Relative Responsibilty for Cartel Damages

Stefan Napel
University of Bayreuth
Universitätsstraße 30, 95447 Bayreuth, Germany
stefan.napel@uni-bayreuth.de

Dominik Welter
University of Bayreuth
dominik.welter@uni-bayreuth.de

September 27, 2021

Abstract

Cartel members are liable jointly and severally: any of the infringing firms may be litigated and forced to compensate victims on behalf of all. EU law then stipulates that the co-infringers must pay internal redress in proportion to their "relative responsibility for the harm caused". We suggest to quantify this responsibility by invoking basic proportioning axioms and the requirement that redress payments reflect causal links between actions and damages. This calls for application of the Shapley value. We prove that even symmetric firms may bear unequal responsibility for individual harm, characterize proportionings for linear market environments, and show that proportioning by market shares typically fails to reflect relative responsibilities.

Keywords: collusion; damage proportioning; Shapley value; relative responsibility

JEL codes: L40; C71; D04; K42

Declarations of interest: none

1. Introduction

Cartel victims have a right to compensation but the pertinent legal hurdles are high. Annually up to 23.3 billion euro of damages used to remain unclaimed from EU-wide cartels according to the European Commission (SWD/2013/203/Final, recital 67). In 2014, this was key motivation for the Commission to advance the *Directive on Antitrust Damages Actions* 2014/104/EU, also known as *Damages Directive*. The position of plaintiffs has since improved and several big cases are pending.

Two provisions for the compensation of cartel victims in the Directive motivate this study. First, the members of a cartel are liable *jointly and severally*: an injured party can litigate any cartel member for the full amount of its damages; if courts confirm the claim, the defendant must compensate the plaintiff on behalf of the entire cartel. This is regardless of whether the plaintiff made its purchases from the sued firm or other ones. Similar provisions apply in Australia, Japan, the UK, or the US.

Second, the sued cartel member is entitled to *internal redress*. Such a rule of contribution existed in EU member states before (incl. the UK) but details differed. It contrasts with the no contribution rule in federal US antitrust cases (cf. Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981) and intermediate arrangements elsewhere.

According to the Directive, cartelists' internal obligations in compensating any external claimant must reflect *"... their relative responsibility for the harm caused by the infringement of competition law"* (Article 11(5)). The Directive is not specific on how this should be operationalized. Our goal is to quantify relative responsibility for cartel damages in an economically sound way.[1]

The analysis focuses on the assessment of economic damage contributions, but is based on the canonical causal conception of legal and moral responsibility for harm. Feinberg (1970, p. 195f) provides a lucid discussion of its three parts: firstly, the defendant was at fault in acting. This clearly applies if, for instance, firm i's manager illegally coordinated its production of some commodity with competitors over dinner, violating antitrust laws. Secondly, the faulty act caused the harm: these conversations resulted in a price increase for the customer. And, finally, the faulty aspects of the

[1] The issue of how alternative norms, such as the no contribution rule in the US, affect incentives for cartel formation, whistleblowing, settlements, etc. is left aside. See, for instance, Landes and Posner (1980) or Hviid and Medvedev (2010).

act were relevant to its causal connection to the harm: illegal coordination by the managers – not, perhaps, just the reaction of commodity investors to observing the meeting – caused the increase. All three parts call for appropriate verification in practical applications.

After this, a systematic approach is warranted to determine each firm's contribution to harm. Asymmetry of cartel members can translate very differently into asymmetric turnover, revenues, or profits. So the simple idea to proportion damages by market shares involves a high degree of arbitrariness. Sound methods should reflect considerations of the following kind: first, a firm has responsibility and should contribute to compensating a given customer only if this customer's damages would have been lower had the firm refused to participate in the cartel. How much lower the respective damage would have been if the firm had stayed legal (and then possibly some others, too) ought to, second, determine the level of the contribution. Third, if cartel membership of two firms had identical effects on harm, both should contribute the same. Finally, a victim's full compensation should be proportioned in a way that neither depends on the unit of account nor on whether multiple damages are dealt with separately or jointly, whether interest has accumulated, etc.

These properties translate into mathematical conditions that are well-known in cooperative game theory as the null player, marginality, symmetry, efficiency and linearity axioms. Classical results by Shapley (1953a) and Young (1985) then imply that the Shapley value of an appropriately defined game provides the best way to split external obligations by relative responsibility of the co-infringers. The Shapley value is an accepted tool for allocating costs or profits in joint ventures.[2] Its use for the division of cartel damages was initially proposed by Schwalbe (2013) and Napel and Oldehaver (2015) to law audiences. We are the first to analyze the pertinent quantitative aspects.

The key feature of Shapley proportionings is that they impute individual responsibility from the ability to influence prices. We revisit the underlying reasoning and a little-known formula that can simplify computations (Section 3) after detailing the problem at hand (Section 2). We prove that even symmetric firms generally have asymmetric obligations with respect to individual claimants (Section 4). For linear market environments, we derive how Shapley proportionings are linked to demand

[2]See, e.g., Shubik (1962), Hart and Moore (1990), Young (1994), or Moretti and Patrone (2008). Dehez and Ferey (2013, 2016) and Huettner and Karos (2017) apply it to sequential liability problems.

and cost parameters and deduce useful bounds (Section 5). In Section 6, we compare Shapley allocations to ad hoc proportioning based on sales, profits, or a flat per head assignment. We find that corresponding suggestions by legal practitioners mostly clash with proportioning by relative responsibility.

2. Cartel Damages and Relative Responsibility

Cartel customers usually suffer two types of damage. First, there is the visible loss: each unit that was purchased involved an overcharge. Second, customers who would have made (additional) purchases and enjoyed surplus on these if prices had been competitive, failed to do so. Respective deadweight losses are acknowledged in the legal literature but have played no role in practice yet (see Argenton et al. 2020). We will hence concentrate on overcharge damages caused by a hardcore cartel. The suggested approach could be generalized, however, by invoking lost profits or money-metric indirect utility, and to analysis of other communal antitrust violations.[3]

A cartel member i having responsibility for damages of a given claimant k requires that k's damages are causally linked to i's cartel membership, i.e., their scale, scope or distribution would have differed without i's illegal action. Identifying the causal links between anticompetitive conduct and harm is generally difficult (see, e.g., Lianos 2015). What makes analysis of responsibility particularly interesting, though, is that even symmetric cost and demand structures may generate asymmetric links to the harm of a specific victim. Namely, price effects of cartel membership differ across cartelists as long as own-price and cross-price elasticities of the respective demands differ. Firms that are symmetric from an industrial organization perspective then can be non-symmetric players from a game theoretic perspective.

For illustration consider n otherwise identical firms on a Salop circle. Think of cement plants that are equally spaced on the shores of an unshippable lake. They sold cement at inflated prices to local construction companies around the lake. The cartel was busted and a customer of firm h sues. The relative responsibilities of this customer's home firm h and of a distant firm j's are tied to the counterfactual price that the customer would have paid had h or respectively j refused to participate. Unless transportation costs are zero, cartel membership of the northernmost vendor has

[3]One can also take a more dynamic view. Harm is often quantified by monthly but-for estimations (e.g., Bernheim 2002) and so might responsibility if membership, demand, or costs varied over time.

smaller effect on overcharges faced by customers in the south than does membership of southern vendors, and vice versa (see, e.g., Levy and Reitzes 1992). The more intensely two firms would have competed in the absence of the cartel, the greater the effect of their collusion. The counterfactual prices that the customer would have paid if h or if j had not joined the cartel, but just best-responded to its practices, thus vary by location. And so do their responsibilities for the customer's harm.

Of course, a symmetric market structure implies that obligations which h and j have in compensating each others' customers are the same. So mutual redress claims cancel if all constructors sue (or a uniform measure of them). However, they do *not* cancel in almost all other situations – e.g., if just some construction companies in the south go to court. A general proportioning method hence requires that responsibility can be attributed to cartel members for the price overcharge on each single product in the cartel's portfolio. Asymmetries in cost or demand make this even more important.

The described problem of proportioning overcharges extends to prices of cartel outsiders who best-responded to the infringement. The respective *umbrella losses*[4] are legally acknowledged in the EU (CJEU C-557/12 2014) and have also been claimed successfully before several US courts. Their compensation is not linked to transactions with any cartelist. Moreover, a firm's relative responsibility stays relevant if litigants settle: claims against co-defendants are reduced by the settling defendant's responsibility for harm (Damages Directive, recital 51).

