Abstract

Cartel members are liable jointly and severally: one may be forced to compensate victims on behalf of all. EU law stipulates that others must pay internal redress in proportion to their “relative responsibility for the harm”. We operationalize this responsibility by evaluating counterfactual damages had one or more cartelists rejected collaboration. Causality and other basic apportioning requirements call for aggregation of counterfactual overcharges via the Shapley value. We prove that even symmetric firms may bear unequal responsibility, characterize damage allocations for linear market environments, and compare Shapley-based redress payments to ad hoc apportioning based on sales or profits.

Keywords: Shapley value; applications of game theory; damage apportioning; relative responsibility

JEL codes: C71; D04; K42

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1. Introduction

Cartels are illegal because they generally harm customers and suppliers of the firms involved, and possibly others. Victims have for long had a right to compensation but the pertinent legal hurdles used to be high. Annually up to 23.3 billion euro of damages remained unclaimed from EU-wide cartels according to the European Commission (SWD/2013/203/Final, recital 67). In 2014, this was a key reason for the Commission to revise rules in its Damages Directive (2014/104/EU). The position of plaintiffs has since improved and several big cases are pending – e.g., against the air cargo, elevator, or truck cartels.

Two provisions for the compensation of cartel victims in the EU Directive motivate this paper. First, the members of a cartel are liable jointly and severally: an injured party can sue any cartel member for the full amount of its damages; if courts confirm the claim, the defendant must compensate the plaintiff on behalf of the entire cartel. This is regardless of whether the plaintiff made its purchases from the sued firm or other ones. Similar provisions apply in Australia, Japan, and the US.

Second, the sued cartel member is entitled to internal redress. Such a rule of contribution existed in the EU before but details differed across member states. It contrasts with the no contribution rule in federal US antitrust cases (cf. Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981) and intermediate arrangements in Australia and Japan.

According to the Directive, cartelists’ internal obligations in compensating any external claimant must reflect “. . . their relative responsibility for the harm caused by the infringement of competition law” (Article 11(5)). The Directive is not specific on how “relative responsibility” should be quantified. It leaves doors open by stating that “. . . determination of that share [of compensation] as the relative responsibility of a given infringer, and the relevant criteria such as turnover, market share, or role in the cartel, is a matter for the applicable national law . . . ” (recital 37). The goal of this paper is to operationalize “relative responsibility” for cartel damages in an economically sound way.

1 The issue of how alternative norms, such as the no contribution rule in the US, affect incentives for cartel formation, whistleblowing, settlements, etc. is here left aside. See, for instance, Landes and Posner (1980) or Hviid and Medvedev (2010).
The paper focuses on the quantification of economic responsibility, but is based on the canonical causal conception of legal and moral responsibility for damages. Feinberg (1970, p. 195f) provides a lucid discussion of its three parts: firstly, the defendant was at fault in acting. This clearly applies if, for instance, firm $i$’s manager illegally coordinated its production of some commodity with competitors over dinner, violating antitrust laws. Secondly, the faulty act caused the harm: these conversations resulted in a price increase for the customer. And, finally, the faulty aspects of the act were relevant to its causal connection to the harm: illegal coordination by the managers – not, perhaps, just the reaction of commodity investors to observing their meeting – caused the increase. All three parts call for appropriate verification in practical applications.

After collective responsibility of the infringers for a given harm has been affirmed, a systematic approach is warranted to determine individual shares and obligations to contribute. The reason is that asymmetry of cartel members can translate very differently into asymmetric turnover, market shares, relative profits, etc. Picking one ad hoc criterion to determine contributions rather than another involves undue arbitrariness in view of Article 11’s explicit reference to relative responsibility.

A systematic rather than an ad hoc approach in our view entails the following considerations: first, a firm has responsibility and should contribute to compensating a given customer only if this customer’s damages would have been lower had the firm refused to participate in the cartel. How much lower the respective damage would have been if the firm had stayed legal (and then possibly some others too) ought to, second, determine the level of the contribution. Third, if cartel membership of two firms had identical effects on harm, both should contribute the same to its remedy. Finally, a victim’s full compensation should be apportioned in a way that neither depends on the unit of account nor on whether multiple damages are dealt with separately or jointly.

These properties translate into mathematical conditions that are well-known in game theory as the null player, marginality, symmetry, efficiency and linearity axioms. Classical results by Shapley (1953a) and Young (1985) hence imply that the Shapley value of an appropriately defined game is the unique best way to split external compensation obligations by relative responsibility of the co-infringers.

