseline parameters considered in panels (a–f) differ from the ones used in the simulations in [15], the asymmetry-dependent 'best' market share heuristics happen to stay unchanged. This suggests that precise parameters are less important for how heuristics perform than the economic asymmetry at hand.

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