3. The Shapley Value as a Tool for Proportioning Damages

3.1. Preliminaries

A damage proportioning problem can be formalized as a *transferable utility* (TU) game where an overcharge damage $v(N)$ caused by a cartel $N = \{1, \ldots, n\}$ is to be divided among the firms involved.[5] For every *coalition* $S \subseteq N$ of players who might collaborate with one another, $v(S)$ describes damage inflicted if firms $i \in S$ coordinate their actions while firms $j \in N \setminus S$ maximize their respective profits in competitive fashion. Mapping $v\colon 2^N \to \mathbb{R}$ is the *characteristic function* of TU game (N, v). When overcharge damages

[4]See Inderst et al. (2014) and Holler and Schinkel (2017) on the theory of umbrella losses. Bos and Harrington (2010) and de Roos and Smirnov (2021) analyze cartels with partial market coverage.

[5]The relevant market may comprise firms $j \notin N$ which did not partake in the cartel. They need not contribute to compensations and matter as exogenous co-determinants of damage rather than players.

for m different goods are to be proportioned, we in principle need to consider m different TU games. So if helpful, we may write (N, v^j) in order to highlight that overcharges for a particular good j are considered.

For strict subsets of N, $v(S)$ (or $v^j(S)$) reflects a counterfactual. This is necessary because responsibility is driven by the fact that the overcharge in question would have differed from $v(N)$ if some firms had refused to collude. Directive 2014/104/EU, recital 46, explicitly acknowledges the role of counterfactual scenarios for the determination of harm $v(N)$. Defining $v(S)$ also for sets $S \subset N$ extends the pertinent logic from quantifying harm to quantifying contributions to harm.

Naturally, $v(S) = 0$ if the set S of collaborators is empty or comprises but a single firm, i.e., if $|S| = 1$. For other coalitions $S \subset N$, a damage estimate $v(S)$ is needed. Intertemporal variation in cartel participation may help obtaining it but the most versatile option is market simulation analysis. This is rather well-established in merger control (cf. Budzinski and Ruhmer 2010): parameters of a structural model of price or quantity competition are estimated based on pre-merger observables; these generate equilibrium predictions for when a subset of firms internalize mutual profit externalities. Analogous analysis of cartel behavior is comparatively rare (see de Roos 2006) but respective calibrations can draw not only on pre-cartel (like pre-merger) observables but also observations during and after the cartel's operation. Former members may have an interest to disclose information if they expect lower contributions than under some ad hoc proportioning. Sensitive data could be pooled by a trusted intermediary who helps settling mutual claims.

We take no stance on how sophisticated estimates $v(S)$ ought to be in practice. For instance, the analysis of a hypothetical scenario with a sub-cartel $S \neq N$ may consider the question of whether S satisfies suitable stability conditions, and put $v(S) = 0$ if not. The illustrations below will keep things simple, but there are reasons to expect that even simpler modeling can still improve on naïve proportioning by market shares.[6] Note that each number $v(S)$ with $i \notin S$ reflects a scenario for how the market might have evolved if there had been no infringement by firm i. It is both possible that firm $j \neq i$ would then have joined the cartel anyhow ($j \in S$) or that it would have stayed legal too ($j \notin S$). These scenarios need not have equal probability. But all partial cartels $S \subseteq N \setminus \{i\}$ are, in principle, relevant in assessing i's contribution to the situation which

[6] See Napel and Welter (2021) on the extent to which even *binary* approximations $\tilde{v}$ of an unknown characteristic function v can identify responsibility better than relative sales, revenues, etc.

calls for compensation, hence i's relative responsibility.[7]

3.2. *Desirable Properties of Responsibility-Based Allocations*

With damages in a factual cartel scenario and related counterfactuals described by (N, v), a *damage proportioning rule* is a mapping Φ from any conceivable cartel damage problem (N, v) to a vector $\Phi(N, v) \in \mathbb{R}^n$, i.e., it is a *value* of the corresponding TU game. The main restriction that the cartel context imposes is that $v(\{i\}) = 0$ for all $i \in N$. As prices of substitute goods are usually higher for bigger cartels,[8] one can take v to be monotonic in S. Component $\Phi_i(N, v)$ denotes the part of the compensation for damages $v(N)$ which cartel member $i \in N$ must contribute.

That a proportioning rule reflects relative responsibilities can be translated into three formal properties of Φ. The first one is straightforward. Suppose that participation or not of a particular firm i would never have made a difference to the damage in question. That is, removing player i if $i \in S$ or adding player i if $i \notin S$ does not change $v(S)$. Then given that i's conduct has no effect on damage, the canonical conditions for i being responsible are not met (see Feinberg 1970). Hence, no responsibility-based obligations to contribute follow. A player i for whom $v(S) = v(S \setminus \{i\})$ for every $S \subseteq N$ is known as a *null player*. The first requirement for rule Φ to be based on relative responsibility hence is the *null player property*:

$$\Phi_i(N, v) = 0 \text{ whenever } i \text{ is a null player in } (N, v). \tag{NUL}$$

Presumably, supply and demand conditions in real markets are rarely compatible with a convicted cartel member being a null player. But (NUL) conducts a valid thought experiment. It also formalizes a certain robustness to misspecification of the relevant market. For instance, a large cartel may have caused damage in several regions with independent costs and demand. If a firm is accidentally included as 'player' in a region where it had no role, (NUL) ensures it need not contribute there.

As responsibility derives from the causal links between cartel membership and the

[7]It is also conceivable that several partial cartels would have formed if i had refused to join. This might be accommodated by considering extensions of the Shapley value to *partition functions* V from the set of partitions $\mathcal{P} = \{P_1, \ldots, P_r\}$ of N to estimated damages $V(\mathcal{P})$. See Ray and Vohra (1999).

[8]See Davidson and Deneckere (1984) and Deneckere and Davidson (1985). *Superadditivity* and *convexity* of v can be natural assumptions, too: to ensure that new members want to join a cartel and existing members accept them, v must be superadditive. For cartels that included all big players, incentives to join have plausibly increased in size (convexity).

harm suffered, a second straightforward requirement is that i's damage share should be determined by these links – and these links alone. Namely, presuming that v correctly describes factual damages as well as the relevant counterfactuals, $\Phi_i(N, v)$ shall be a function only of i's *marginal contributions* $v(S) - v(S \setminus \{i\})$ in (N, v). The corresponding formal property of *marginality*, introduced by Young (1985), is

$$\Phi_i(N, v) = \Phi_i(N, v') \text{ whenever } v(S) - v(S \setminus \{i\}) = v'(S) - v'(S \setminus \{i\}) \text{ holds for all } S \subseteq N. \tag{MRG}$$

Marginality does not pin down *how* $\Phi_i(N, v)$ should depend on the differences that i makes to various coalitions. For instance, (MRG) does not imply (NUL).

If contributions of two firms i and j to the generation of damages are symmetric, i.e., $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition $S \subseteq N \setminus \{i, j\}$, their responsibilities are the same. So Φ should satisfy *symmetry*:[9]

$$\Phi_i(N, v) = \Phi_j(N, v) \text{ whenever } i \text{ and } j \text{ are symmetric in } (N, v). \tag{SYM}$$

Irrespective of whether a division of damages reflects responsibility of the involved players or follows alternative principles, firms' contributions should add up to $v(N)$. In the context of TU games, this is called *efficiency* of a value:

$$\sum_{i \in N} \Phi_i(N, v) = v(N). \tag{EFF}$$

Efficiency and symmetry imply $\Phi_1(N, v) = \Phi_2(N, v) = \frac{1}{2}v(N)$ if $N = \{1, 2\}$, i.e., participants to any 2-firm cartel (see, e.g., Argenton 2019) must contribute equally. This may at first seem counterintuitive when market shares, costs, or profits are asymmetric, but exit by either firm would have restored duopolistic competition.