The Shapley value is a well-established tool for allocating costs and profits in joint
ventures. The corresponding rationale applies to joint liability at least as well. It has been adapted by Dehez and Ferey (2013, 2016) or Huettner and Karos (2017) to sequential liability games, which reflect incremental harm caused by chronologically ordered acts of negligence. Use of the Shapley value for the allocation of cartel damages was first proposed by Schwalbe (2013) and Napel and Oldehaver (2015) to law audiences, and has been taken up, e.g., by Bornemann (2018). This paper is the first to analyze the quantitative aspects of apportioning cartel damages by relative responsibility in detail.

A Shapley apportionment imputes responsibility from individual abilities of the conspiring firms to influence prices. After a more detailed discussion of cartel damages (Section 2), we highlight the properties which make the Shapley value the right tool and investigate a widely unknown but useful decomposition of the Shapley value (Section 3). We prove that even symmetric firms can have asymmetric obligations to compensate an individual claimant (Section 4). For linear market environments, we then analyze how a Shapley apportionment is linked to demand and cost parameters, with a closed form representation of the implied Shapley value (Section 5). We derive potentially useful bounds on a given firm’s responsibility for its own overcharges and those by other cartel members, and compare Shapley allocations to ad hoc apportioning based on market shares, profits, or a flat per head assignment. Corresponding heuristics have been suggested by law practitioners and we illustrate the extent to which they conflict with relative responsibility. We also point to weighted Shapley values as an option for accommodating immunity of a co-defendant or ringleaders (Section 6).

2. Cartel Damages and Relative Responsibility

Antitrust victims have a right to be compensated: they should be put in the financial position that they would have been in without the infringement. A cartel’s customers have usually suffered two types of damage. The first is the visible loss due to higher prices: each unit that was purchased involved an overcharge damage. Further harm relates to deadweight losses: customers who would have made (additional) purchases,
and thus would have enjoyed surplus had prices been competitive, failed to do so. This is acknowledged as *lucrum cessans* in the legal literature but has played little role in practice yet (see Laborde 2017).

We concentrate on overcharge damages and assume that they were caused by a hardcore cartel that fixed quantities, sales areas, or prices of differentiated goods. This leaves aside other kinds of infringements, deadweight losses and possible general equilibrium effects (see Eger and Weise 2015). The suggested approach can be generalized, however. Changes in prices could, e.g., be replaced by lost downstream profits or money-metric indirect utility. The scale invariance of the Shapley value also allows to deal naturally with the payment of interest, which is an essential part of compensation (Damages Directive, recital 12).

Cartel overcharges are often quantified on the basis of a monthly but-for estimation (see, e.g., Bernheim 2008). Our presentation adopts a static perspective but, subject to availability of data, it seems desirable to apportion damages based on dynamic estimations, too. A firm’s relative responsibility for harm may vary as other firms join or leave the cartel, or when demand and costs change individual influence on prices.

A cartel member *i* having responsibility for damages of a given claimant *k* requires that *k*’s damages are causally linked to *i*’s cartel membership, i.e., their scale, scope or distribution would have differed without *i*’s illegal action. Identifying the causal links between anticompetitive conduct and harm is generally fraught with difficulties (see, e.g., Lianos 2015). What makes economic analysis of responsibility for cartel damages particularly interesting, however, is that even symmetric cost and demand structures may generate asymmetric links to harm suffered by a specific victim. Namely, price effects of individual cartel membership differ across cartelists as long as own-price and cross-price elasticities of the respective demands differ. Firms that are symmetric from an industrial organization perspective then are non-symmetric from a game theory perspective.

For illustration consider *n* otherwise identical firms on a Salop circle. Think of cement plants that are equally spaced on the shores of an unshippable lake. They sold their cement at inflated prices to local construction companies around the lake. Their cartel was busted and a customer of firm *i* sues. Firm *i*’s and another firm *j*’s relative responsibilities for this customer’s damages are tied to the counterfactual price that the customer would have paid had *i* or respectively *j* refused to participate.
Unless transportation costs are zero, and thus all products perfect substitutes, cartel membership of the northernmost vendor has smaller effect on overcharges faced by customers in the south than does membership of southern vendors, and vice versa (see, e.g., Levy and Reitzes 1992). The closer two firms are located and hence the more intensely they would have competed in the absence of the cartel, the greater the price effect of their illegal coordination.

So counterfactual prices that the suing customer would have paid if $i$ or if $j$ had not joined the cartel, but just best-responded to its practices, vary according to $i$’s and $j$’s locations. Differential effects of cartel membership imply differential responsibilities for a specific customer’s damage; hence different obligations for compensation.