Firms' shares should not depend on whether damages are proportioned for one or many units, expressed in US dollar or euro, whether they are trebled, already include interest, etc. Moreover, if the same cartel N caused harm to customers in several markets – reflected by a characteristic function v for market 1, v' for market 2, etc. – then the total contribution of firm $i \in N$ should not depend on whether the proportioning rule is applied to damages in one market at a time or simultaneously. Different 'markets' could here refer to different plaintiffs, different products in the cartel's portfolio, or distinct quantities of the same product. Scale invariance and

[9]Deviations from symmetry may be necessary if firms played highly asymmetric roles in the organization of the cartel (e.g. as ringleader) or when leniency provisions provide liability exemptions. One can then turn to *weighted Shapley values* (Shapley 1953b; Kalai and Samet 1987).

additivity combine to *linearity*:

$$\Phi(N, \lambda \cdot v + \lambda' \cdot v') = \lambda \cdot \Phi(N, v) + \lambda' \cdot \Phi(N, v') \tag{LIN}$$

for any scalars $\lambda, \lambda' \in \mathbb{R}$ and any characteristic functions v, v'.

3.3. *Shapley Value and Decomposition by Average Damage Increments*

Above properties imply that a responsibility-based proportioning method must lead to contributions $\Phi_i(N, v)$ that equal

$$\varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \Big[v(S) - v(S \setminus \{i\}) \Big] \tag{1}$$

for each $i \in N$ and $s = |S|$. $\varphi(N, v)$ is the *Shapley value* of (N, v). Shapley (1953a) showed that any allocation rule that satisfies (NUL), (SYM), (EFF) and (LIN) is equivalent to φ. Young (1985) proved that the same is true if (MRG), (SYM) and (EFF) are satisfied.[10] Formula (1) may look unwieldy but weights $(s-1)!(n-s)!/n!$ on marginal contributions are a logical consequence of the desired properties.

It is little-known – but will below be very practical – that an equivalent way of writing eq. (1) is

$$\varphi_i(N, v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=1}^{n-1} \Big[\bar{v}^{i}(s) - \bar{v}^{\not i}(s) \Big] \tag{2}$$

where

$$\bar{v}^{i}(s) := \binom{n-1}{s-1}^{-1} \sum_{S \ni i,\, |S|=s} v(S) \tag{3}$$

captures the *average damages* caused by coalitions of size s which *include* firm i and

$$\bar{v}^{\not i}(s) := \binom{n-1}{s}^{-1} \sum_{S \not\ni i,\, |S|=s} v(S). \tag{4}$$

those which *exclude* firm i.[11] Abbreviating $\kappa(s) := (s-1)!(n-s)!/n! = \frac{1}{n} \cdot \binom{n-1}{s-1}^{-1}$, eq. (2)

[10] See, e.g., Maschler et al. (2013, ch. 18). To verify that Shapley's uniqueness result extends to the class of damage proportioning problems, note that cartels in which $i \in T \subseteq N$ produce perfect substitutes with competitive price $p^* = 0$ and cartel price $p^c = 1$ while all $j \notin T$ operate in unrelated markets define the required carrier games (N, u_T).

[11] The decomposition in eq. (2) is distinct from those suggested by Kleinberg and Weiss (1985) and Rothblum (1988). It may here be stated for the first time.

follows from

$$\varphi_i(N,v) = \sum_{S\subseteq N} \kappa(s)\cdot\Big[v(S) - v(S\setminus\{i\})\Big] = \sum_{\substack{S\subseteq N\\ S\ni i}} \kappa(s)v(S) - \sum_{\substack{S\subseteq N\\ S\not\ni i}} \kappa(s+1)v(S) \tag{5}$$

$$= \kappa(n)v(N) + \sum_{s=1}^{n-1}\Bigg[\sum_{\substack{S\ni i\\ |S|=s}} \kappa(s)v(S) - \sum_{\substack{S\not\ni i\\ |S|=s}} \kappa(s+1)v(S)\Bigg] = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=1}^{n-1}\Big[\bar{v}^{i}(s) - \bar{v}^{\not{i}}(s)\Big].$$

Equation (2) simplifies further because a 'cartel' of size $s = 1$ leaves prices constant, i.e., $\bar{v}^{i}(1) = \bar{v}^{\not{i}}(1) = 0$ for each $i \in N$. We thus obtain:

Shapley proportioning Rule *Firm i must contribute*

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=2}^{n-1}\Big[\bar{v}^{i}(s) - \bar{v}^{\not{i}}(s)\Big] \tag{6}$$

to compensation of a cartel damage $v(N)$ in order to reflect i's relative responsibility and to share harm in a scale-invariant additive way.

Proportioning by relative responsibility of the infringers – formalized by (NUL), (MRG) and (SYM), plus (EFF) and (LIN) – thus means: start out with *equal shares per head*; then add an n-th of the *average size-specific damage increments* that arise due to a given firm i's participation. This summand accounts for asymmetric effects on harm, which can arise even in symmetric market environments (cf. Section 4).

Equation (6) provides a useful perspective on φ_i but can also facilitate its calculation: possible symmetry among players reduces the sum of 2^n differences in eq. (1) to less than n ones in (6). This extends when asymmetries are such that i's increments for specific coalition sizes s can be expressed as a function of 'aggregate asymmetry' among other firms (see Subsection 5.3). The calculation is further simplified if cartel sizes s below some threshold $\tilde{s}$ are unstable (Bos and Harrington 2010), or if exchangeability of firms implies that $\bar{v}^{i}(s) - \bar{v}^{\not{i}}(s)$ is zero. For instance, the summand vanishes in a homogeneous Bertrand or Cournot oligopoly or when $n = 2$; equal shares follow.

4. Unequal Responsibility of Symmetric Differentiated Firms

By contrast, unequal responsibility and Shapley shares in compensation follow when $n > 2$ symmetric firms produce differentiated goods. This holds even for a very strong form of symmetry, where differentiation generates greater own-price than cross-price effects but firms are otherwise identical.

Specifically, let firms $1, \dots, n$ simultaneously choose strategies $y_1, \dots, y_n$ that jointly determine prices p_i and profits Π_i for all $i \in N$. $p = (p_1, \dots, p_n)$ and $\Pi = (\Pi_1, \dots, \Pi_n)$ are taken to be smooth functions of $y = (y_1, \dots, y_n)$. The two focal cases are differentiated price competition where $p_i(y) \equiv y_i$ and quantity competition where y_i denotes firm i's output. However, y_i might also refer to some marketing activity, product characteristic, firm i's geographic radius of operation, etc. We make the following general assumptions:

A1. Price p_i and profits Π_i are affected identically by own strategy y_i for all firms $i \in N$ and also by all strategy choices y_j of the respective other firms $j \neq i$, that is

$$p_i(y_1, \dots, y_n) \equiv p_j(y_{\varrho(1)}, \dots, y_{\varrho(n)}) \text{ and } \Pi_i(y_1, \dots, y_n) \equiv \Pi_j(y_{\varrho(1)}, \dots, y_{\varrho(n)}) \tag{I}$$

for each $i \neq j$ and all permutations $\varrho \colon N \to N$ with $\varrho(i) = j$ and $\varrho(j) = i$.[12]

A2. For all $i \neq j \in N$,

$$\left|\frac{\partial p_i}{\partial y_i}\right| > \left|\frac{\partial p_i}{\partial y_j}\right| \tag{IIa}$$

and

$$\frac{\partial \Pi_j}{\partial y_i} \cdot \frac{\partial p_i}{\partial y_i} > 0. \tag{IIb}$$

Condition (IIa) is trivially satisfied for price competition and otherwise formalizes that inverse demand responds more to changes of the quantity (or product characteristic, delivery range, etc.) of the variety in question than to that of others. Condition (IIb) requires y_i to change own price p_i and other firms' profits Π_j in the same direction: e.g. for quantity competition, greater output y_i lowers p_i as well as the profits of firms $j \neq i$; for price competition, higher prices $y_i = p_i$ raise profits of j.

A3. For all $S \subseteq N$ there exists a unique Nash equilibrium $y^S = (y_1^S, \dots, y_n^S)$ such that

– $y_i^S = y^c$ if $i \in S$, where y^c solves the first order condition

$$\frac{d\Pi_S}{dy_i} = \sum_{j \in S} \frac{\partial \Pi_j}{\partial y_i} = 0$$

for maximization of joint profit $\Pi_S(y) = \sum_{k \in S} \Pi_k(y)$ by cartel S;

[12]For instance, each variety i could be the personal favorite of an equal share of consumers who regard varieties $j \neq i$ as equally close substitutes. The symmetry in A1 is stronger than in the Salop model: some permutation ϱ with $\varrho(i) = j$ and $\varrho(j) = i$ satisfies (I) there, but not all do.

– $y_i^S = y^o$ if $i \notin S$, where y^o solves the first order condition

$$\frac{\partial \Pi_i}{\partial y_i} = 0$$

for individual profit maximization by an outsider to cartel S.

Sufficient conditions for the equilibrium in A3 to exist are provided in Section 5.

Proposition 1. *Given A1–A3, let $p^i(s)$ ($p^{\not i}(s)$) equal the equilibrium price for good i if firm i is (is not) part of a cartel with $s \in \{2, \ldots, n-1\}$ members. Then $p^i(s) > p^{\not i}(s)$.*

Proof. Inequality (IIb) implies that firms' strategies either lower their own prices, $\partial p_i/\partial y_i < 0$, and have a negative externality on each other's profits, $\partial \Pi_j/\partial y_i < 0$, as for quantity competition; or that $\partial p_i/\partial y_i > 0$ and $\partial \Pi_j/\partial y_i > 0$. In the former case, internalization of the negative profit externality in a cartel with $s \in \{2, \ldots, n-1\}$ members implies a smaller individual action or output choice $y^c < y^o$ for cartel members than outsiders (see A3); otherwise $y^c > y^o$.

We first address $y^c < y^o$ with $\partial p_i/\partial y_i < 0$ and $\partial \Pi_j/\partial y_i < 0$. Let $S = \{1, \ldots, s\}$ w.l.o.g. and consider the straight line L which connects $\hat{y} = (y^o, y^c, \ldots, y^c, y^o, \ldots, y^o, y^c)$ to $y^S = (y^c, y^c, \ldots, y^c, y^o, \ldots, y^o, y^o)$ in the space of output choices. L can be parameterized by

$$r(t) = (\underbrace{y^o - t, y^c, \ldots, y^c}_{s \text{ terms}}, \underbrace{y^o, \ldots, y^o, y^c + t}_{n-s \text{ terms}}) \tag{7}$$

with $t \in [0, y^o - y^c]$, i.e., we simultaneously decrease firm 1's action and increase firm n's action by identical amounts as we move along L. The gradient $\nabla p_n = \left(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\right)$ of function p_n can be used in order to evaluate the price change caused by switching from $\hat{y}$ to y^S. In particular, the (Stokes) gradient theorem for line integrals (see, e.g., Protter and Morrey 1991, Thm. 16.15) implies

$$\begin{aligned}
p_n(y^S) - p_n(\hat{y}) &= \int_L \nabla p_n \, dr = \int_0^{y^o - y^c} \nabla p_n(r(t)) \cdot r'(t) \, dt &(8)\\
&= \int_0^{y^o - y^c} \left(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\right)\Bigg|_{y=r(t)} \cdot \Big(-1, 0, \ldots, 0, 1\Big) \, dt &(9)\\
&= \int_0^{y^o - y^c} \left[\frac{\partial p_n(r(t))}{\partial y_n} - \frac{\partial p_n(r(t))}{\partial y_1}\right] dt < 0. &(10)
\end{aligned}$$

The inequality follows from (IIa): firm n's own strategy changes have bigger price effects than changes by competitor firm 1.

A1 then implies

$$\begin{aligned} p^1(s) := p_1(y^c, y^c, \dots, y^c, y^o, \dots, y^o, y^o) = p_n(y^o, y^c, \dots, y^c, y^o, \dots, y^o, y^c) \\ = p_n(\hat{y}) > p_n(y^S) = p_n(y^c, y^c, \dots, y^c, y^o, \dots, y^o, y^o) := p^{\not n}(s). \end{aligned} \tag{11}$$

That is, the price $p^1(s)$ of good 1 when its producer is one of s exchangeable cartel members exceeds the price $p^{\not n}(s)$ of good n when firm n is *not* part of a cartel with s members. And, also by A1, we have $p^{\not 1}(s) = p^{\not n}(s)$ and $p^1(s) = p^n(s)$. So we can conclude $p^1(s) > p^{\not 1}(s)$ from (11). The same applies to any other firm i, and we obtain $p^i(s) > p^{\not i}(s)$ for all $s \in \{2, \dots, n-1\}$ as claimed.

For $y^c > y^o$, the specific case $p_i(y) = y_i$ directly implies the claim. The general case of $\partial p_i / \partial y_i > 0, \partial \Pi_j / \partial y_i > 0$ is analogous to $y^c < y^o$: reversed orientation of the integral from $t = 0$ to $y^o - y^c < 0$ in (10) and the reversed sign of integrand $\partial p_n / \partial y_n - \partial p_n / \partial y_1$ cancel. □

Now focus on the per unit damage $v^h(N)$ that accrued to a customer who bought her home product h and paid cartel price $p^c = p_h(y^N)$ instead of competitive price $p^* = p_h(y^{\varnothing})$. The counterfactual average damages implied by partial cartels of size s that include and exclude firm h are $\bar{v}^h(s) = p^h(s) - p^*$ and $\bar{v}^{\not h}(s) = p^{\not h}(s) - p^*$, respectively. Proposition 1 implies

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) > 0 \text{ for any } s = 2, \dots, n-1. \tag{12}$$

So from eq. (6) we can conclude

Proposition 2. *Given A1–A3, consider an overcharge damage $v^h(N)$ that was suffered on purchases from firm $h \in N$ after $n \geq 3$ symmetric producers of differentiated goods formed cartel N. Then*

$$\varphi_i(N, v^h) \begin{cases} > \frac{v^h(N)}{n} & \text{if } i = h, \\ < \frac{v^h(N)}{n} & \text{if } i \neq h. \end{cases} \tag{13}$$

That is, firm h is always responsible for more than 1/n-th of harm to its own (home) customers.

5. Proportioning by Responsibility in Linear Market Settings

Shapley proportionings require estimates of counterfactual damages for all conceivable partial cartels (see, e.g., de Roos 2006). We illustrate this here for situations in which the costs and demand for differentiated goods are linear. We conjecture that parameter restrictions in analogy to, e.g., the proportionality condition of Epstein and Rubinfeld (2001) could reduce the data requirements in practical cases sufficiently to be applicable. If the producers of differentiated products face at most one kind of asymmetry, closed-form expressions for the Shapley shares can be derived via eq. (6).[13] This is often impossible in other applications of the Shapley value. The parametric solutions allow to derive upper and lower bounds for the responsibility-based contribution by a firm to harm of its own and of other firms' customers, respectively. They also facilitate assessing the degree to which, e.g., cartel-period revenue shares might serve as proxies of relative responsibility in Section 6.

5.1. *Model*

Consider a cartel of $n \geq 3$ suppliers where each firm $i \in N = \{1, \ldots, n\}$ produces a single good. Firm i's costs are given by

$$C_i(q_i) = \gamma_i q_i \text{ for } \gamma_i \geq 0. \tag{14}$$

Demand at price vector $p = (p_1, \ldots, p_n)$ is described by

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \text{ for } a_i > \gamma_i,\ d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i. \tag{15}$$

We presume $D_i(\gamma) > 0$, i.e., demand is positive when all firms price at cost. This amounts to assuming $a + (n-1)b\gamma > d\gamma$ in the symmetric case with $\gamma_i = \gamma$, $a_i = a$, $d_i = d$ and $b_{ij} = b$ for all $i \neq j \in N$. Firms set prices simultaneously à la Bertrand. If some group $S \subseteq N$ forms a cartel, outsiders $j \notin S$ are assumed to somehow become aware of this and to best-respond to cartel prices, which is already anticipated by S.