Of course, a symmetric market structure implies that obligations which $i$ and $j$ have in compensating each others’ customers are the same. So mutual redress claims cancel out if all constructors sue, or if equal measures of them do everywhere. However, they do not cancel in almost all other situations – e.g., if just some construction companies in the south go to court. A general analysis hence requires that responsibility can be attributed to cartel members for the price overcharge on each single product in the cartel’s portfolio. Asymmetric market structures make a focus on individual products even more important.

A sound procedure for apportioning damages matters also for losses to victims that purchased from cartel outsiders at prices whose elevated level derived from the infringement. These so-called *umbrella losses*[^1] have successfully been claimed before several US courts (but not all) and are generally acknowledged in the EU (CJEU C-557/12 2014). Since their compensation is not linked to transactions with any cartelist, apportionment based on one of the typically conflicting notions of market shares – by sales, revenues or profits; during the cartel period, before, thereafter – would be even more ad hoc than for damages to direct customers of the cartel.

Finally note that, in the EU, a firm’s relative responsibility matters also if litigants settle. An injured party’s claim against remaining co-defendants is reduced by the settling defendant’s responsibility for harm irrespective of the amount of the settlement (Damages Directive, recital 51). A reliable identification of responsibility thus matters for incentives to pursue further private action (see Bourjade et al. 2009).

[^1]: See Inderst et al. (2014) and Holler and Schinkel (2017)
3. The Shapley Value as a Tool for Apportioning Damages

We propose several conditions that a rule for apportioning damages in line with relative responsibilities should satisfy. It turns out that all are verified if proportioning is based on the Shapley value whereas other rules violate at least one condition.

3.1. Preliminaries

A damage apportioning problem can be seen as a transferable utility (TU) game where the damage caused by a cartel \( N = \{1, \ldots, n\} \) is to be divided among the firms involved.\(^4\) For every coalition \( S \subseteq N \) of players who might collaborate with one another, \( v(S) \) describes damage inflicted if firms \( i \in S \) coordinate their actions while firms \( j \in N \setminus S \) maximize their respective profits in competitive fashion. Mapping \( v : 2^N \to \mathbb{R} \) is the characteristic function of TU game \( (N, v) \).

For strict subsets of \( N \), \( v(S) \) reflects a counterfactual. This is necessary: responsibility is driven by the fact that overcharges would have differed from the observed damage \( v(N) \) if conducts had differed, i.e., if some firms had stayed out. The Damages Directive explicitly acknowledges the role of counterfactual scenarios: “... quantifying harm means assessing how the market in question would have evolved had there been no infringement. This assessment implies a comparison with a situation which is by definition hypothetical ...” (recital 46). Defining \( v(S) \) for every set \( S \subseteq N \) extends this logic from quantifying harm to quantifying contributions to harm.

Naturally, \( v(S) = 0 \) if the set \( S \) of collaborators is empty \( (S = \emptyset) \) or comprises but a single firm, i.e., if \( |S| = 1 \). For other coalitions \( S \subset N \), a damage estimate \( v(S) \) is needed. Intertemporal variation in cartel participation may help obtaining it but the option likely to work best is market simulation analysis, which is rather well-established in merger control.\(^5\) There, parameters of a structural model of price or quantity competition are estimated based on pre-merger observables; these generate equilibrium predictions for when a subset of firms merge and internalize mutual

\(^4\)The relevant market may comprise firms \( j \notin N \) which did not partake in the cartel. They need not contribute to compensations and matter as exogenous co-determinants of damage rather than players.

profit externalities, just as cartel members do. Analogous analysis of cartel behavior is still comparatively rare (see Roos 2006) and its use for the estimation of function \( v \) is more tedious: many scenarios rather than just a single proposed merger need to be evaluated. The model’s calibration could, however, draw not only on pre-cartel (like pre-merger) observables but also observations during and after the cartel’s operation. Former members may have an interest to disclose cost or demand information if they expect lower contributions than under an ad hoc apportionment. Sensitive data could be pooled by a trusted intermediary who aids in settling mutual redress claims (e.g., auditing or law firms).