Members of $S \subseteq N$ maximize the sum of their profits

$$\Pi_S(p) = \sum_{i \in S} (p_i - \gamma_i) D_i(p) \tag{16}$$

with corresponding first-order conditions

$$\frac{\partial \Pi_S(p)}{\partial p_j} = D_j(p) + \sum_{i \in S} (p_i - \gamma_i) \frac{\partial D_i(p)}{\partial p_j} \text{ for all } j \in S. \tag{17}$$

Analogous expressions hold if j is a cartel outsider. It is sufficient for existence and uniqueness of a Nash equilibrium that a uniform increase of all prices and

[13] Quadratic costs do not change findings much: Proposition 3 below then involves cost parameter γ but remains independent of a. Later expressions get significantly more unwieldy, however.

a unilateral increase of any single price respectively decrease individual and total demand.[14] Formally, this requires $\sum_{j=1}^{n} \partial D_i/\partial p_j < 0$ and $\sum_{j=1}^{n} \partial D_j/\partial p_i < 0$, i.e., we will assume

$$\alpha_i := d_i/\sum_{j\neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j\neq i} b_{ji} \text{ for all } i \in N. \tag{18}$$

This simplifies to $\alpha := \frac{d}{(n-1)b} > 1$ in the symmetric case ($\gamma_i = \gamma$, $a_i = a$, $d_i = d$, $b_{ij} = b$).

Products are relatively good substitutes when α_i is small; then price increases by one firm significantly raise profits for other firms. A cartel internalizes this. So the price p_i set by cartel member i will be the higher, the smaller α_i.

For any $S \subseteq N$, the (unique) Nash equilibrium p^S summarizes equilibrium prices p_i^S of all products $i \in N$ assuming firms in S coordinate and the rest acts competitively. The per unit overcharge suffered by a customer who bought product i is denoted by $v^i(N) = p_i^N - p_i^{\emptyset}$.

5.2. *Symmetric Case*

Own and cross price elasticities for the considered goods vary even under symmetry given $\alpha > 1$. We hence distinguish the home firm h that produced the good for which a fixed customer suffered harm from those cartel members $j \neq h$ that were not part of their transaction. We focus on the per unit overcharge $v^h(N)$ for good h. After solving for the Nash equilibria p^S implied by (14) – (18) for all $S \subseteq N$, the percentages of $v^h(N)$ for which firms h and j are respectively responsible, $\rho_h^{h*} := \varphi_h(N, v^h)/v^h(N)$ and $\rho_j^{h*} := \varphi_j(N, v^h)/v^h(N)$, can be determined in closed form (see the Appendix):

Proposition 3. *Suppose firms are symmetric in the linear market environment defined by equations (14), (15) and (18). Let h be the producer of the good for which overcharge damages are to be proportioned, and j be any of h's $n-1$ competitors. The relative responsibilities for harm then are*

$$\rho_h^{h*} = \frac{1}{n} + \frac{n-1}{n}\sum_{s=2}^{n-1} \frac{(s-1)\cdot(4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n-1)^2 - 2(n+s-3)(n-1)\alpha + s(n-s) - 2(n-1)}$$

and $\rho_j^{h*} = (1 - \rho_h^{h*})/(n-1)$.

The common unit cost γ and demand intercept a have no effect on h's Shapley share. It is determined only by the ratio α of own and cross-price parameters. If this

[14] See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6).

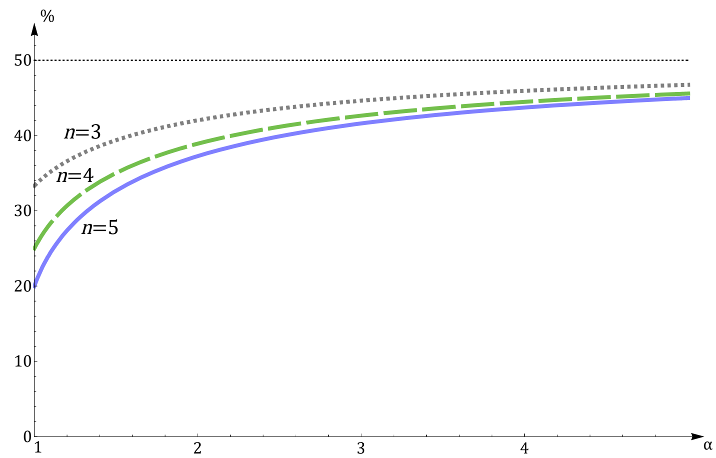

Figure 1: Share ρ_h^{h*} of overcharge damages on good h attributed to firm h for given differentiation parameter α (assuming $d = 2$, $b = 2/[(n-1)\alpha]$)

degree of differentiation α is low, cartel participation by all firms is important. In the limit, each firm is essential for maintaining an overcharge and affects damage equally:

$$\lim_{\alpha\to 1} \rho_h^{h*} = \frac{1}{n} \quad \text{and} \quad \lim_{\alpha\to 1} \rho_j^{h*} = \frac{1}{n} \text{ for } j \neq h. \tag{19}$$

If, in contrast, firms produce highly differentiated goods, we have

$$\lim_{\alpha\to\infty} \rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} (s-1) = \frac{1}{2}. \tag{20}$$

One can check that ρ_h^{h*} is strictly increasing in α. We therefore obtain:

Corollary 1. *If $v^h(N)$ is the compensation obtained by a customer of firm $h \in N$, then proportioning by relative responsibility calls for firm i to contribute*

$$\varphi_i(N, v^h) \in \begin{cases} \left(\frac{v^h(N)}{n}, \frac{v^h(N)}{2}\right) & \text{if } i = h, \\ \left(\frac{v^h(N)}{2(n-1)}, \frac{v^h(N)}{n}\right) & \text{if } i \neq h. \end{cases} \tag{21}$$

Figure 1 illustrates the behavior of ρ_h^{h*} for intermediate degrees of differentiation.

5.3. Asymmetric Case

The bounds in Corollary 1 provide guidance for mildly asymmetric markets by continuity. When firms are sufficiently heterogeneous, though, it is possible that home firm h has *lower* responsibility for harm of its customers and will be assigned a *smaller* share of compensation than its competitors, i.e., $\varphi_h(N, v^h) < v^h(N)/n$. This happens when the cross-price effects involving firm h are sufficiently smaller than those between other cartel members. We can, e.g., have three firms such that demands of firm 1 and 2 involve high mutual cross-price reactions b_{12} and b_{21}, while there are only small linkages b_{i3} and b_{3i} with firm 3 ($i \neq 3$). Firm 3's cartel participation matters for overcharges on p_1, p_2 and p_3 but a significant increase of p_3 would have occurred even if firm 3 had not been part of the cartel and had just best-responded. This part of $v^3(N)$ is caused by price increases on goods 1 and 2, which are mostly driven by shutting down competition between firms 1 and 2. The latter hence had greater influence on $v^3(N)$ than firm 3 itself.[15] Therefore, asymmetry in cross-price effects does not come with useful bounds on responsibilities.

Asymmetry in demand parameters a_i or costs γ_i can be dealt with better, although calculations become tedious. For instance, supposing $\gamma = 0$ and that firm-specific demand intercepts a_i are the only asymmetry at hand, we have:

Proposition 4. *Suppose firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (14), (15) and (18) with $\gamma = 0$. Firm h's Shapely share then is*

$$\rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} \frac{(s-1)\Big[6\alpha(n-1) + (s+4-n) + \Big(4\alpha^2(n-1)^2 + \tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big](\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big)\Big[(3\alpha + \frac{2-n}{n-1}) + (2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]}$$

with $\bar{a}_{-h} := \sum_{i \neq h}^{n} a_i/(n-1)$, $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$.

Ratio $\bar{a}_{-h}/a_h$ relates the market sizes of firm h and its competitors: a large ratio means firm h is comparatively small, a ratio close to zero that h's market is big. It can be checked that ρ_h^{h*} is maximized when (*i*) firms produce highly differentiated goods, i.e, $\alpha \to \infty$, and (*ii*) when firm h's market size is massive, that is, $\lim a_h/\bar{a}_{-h} \to 0$. Then firm h is responsible for half of the damage to its customers. Contrary, ρ_h^{h*} is minimized when goods are close substitutes and firm h's market size is small. Then, firm h is responsible for around $1/n$-th of its own customers' damage. Similar reasoning for firms $j \neq h$ yields

Corollary 2. *Suppose firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (14), (15) and (18) with $\gamma = 0$. If $v^h(N)$*

[15]For instance, assume $a_i = 10$, $d_i = 3$, $\gamma_i = 0$ for $i \in \{1, 2, 3\}$, $b_{12} = b_{21} = 2$, $b_{13} = b_{23} = b_{31} = b_{32} = 0.5$ and consider $v^3(N)$. Then Shapley shares evaluate to $\rho_1^{3*} = \rho_2^{3*} \approx 35.1\% > \rho_3^{3*} \approx 29.8\%$.

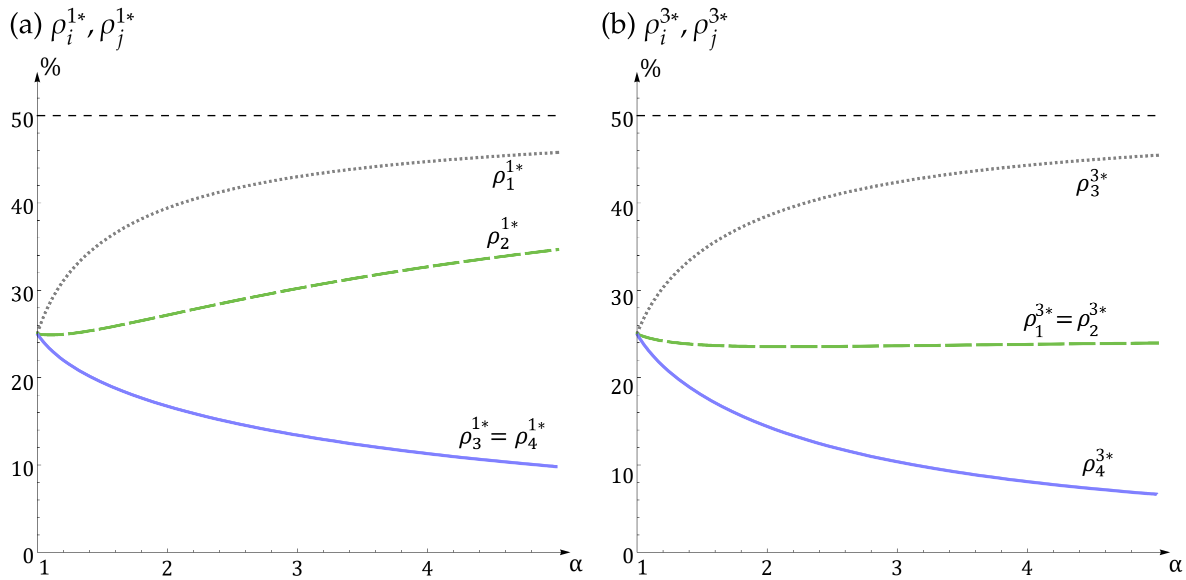

Figure 2: Shares ρ_i^{h*} for cost leaders $i \in \{1,2\}$ and laggards $j \in \{3,4\}$

reflects damages to a customer of firm $h \in N$, then

$$\varphi_i(N, v^h) \in \begin{cases} \left(\frac{v^h(N)}{n}, \frac{v^h(N)}{2}\right) & \textit{if } i = h, \\ \left(0, \frac{v^h(N)}{2}\right) & \textit{if } i \neq h. \end{cases} \tag{22}$$

The bounds in eq. (22) also apply to firms which are symmetric in all but technology. This case is illustrated in Figure 2. It considers responsibility for per unit overcharges of $v^1(N)$ and $v^3(N)$ for a cartel of two low-cost firms (1 and 2) and two high-cost producers (3 and 4) with common parameters $a = 10$, $d = 2$, and $b = 2/(3\alpha)$. No matter whether the selling firm has (a) low costs $\gamma_1 = \gamma_2 = 1$ or (b) high costs $\gamma_3 = \gamma_4 = 5$, it bears responsibility for 25% to 50% of overcharges on its product, and always the greatest share.

6. Comparison to Proportioning by Market Shares

A simple and reliable proportioning heuristic could save the effort of above calculations. Perhaps market shares, which are comparatively easy to obtain, are a good proxy for whose cartel participation is responsible for which proportion of damages, at least under some identifiable circumstances? If yes, should we use sales or revenues? From the cartel or competitive regime? Or perhaps better use a profit measure?

We will address these questions by a range of numerical simulations. We already know from the above analysis of symmetric situations that respective (symmetric) market shares clash with firms' asymmetric responsibilities for harm of a single customer who purchased only one good (e.g. compare market share $1/n$ to ρ_h^{h*} in Figure 1 if $\alpha > 1$). So to give proportioning heuristics a good shot we will assume that *all customers* of the detected cartel received full compensation. Firms' over and under-contributions relative to the product-specific Shapley shares can then cancel out for a given heuristic across products. In particular, a division by heads perfectly matches relative responsibility in the aggregate if firms are symmetric.

So let us consider asymmetric firms. Our benchmark are aggregate payments under Shapley proportioning for each firm $i \in N$,

$$\Phi_i := \sum_{j \in N} \varphi_i(N, v^j) = \sum_{j \in N} q_j^c \cdot v^j(N) \cdot \rho_i^{j*}(N, v^j), \tag{23}$$

and we compare this to firm i's payment H_i^ρ if the total damage

$$\mathcal{D} := \sum_{i \in N} q_i^c \cdot v^i(N), \tag{24}$$

is proportioned by a market share measure ρ, i.e. to $H_i^\rho := \rho_i \cdot \mathcal{D}$ (with q_i^c denoting firm i's cartel sales). Firms' respective over and under-payments are summed and normalized to give an index of *aggregate mis-allocation of damages*

$$M^\rho := \sum_{i \in N} |\, \Phi_i - H_i^\rho \,| \Big/ \mathcal{D}. \tag{25}$$

This index is proportional to the expected mis-allocation of compensation for a unit purchase by a randomly drawn customer, for a customer who made purchases from all firms in proportion to their cartel sales, or when all customers go after the cartel with identical positive probability.

In Figure 3, we start from the baseline scenario $a = 10$, $\gamma = 1$, $d = 2$, $b = d/(3\alpha)$ and break symmetry for one parameter at a time. The two top panels consider heterogeneity in firm-specific market sizes a_i. Panel (a) involves two large and two small firms; in panel (b) all firms differ. An *equal per head allocation* ρ^0 non-surprisingly performs well when differentiation is very low. It soon loses out, though, to allocating damages in proportion to *market shares based on competitive sales* ρ^4 and to *market shares based on cartel sales* ρ^2. Market shares determined by *cartel revenues* ρ^1 or *competitive revenues* ρ^3 produce very high mis-allocations at all levels of differentiation. Only proportioning by *cartel profits* ρ^5 is worse.

Panels (c) and (d) assume an intermediate and a big cost asymmetry between firms 1