We take no stance here on how sophisticated estimates \( v(S) \) ought to be in practice. For instance, the analysis of a hypothetical scenario with a sub-cartel \( S \neq N \) may consider the question of whether \( S \) satisfies suitable stability conditions, and put \( v(S) = 0 \) if not. The illustrations below will keep things simple. But note that each number \( v(S) \) with \( i \notin S \) reflects a scenario for how the market might have evolved if there had been no infringement by firm \( i \). It is both possible that firm \( j \neq i \) would then have joined the cartel anyhow (\( j \in S \)) or that it would have stayed legal too (\( j \notin S \)). These scenarios need not have equal probability. But all partial cartels \( S \subseteq N \setminus \{i\} \) are, in principle, relevant in assessing \( i \)'s contribution to the situation which calls for compensation, hence \( i \)'s relative responsibility.

3.2. Desirable Properties of Responsibility-Based Allocations

With damages in the factual cartel scenario and related counterfactuals described by \((N,v)\), a damage apportioning rule is a mapping \( \Phi \) from any conceivable cartel damage problem \((N,v)\) to a vector \( \Phi(N,v) \in \mathbb{R}^n \), i.e., it is a value of the corresponding TU game. The main restriction that the cartel context imposes is that \( v(\{i\}) = 0 \) for all \( i \in N \). As prices of substitute goods are usually higher for bigger cartels (see Davidson and Deneckere 1984, Deneckere and Davidson 1985), one can take \( v \) to be monotonic in \( S \), but we cannot generally impose more structure. The \( i \)-th component \( \Phi_i(N,v) \) denotes

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\(^6\)It is also conceivable that several partial cartels would have formed if \( i \) had refused to join. This could be accommodated by considering extensions of the Shapley value to partition functions \( V \) from the set of partitions \( \mathcal{P} = \{P_1, \ldots, P_r\} \) of \( N \) (satisfying \( \bigcup_l P_l = N \) and \( P_l \cap P_k = \emptyset \) for any \( l, k \in \{1, \ldots, r\} \)) to estimated damages \( V(\mathcal{P}) \). See Ray and Vohra (1999).

\(^7\)Superadditivity and convexity of \( v \) are natural further assumptions, however. To ensure that new members want to join a cartel and existing members accept them, \( v \) must be superadditive. For cartels
the part of the compensation for damages $v(N)$ which cartel member $i \in N$ must contribute.

That an apportioning rule reflects relative responsibilities can be translated into three formal properties of a rule $\Phi$. The first one is straightforward. Suppose that participation or not of a particular firm $i$ would never have made a difference to the damage in question. That is, removing player $i$ if $i \in S$ or adding player $i$ if $i \notin S$ does not change $v(S)$. Then given that $i$’s conduct has no effect on damage, the conditions for $i$ being responsible are not met (see Feinberg 1970). Hence, no responsibility-based obligations to contribute follow. A player $i$ for whom $v(S) = v(S \setminus \{i\})$ for every $S \subseteq N$ is known as a null player. The first requirement for rule $\Phi$ to be based on relative responsibility hence is the null player property:

$$\Phi_i(N, v) = 0 \text{ whenever } i \text{ is a null player in } (N, v).$$

(NUL)

Presumably, the supply and demand conditions in real markets are rarely compatible with a convicted cartel member being a null player. But (NUL) conducts a valid thought experiment. It also formalizes a certain robustness to misspecification of the relevant market. For instance, a large cartel may have caused damage in several regions with independent costs and demand. If a firm is accidentally included as ‘player’ in a region where it had no role, (NUL) ensures it need not contribute there.

As responsibility derives from the causal link between cartel membership and harm suffered, a second straightforward requirement is that $i$’s damage share $\Phi_i(N, v)$ should be determined by this link – and this link alone. Namely, presuming that $v$ correctly describes factual damages as well as the relevant counterfactuals, $\Phi_i(N, v)$ shall be a function only of $i$’s marginal contributions $v(S) - v(S \setminus \{i\})$ in $(N, v)$. The corresponding formal property of marginality, introduced by Young (1985), demands that $i$’s shares in two apportioning problems $(N, v)$ and $(N, v')$ ought to coincide whenever $i$’s marginal contributions do:

$$\Phi_i(N, v) = \Phi_i(N, v') \text{ whenever } v(S) - v(S \setminus \{i\}) = v'(S) - v'(S \setminus \{i\}) \text{ holds for all } S \subseteq N.$$  

(MRG)

Marginality does not pin down how $\Phi_i(N, v)$ should depend on the differences that $i$ makes to various coalitions. For instance, imposing (MRG) does not imply (NUL); the
properties formalize different aspects of $\Phi$ reflecting firms’ responsibilities.