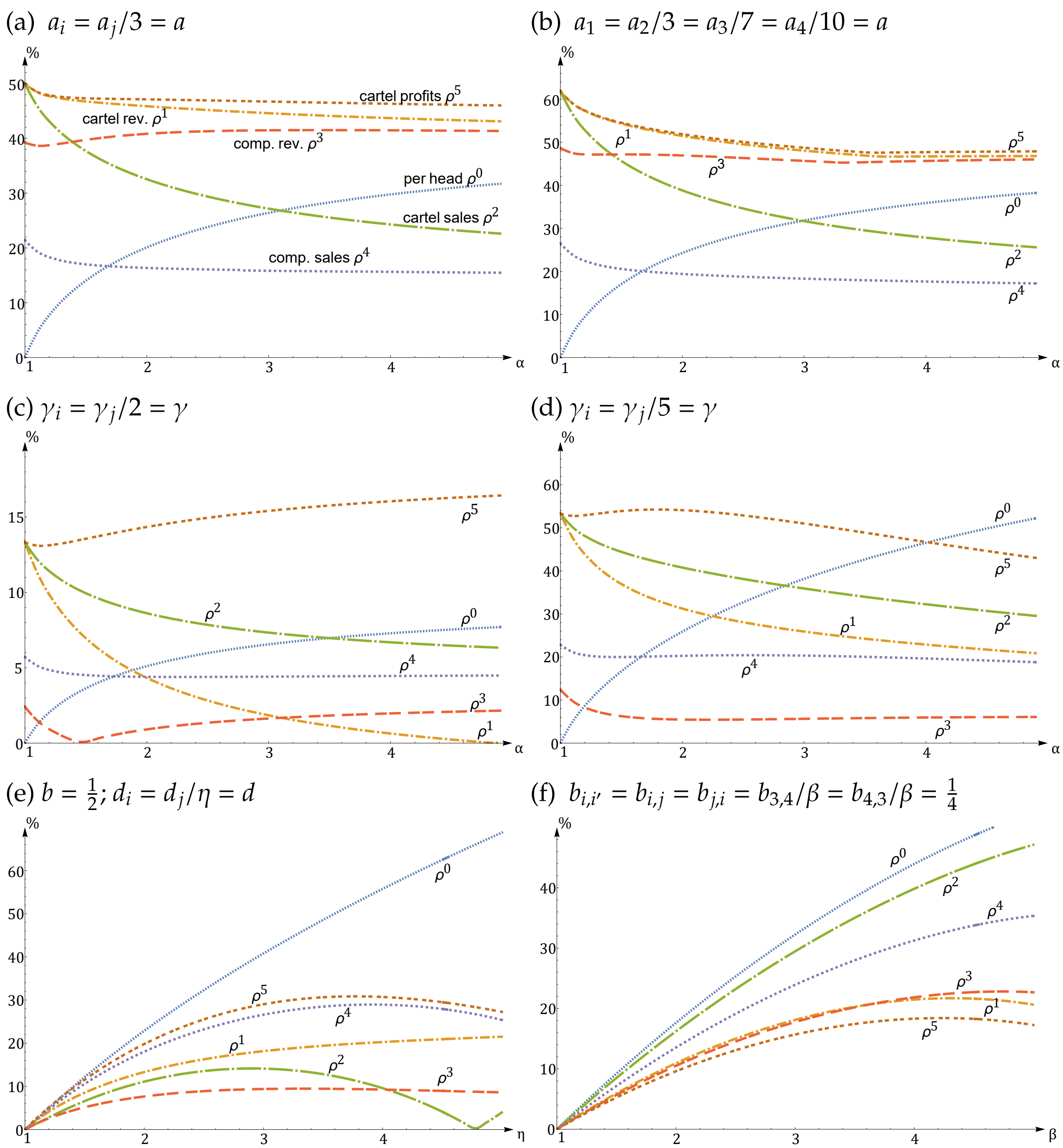

Figure 3: Mis-allocation M^ρ by different heuristics considering $i = 1, 2$ and $j = 3, 4$

and 2 vs. firms 3 and 4. The deviations from the Shapley payments, aggregated for each firm, are significantly higher for the big asymmetry in (d) than the smaller one in (c).[16] Revenue-based market shares ρ^1 or ρ^3 and sales-based competitive market shares ρ^4 here perform the best.

Panel (e) assumes firms 3 and 4 face bigger own-price elasticities than firms 1 and 2. Market shares ρ^2 and ρ^3 based on cartel sales or competitive revenues then are closest

[16]The kink that is visible in panel (c) for ρ^3 – or ρ^2 in (e) – results from cancellation of product-specific deviations at the firm level when these switch from having opposite to identical signs.

to the Shapley benchmark. The final panel (f) assumes heterogeneity in cross-price effects: firms 1 and 2 face a fixed cross-price parameter of $1/4$, competition between firms 3 and 4 is more intense by some factor β. In contrast to the five environments (a) – (e) in which its relative ranking was consistently low, proportioning by cartel profits ρ^5 here comes closest to reflecting the Shapley shares.

The key message of the ups and downs and, notably, the changing ranking of $M^{\rho^0}, \ldots, M^{\rho^5}$ in Figure 3 is that *no* market share heuristic provides a reliable short-cut to relative responsibilities for harm. This holds even when contributions are evaluated at the aggregate market level rather than for harm suffered by an individual litigant. So tempting as proportioning compensation payments by cartel sales, revenues, or profits may be, we see that market shares generally fail to reflect responsibility shares.

7. Concluding Remarks

The results in this paper have been obtained under the assumption that damages in "What if some cartel members had refused to participate?"-scenarios can somehow be quantified. This is a limitation. But counterfactuals provide the basis of any causality-based ascription of responsibility, as well as of the quantification of harm and a victim's compensation in the first place (see Directive 2014/104/EU, recital 46).

Depending on the case at hand, refined estimates of per unit overcharges may be obtained from a structural market model that has been calibrated to a sufficiently rich panel of data (see de Roos 2006). The assessment by an experienced practitioner is still skeptical: "... for almost all real-life cases, such a data panel will be exceedingly difficult or downright impossible to obtain" (Bornemann 2018, recital 124).

In our view, it is nonetheless relevant to study ideal worlds with accurate assessments $v^j(S)$ for counterfactual sub-cartels S: one can gain structural insights (such as the bounds derived above) and, importantly, assess the quality of more pragmatic suggestions. Without a sound benchmark it is unclear why proportioning by "... sales of the product during the conspiracy ...", as proposed by Baker (2004) early on, should reflect relative responsibilities any better than, say, profit shares or a division by heads. The numerical analysis in Section 6 demonstrates, alas, that *any* market share heuristic provides a blurred reflection of responsibility at best.

A possible way forward is to anyhow proportion by (an arbitrary choice of) market shares but to stop pretending that robust links to causality-based responsibility exist.

Another and our preferred alternative would be to capture causal links between actions and harm by applying the Shapley value at least to first approximations of applicable counterfactuals. In a companion paper, approximations of $v^j(S)$ that partition partial cartels $S \subset N$ into binary categories (namely, S is either able to sustain significant overcharges or not) turn out to perform surprisingly well (Napel and Welter 2021). We conjecture that modestly finer classifications – such as S causing 'significant' vs. 'intermediate' vs. 'insignificant' harm to buyers of good j – are still tractable but come very close to implementing Directive 2014/104/EU's provision: when cartel victims are compensated, jointly liable co-infringers need to contribute according to their "relative responsibility for the harm caused".

A. Appendix – Proofs of Propositions 3 and 4

Proof of Proposition 3. Suppose $n \geq 3$ firms are symmetric in the linear market environment defined by equations (14), (15) and (18). The cartel price then evaluates to

$$p^c := p_i^N = \left(\frac{a}{d-(n-1)b} + \gamma\right)\Big/2 \tag{A.1}$$

for each differentiated product $i \in N$.[17] Corresponding competitive Bertrand prices are

$$p^B := p_i^{\varnothing} = \frac{a + d\gamma}{2d-(n-1)b} \text{ for all } i \in N. \tag{A.2}$$

This implies per unit cartel overcharges of

$$v^i(N) = p^c - p^B = \frac{a/d - \gamma(1-\frac{1}{\alpha})}{4\alpha - 6 + 2/\alpha} \quad \text{with} \quad \alpha = \frac{d}{(n-1)b} > 1 \tag{A.3}$$

for each product $i \in N$. They are homogeneous of degree one in (a, γ) and strictly decreasing in differentiation parameter α as well as in unit costs γ.

If there is a partial cartel S of size $s = 2, \ldots, n-1$, equilibrium prices are

$$p_i^S = \begin{cases} \dfrac{a(2d+b) + \gamma\Big(2d^2 + bd(3-2s) + b^2(ns - n - s^2 + 1)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \in S, \\[2ex] \dfrac{a(2d - sb + 2b) + \gamma\Big(2d^2 - bd(s-2) - b^2(s^2 - s)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \notin S \end{cases} \tag{A.4}$$

with $\eta_s = s(n-s) - 2(n-1) \geq -(n-1)$.

Comparing the price p_h^S of the home product $h \in N$ paid by a suing customer in case that the respective producer h is part of a cartel with s members, i.e., for $h \in S$, to the respective price p_h^S if h is not, i.e., for $h \notin S$, yields[18]

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) = \frac{b(s-1)\Big(a + (n-1)b\gamma - d\gamma\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} > 0. \tag{A.5}$$

Inserting this into eq. (6) gives the Shapley proportioning $\varphi_h(N, v^h)$ in absolute terms. Dividing the latter by $v^h(N)$ yields h's claimed Shapley share ρ_h^{h*}. □

[17] The detailed algebraic manipulations omitted here are available upon request.

[18] The three factors in the numerator are strictly positive. Invoking $s \leq n-1$ and $\eta_s \geq -(n-1)$ first, and $d > (n-1)b$ next, the denominator can be bounded below by $2d[2d - 2(n-2)b] - b^2(n-1) > 2d[2(n-1)b - 2(n-2)b] - bd = 3bd > 0$. Hence $p^h(s) - p^{\not h}(s) > 0$.