A third such property refers to situations in which roles of firms $i$ and $j$ in determining damages $v(S)$ are symmetric. Players $i$ and $j$ are called symmetric if $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition $S \subseteq N \setminus \{i, j\}$. When adding $i$ to a sub-cartel $S$ has the same damage implications as adding $j$ whenever $S$ previously contained neither, their responsibilities are the same. So $\Phi$ should also satisfy symmetry:

$$\Phi_i(N, v) = \Phi_j(N, v) \text{ whenever } i \text{ and } j \text{ are symmetric in } (N, v).$$  \hfill (SYM)

Irrespective of whether a damage apportionment reflects responsibility of the involved players or follows alternative principles, individual contributions of all firms $i \in N$ should add up to $v(N)$. In the context of TU games, this is the efficiency of a value:

$$\sum_{i \in N} \Phi_i(N, v) = v(N).$$  \hfill (EFF)

In 2-player games with $N = \{1, 2\}$, efficiency and symmetry jointly imply that $\Phi_1(\{1, 2\}, v) = \Phi_2(\{1, 2\}, v) = \frac{1}{2}v(\{1, 2\})$, i.e., participants to any 2-firm cartel must contribute equally.[8] This may seem counterintuitive if two duopolists have asymmetric market shares, costs and profits. But it makes good sense: both are equally responsible for harm because exit by either firm would have restored competition.

Scale invariance is another natural requirement: firms’ shares of compensation should not depend on whether damages are expressed in US dollar or euro, on whether they are trebled or not, nor on whether they already include interest payments or not. So multiplying all numbers $v(S)$ by an exchange rate, interest or other factor $\lambda > 0$ should re-scale firms’ contributions by the same factor. Moreover, if the same cartel $N$ caused damages to suing customers in several markets – reflected by a characteristic function $v^1$ for market 1, by $v^2$ for market 2, etc. – then the total responsibility-based contribution of firm $i \in N$ should not depend on whether the apportioning rule is applied to damages $v^l$ in one market $l$ at a time, or in one go to the total $v = v^1 + v^2 + \ldots$ Different ‘markets’ could here refer to different plaintiffs or subsidiaries of the same plaintiff, to different products in the cartel’s portfolio, or distinct quantities of the same

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[8] A value $\Phi$ that satisfies this 2-player property is called standard by Hart and Mas-Colell (1989). They showed the Shapley value to be the only standard one with a particular consistency property, which echoes that the order in which firms might settle sequentially does not change relative responsibility.
product. The respective additivity\(^9\) combines with scale invariance to \textit{linearity}:

\[
\Phi(N, \lambda \cdot v + \lambda' \cdot v') = \lambda \cdot \Phi(N, v) + \lambda' \cdot \Phi(N, v') \tag{LIN}
\]

for any scalars \(\lambda, \lambda' \in \mathbb{R}\) and any characteristic functions \(v, v'\).

3.3. \textit{Shapley Value and Decomposition by Average Damage Increments}

Above properties are more than is needed to conclude that a responsibility-based apportioning rule should have a particular form:

\textbf{Shapley-Young Theorem} \hspace{1em} \textit{The following statements about a damage apportioning rule} \(\Phi\) \textit{are equivalent}:

(I) \(\Phi\) satisfies \(\{\text{NULL}\}, \{\text{SYM}\}, \{\text{EFF}\}\) and \(\{\text{LIN}\}\).

(II) \(\Phi\) satisfies \(\{\text{MRG}\}, \{\text{SYM}\}\) and \(\{\text{EFF}\}\).

(III) \[
\Phi_i(N, v) = \varphi_i(N, v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \left[ v(S) - v(S \setminus \{i\}) \right] \tag{1}
\]

where \(s = |S|\).

\(\varphi(N, v)\) is the \textit{Shapley value} of \((N, v)\). Equivalence of (I) and (III) was established by Shapley (1953a)\(^9\) equivalence of (II) and (III) by Young (1985). See, e.g., Maschler et al. (2013, ch. 18).\(^10\) Formula (I) might look unwieldy but weights \((s - 1)!(n - s)!/n!\) on marginal contributions are a logical consequence of the desirable properties discussed.

It is little-known – but can generally be practical in cost or profit allocation scenarios such as the damage apportioning problem – that an equivalent way of writing eq. (I) is

\[
\varphi_i(N, v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=1}^{n-1} \left[ \vartheta(s) - \vartheta^i(s) \right] \tag{2}
\]

where

\[
\vartheta(s) := \binom{n-1}{s-1} \sum_{|S|=s} v(S) \quad \text{and} \quad \vartheta^i(s) := \binom{n-1}{s} \sum_{|S|=s} v(S). \tag{3}
\]

\(^9\)Additivity applies also to different \textit{types} of damages described by \(v\) and \(v'\) – for instance, overcharge damages and deadweight losses.

\(^{10}\)To verify that Shapley’s uniqueness result extends to the class of damage apportionment problems, note that cartels in which \(i \in T \subseteq N\) produce perfect substitutes with competitive price \(p^* = 0\) and cartel price \(p^c = 1\) while all \(j \notin T\) operate in unrelated markets define the required carrier games \((N, u_T)\).
\( \bar{\sigma}(s) \) captures the average damages caused by coalitions of size \( s \) which include firm \( i \); \( \bar{\sigma}^k(s) \) represents average damages caused by same size coalitions which exclude firm \( i \). 

Abbreviating \( \kappa(s) := (s-1)!((n-s)!/n! = \frac{1}{n} \cdot \binom{n-1}{s-1} \), eq. (2) follows from

\[
\varphi_i(N, v) = \sum_{S \subseteq N} \kappa(s) \cdot [v(S) - v(S \setminus \{i\})] = \sum_{S \subseteq N} \kappa(s)v(S) - \sum_{S \subseteq N} \kappa(s+1)v(S)
\]

\[= \kappa(n)v(N) + \sum_{s=1}^{n-1} \left[ \sum_{S \ni i \mid |S| = s} \kappa(s)v(S) - \sum_{S \ni i \mid |S| = s} \kappa(s+1)v(S) \right] = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=1}^{n-1} \left[ \bar{\sigma}(s) - \bar{\sigma}^k(s) \right]. \tag{4} \]

Equation (4) simplifies further because a degenerate ‘cartel’ of size \( s = 1 \) leaves prices constant, i.e., \( \bar{\sigma}(1) = \bar{\sigma}^k(1) = 0 \) for each \( i \in N \). So in summary we obtain:

**Shapley Apportioning Rule**  
Firm \( i \) must contribute

\[
\varphi_i(N, v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=2}^{n-1} \left[ \bar{\sigma}(s) - \bar{\sigma}^k(s) \right] \tag{5}
\]

to compensation of cartel damage \( v(N) \) in order to reflect \( i \)'s relative responsibility and to share \( v(N) \) in a scale-invariant additive way.

Apportionment by relative responsibility of the infringers – formalized by \( \text{[NUL]} \), \( \text{[MRG]} \) and \( \text{[SYM]} \), plus \( \text{[EFF]} \) and \( \text{[LIN]} \) – thus means: start out with equal shares per head; then add an \( n \)-th of the average size-specific damage increments that arise due to a given firm \( i \)'s participation. The latter accounts for asymmetric effects on harm that can arise even in highly symmetric market environments (cf. Section 4).

The decomposition in eq. (5) provides a useful perspective on \( \varphi_i \), and can facilitate its calculation: possible symmetry among players reduces the sum of \( 2^n \) differences in eq. (1) to less than \( n \) ones in (5). This extends when asymmetries are such that \( i \)'s increments for specific coalition sizes \( s \) can be written as a function of ‘aggregate asymmetry’ among the other firms (see Subsection 5.3). The calculation is further simplified if cartels of size \( s \) below some threshold \( \tilde{s} \) are unstable (Bos and Harrington 2010), or if perfect exchangeability of the firms implies that all damage increments \( \bar{\sigma}(s) - \bar{\sigma}^k(s) \) are zero. For instance, the second summand in (5) vanishes in an undifferentiated Bertrand or Cournot oligopoly of symmetric firms or whenever \( n = 2 \). Then equal responsibility and Shapley shares follow.

11The decomposition is related to those discussed by Kleinberg and Weiss (1985) and Rothblum (1988) but all three differ. We may here be the first to note eq. (2) explicitly.
4. Unequal Responsibility of Symmetric Firms

By contrast, unequal responsibility and Shapley shares in compensation follow when three or more symmetric firms produce differentiated substitutes. This holds even for a particularly strong form of symmetry, assuming that differentiation generates greater own-price than cross-price effects but letting firms otherwise be identical.