Proof of Proposition 4. Suppose $n \geq 3$ firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (14), (15) and (18) with $\gamma = 0$. Then, firm h's cartel price is

$$p_h^C = \frac{a_h d - (n-2)a_h b + b(n-1)\bar{a}_{-h}}{2(b+d)(d+b-bn)} \tag{A.6}$$

with $\bar{a}_{-h} = \sum_{l=1\neq h}^{n} a_l/(n-1)$. Firm h's corresponding competitive price is

$$p_h^B = \frac{2a_h d - (n-2)a_h b + b(n-1)\bar{a}_{-h}}{(2d+b)(2d+b-bn)}. \tag{A.7}$$

A customer's per unit cartel overcharge by the product h then is

$$v^h(N) = p_h^C - p_h^B = \frac{b(n-1)[b(3d+2b-bn)a_h + (2d^2+b^2n-b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)}. \tag{A.8}$$

It rises in the saturation level a_h of firm h's demand as well as in the average saturation quantity $\bar{a}_{-h}$ of firms $l \neq h$. The corresponding Shapley value of firm h in proportioning $v^h(N)$ is

$$\varphi_h = \frac{v^h(N)}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{b(s-1)[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(4d^2-(2n-6+2s)db+\eta_s b^2)} \tag{A.9}$$

with $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$. Dividing φ_h by $v^h(N)$ and substituting $\alpha = \frac{d}{b(n-1)}$ gives

$$\rho_h^{h*} = \frac{1}{n} + \frac{1}{n(n-1)}\sum_{s=2}^{n-1} \frac{(s-1)\Big[6\alpha(n-1) + (s+4-n) + \Big(4\alpha^2(n-1)^2+\tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big](\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big)\Big[(3\alpha+\frac{2-n}{n-1}) + (2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]} \tag{A.10}$$

as claimed. □

References

Argenton, C. (2019). Colluding on excluding. *European Economic Review 113*, 194–206.

Argenton, C., D. Geradin, and A. Stephan (2020). *EU Cartel Law and Economics*. Oxford: Oxford University Press.

Baker, D. I. (2004). Revisiting history: What have we learned about private antitrust enforcement that we would recommend to others? *Loyola Consumer Law Review 16*(4), 379–408.

Bernheim, D. B. (2002). In RE: Vitamins Antitrust Litigation, M.D.L. No. 1285, United States District Court for the District of Columbia. Expert report.

Bornemann, B. (2018, last accessed August 26, 2021). *Cartel Damages: Liability and Settlement*. `https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3208840`.

Bos, I. and J. E. Harrington, Jr. (2010). Endogenous cartel formation with heterogeneous firms. *RAND Journal of Economics 41*(1), 92–117.

Budzinski, O. and I. Ruhmer (2010). Merger simulation in competition policy: A survey. *Journal of Competition Law and Economics 6*(2), 277–319.

Davidson, C. and R. Deneckere (1984). Horizontal mergers and collusive behavior. *International Journal of Industrial Organization 2*(2), 117–132.

de Roos, N. (2006). Examining models of collusion: The market for lysine. *International Journal of Industrial Organization 24*(6), 1083–1107.

de Roos, N. and V. Smirnov (2021). Collusion, price dispersion, and fringe competition. *European Economic Review 132*, #103640.

Dehez, P. and S. Ferey (2013). How to share joint liability: A cooperative game approach. *Mathematical Social Sciences 66*(1), 44–50.

Dehez, P. and S. Ferey (2016). Multiple causation, apportionment, and the Shapley value. *Journal of Legal Studies 45*(1), 143–171.

Deneckere, R. and C. Davidson (1985). Incentives to form coalitions with Bertrand competition. *RAND Journal of Economics 16*(4), 473–486.

Epstein, R. J. and D. Rubinfeld (2001). Merger simulation: A simplified approach with new applications. *Antitrust Law Journal 69*(3), 883–919.

European Commission (2013). Impact Assessment Report. Damages actions for breach of the EU antitrust rules. Commission Staff Working Document 203.

European Commission (2014). On certain rules governing actions for damages under national law for infringements of the competition law provisions of the Member States and of the European Union. Directive 104/EU.

Federgruen, A. and M. Pierson (2011). The impact of mergers and acquisitions in price competition models. Working paper, Columbia Business School.

Feinberg, J. (1970). *Doing and Deserving*. Princeton, NJ: Princeton University Press.

Hart, O. and J. Moore (1990). Property rights and the nature of the firm. *Journal of Political Economy 98*(6), 1119–1158.

Holler, E. and M. P. Schinkel (2017). Umbrella effects: Correction and extension. *Journal of Competition Law and Economics 13*(1), 185–189.

Huettner, F. and D. Karos (2017). Liability situations with joint tortfeasors. Working paper, ESMT Berlin and Maastricht University.

Hviid, M. and A. Medvedev (2010). The role of contribution among defendants in private antitrust litigation. *International Review of Law and Economics 30*(4), 306–316.

Inderst, R., F. P. Maier-Rigaud, and U. Schwalbe (2014). Umbrella effects. *Journal of Competition Law and Economics 10*(3), 739–763.

Kalai, E. and D. Samet (1987). On weighted Shapley values. *International Journal of Game Theory 16*(3), 205–222.

Kleinberg, N. L. and J. H. Weiss (1985). A new formula for the Shapley value. *Economics Letters 17*(4), 311–315.

Landes, W. M. and R. A. Posner (1980). Joint and multiple tortfeasors: An economic analysis. *Journal of Legal Studies 9*(3), 517–555.

Levy, D. T. and J. D. Reitzes (1992). Anticompetitive effects of mergers in markets with localized competition. *Journal of Law, Economics, and Organization 8*(2), 427–440.

Lianos, I. (2015). Causal uncertainty and damages claims for the infringement of competition law in Europe. *Yearbook of European Law 34*(1), 170–231.

Maschler, M., E. Solan, and S. Zamir (2013). *Game Theory*. Cambridge: Cambridge University Press.

Moretti, S. and F. Patrone (2008). Transversality of the Shapley value. *TOP 16*(1), 1–41.

Napel, S. and G. Oldehaver (2015). Kartellschadensersatz und Gesamtschuldnerausgleich – Ökonomisch faire Schadensaufteilung mit dem Shapley-Wert. *Neue Zeitschrift für Kartellrecht 3*, 135–140.

Napel, S. and D. Welter (2021). Simple voting games and cartel damage proportioning. Discussion paper, University of Bayreuth.

Protter, M. H. and C. B. Morrey (1991). *A First Course in Real Analysis* (2nd ed.). New York, NY: Springer.

Ray, D. and R. Vohra (1999). A theory of endogenous coalition structures. *Games and Economic Behavior 26*(2), 286–336.

Rothblum, U. G. (1988). Combinatorial representations of the Shapley value based on average relative payoffs. In A. E. Roth (Ed.), *The Shapley Value – Essays in Honor of Lloyd S. Shapley*, pp. 121–126. Cambridge: Cambridge University Press.

Schwalbe, U. (2013). Haftungsquotierung bei Kartellschäden – Ein Ansatz aus der Theorie kooperativer Spiele. Discussion Paper, University of Hohenheim.

Shapley, L. S. (1953a). A value for n-person games. In H. Kuhn and A. Tucker (Eds.), *Contributions to the Theory of Games*, Volume II of *Ann. Math. Studies 28*, pp. 307–17. Princeton, NJ: Princton University Press.

Shapley, L. S. (1953b). *Additive and Non-Additive Set Functions*. Ph.D thesis, Princeton University.

Shubik, M. (1962). Incentives, decentralized control, the assignment of joint costs and internal pricing. *Management Science 8*(3), 325–343.

Vives, X. (1999). *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, MA: MIT Press.

Young, H. P. (1985). Monotonic solutions of cooperative games. *International Journal of Game Theory 14*(2), 65–72.

Young, H. P. (1994). Cost allocation. In R. Aumann and S. Hart (Eds.), *Handbook of Game Theory with Economic Applications*, Volume 2, Chapter 34, pp. 1193–1235. Amsterdam: Elsevier.