Specifically, let firms 1, ..., n simultaneously choose strategies $y_1, \ldots, y_n$ that jointly determine prices $p_i$ and profits $\Pi_i$ for all $i \in N$. $p = (p_1, \ldots, p_n)$ and $\Pi = (\Pi_1, \ldots, \Pi_n)$ are taken to be smooth functions of $y = (y_1, \ldots, y_n)$. The two focal cases are differentiated price competition where $p_i(y) \equiv y_i$ and quantity competition where $y_i$ denotes firm $i$’s output. However, $y_i$ might also refer to some marketing activity or product parameter, the geographic radius of operation, etc. We make the following rather general assumptions:

A1. Price $p_i$ and profits $\Pi_i$ are affected identically by own strategy $y_i$ for all firms $i \in N$ and also by all strategy choices $y_j$ of the respective other firms $j \neq i$, that is

$$p_i(y_1, \ldots, y_n) \equiv p_j(y_{\varphi(1)}, \ldots, y_{\varphi(n)}) \quad \text{and} \quad \Pi_i(y_1, \ldots, y_n) \equiv \Pi_j(y_{\varphi(1)}, \ldots, y_{\varphi(n)})$$

(I)

for each $i \neq j$ and all permutations $\varphi \colon N \rightarrow N$ with $\varphi(i) = j$ and $\varphi(j) = i$.\(^{12}\)

A2. For all $i \neq j \in N$,

$$\left| \frac{\partial p_i}{\partial y_i} \right| > \left| \frac{\partial p_i}{\partial y_j} \right|$$

(IIa)

and

$$\frac{\partial \Pi_i}{\partial y_i} \cdot \frac{\partial p_i}{\partial y_i} > 0.$$  

(IIb)

Condition (IIa) is trivially satisfied for price competition and otherwise formalizes that inverse demand responds more to changes of the quantity (or product parameter, delivery range, etc.) of the variety in question than to that of others. Condition (IIb) takes products to be differentiated substitutes where $y_i$ changes own price $p_i$ and other firms’ profits $\Pi_j$ in the same direction: e.g. for quantity competition, greater output $y_i$ lowers $p_i$ as well as the profits of firms $j \neq i$; for price competition, higher prices $y_i = p_i$

\(^{12}\)For instance, each variety $i$ could be the personal favorite of an equal share of consumers who regard varieties $j \neq i$ as equally close substitutes. Note that the symmetry in A1 is stronger than in the Salop model: some permutation $\varphi$ with $\varphi(i) = j$ and $\varphi(j) = i$ satisfies (I) there, but not all do.
raise profits of j.

A3. For all \( S \subseteq N \) there exists a unique Nash equilibrium \( y^S = (y^S_1, \ldots, y^S_n) \) such that

- \( y^S_i = y^c \) if \( i \in S \), where \( y^c \) solves the first order condition

\[
\frac{d \Pi_S}{dy_i} = \sum_{j \in S} \frac{\partial \Pi_j}{\partial y_i} = 0
\]

for maximization of joint profit \( \Pi_S(y) = \sum_{k \in S} \Pi_k(y) \) by cartel \( S \);

- \( y^S_i = y^o \) if \( i \notin S \), where \( y^o \) solves the first order condition

\[
\frac{\partial \Pi_i}{\partial y_i} = 0
\]

for individual profit maximization by an outsider to cartel \( S \).

Sufficient conditions for the equilibrium in A3 to exist are provided in Section 5.

**Proposition 1.** Given A1–A3, let \( p^i(s) (p^k(s)) \) equal the equilibrium price for good \( i \) if firm \( i \) is (is not) part of a cartel with \( s \in \{2, \ldots, n-1\} \) members. Then \( p^i(s) > p^k(s) \).

**Proof.** Inequality (IIb) implies that firms’ strategies either lower their own prices, \( \partial p_i/\partial y_i < 0 \), and have a negative externality on each other’s profits, \( \partial \Pi_j/\partial y_i < 0 \), as for quantity competition; or that \( \partial p_i/\partial y_i > 0 \) and \( \partial \Pi_j/\partial y_i > 0 \). In the former case, internalization of the negative profit externality in a cartel with \( s \in \{2, \ldots, n-1\} \) members implies a smaller individual action or output choice \( y^c < y^o \) for cartel members than outsiders (see A3); otherwise \( y^c > y^o \) with the specific case of price competition, \( p_i(y) = y_i \), directly implying the result.

So take \( \partial p_i/\partial y_i < 0 \) and \( \partial \Pi_j/\partial y_i < 0 \) with \( y^c < y^c' \). Let \( S = \{1, \ldots, s\} \) w.l.o.g. and consider the straight line \( L \) which connects \( \hat{y} = (y^c, y^c, \ldots, y^c, y^c, y^c) \) to \( y^S = (y^c', y^c', \ldots, y^c', y^c', y^c) \) in the space of output choices. \( L \) can be parameterized by

\[
r(t) = (y^o - t, y^c', \ldots, y^c', y^c', \ldots, y^c', y^c + t)
\]

with \( t \in [0, y^c' - y^c] \), i.e., we simultaneously decrease firm 1’s action and increase firm \( n \)’s action by identical amounts as we move along \( L \). The gradient \( \nabla p_n = \left(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\right) \) of function \( p_n \) can be used in order to evaluate the price change caused by switching from
\( \hat{y} \) to \( y^S \). In particular, the (Stokes) gradient theorem for line integrals (see, e.g., [Protter and Morrey 1991, Thm. 16.15]) implies

\[
p_n(y^S) - p_n(\hat{y}) = \int_L \nabla p_n \cdot dr = \int_0^{y^S - y} \nabla p_n(r(t)) \cdot r'(t) \, dt \tag{7}
\]

\[
= \int_0^{y^S - y} \left( \frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n} \right) \bigg|_{y=r(t)} \cdot \left( -1, 0, \ldots, 0, 1 \right) \, dt \tag{8}
\]

\[
= \int_0^{y^S - y} \left[ \frac{\partial p_n(r(t))}{\partial y_n} - \frac{\partial p_n(r(t))}{\partial y_1} \right] \, dt < 0. \tag{9}
\]

The inequality follows from (IIa): firm \( n' \)'s own strategy changes have bigger price effects than changes by competitor firm 1.

A1 then implies

\[
p^1(s) := p_1(y^f, y^f, \ldots, y^f, y^f) = p_n(y^f, y^f, \ldots, y^f, y^f) = p_n(y^S) = p_n(\hat{y}) > p_n(y^S) = p_n(y^f, y^f, \ldots, y^f, y^f) = p^h(s).
\]

That is, the price \( p^1(s) \) of good 1 when its producer is one of \( s \) exchangeable cartel members exceeds the price \( p^h(s) \) of good \( n \) when firm \( n \) is not part of a cartel with \( s \) members. And, also by A1, we have \( p^1(s) = p^h(s) \) and \( p^1(s) = p^n(s) \). So we can conclude \( p^1(s) > p^h(s) \) from (10). The same applies to any other firm \( i \), and we obtain \( p^i(s) > p^h(s) \) for all \( s \in \{2, \ldots, n-1\} \) as claimed. \( \square \)

Now consider the per unit damage \( v(N) \) that accrued to a customer of “home” firm \( h \) who paid cartel price \( p^c = p_h(y^N) \) instead of competitive price \( p^* = p_h(y^o) \). The counterfactual average damages implied by partial cartels of size \( s \) that include and exclude firm \( h \) are \( \bar{\sigma}^h(s) = p^h(s) - p^* \) and \( \bar{\sigma}^k(s) = p^k(s) - p^* \), respectively. Proposition 1 implies

\[ \bar{\sigma}^h(s) - \bar{\sigma}^k(s) = p^h(s) - p^k(s) > 0 \text{ for any } s = 2, \ldots, n-1. \tag{11} \]

So from eq. (5) we can directly conclude

**Proposition 2.** Given A1–A3, consider an overcharge damage \( v(N) \) that was suffered on purchases from firm \( h \in N \) after \( n \geq 3 \) symmetric producers of differentiated goods formed

Note that for \( \partial p_n/\partial y_1 > 0 \) and \( \partial \Pi / \partial y_1 > 0 \) with \( y^f > y^* \), the sign switch from reversed orientation of the integral from \( t = 0 \) to \( y^f - y^f < 0 \) in (6) and the reversed sign of integrand \( \partial p_n/\partial y_n - \partial p_n/\partial y_1 \) cancel.
cartel $N$. Then

$$\varphi_i(N,v) \begin{cases} > \frac{v(N)}{n} & \text{if } i = h, \\ < \frac{v(N)}{n} & \text{if } i \neq h, \end{cases}$$

(12)

i.e., firm $h$ is responsible for more than $1/n$-th of harm to its own customers.

5. Apportionment by Responsibility in Linear Market Settings

The concrete application of the Shapley apportioning rule requires estimates of counterfactual damages for all conceivable partial cartels (see, e.g., Roos 2006). We illustrate this here for situations in which the costs and demand for differentiated goods are described, in acceptably good approximation, by linear functions. We conjecture that parameter restrictions in analogy to, e.g., the proportionality condition of Epstein and Rubinfeld (2001) could reduce the data requirements in practical cases sufficiently to be applicable. If the producers of differentiated products face at most one kind of asymmetry, closed-form expressions for the Shapley shares can be derived via eq. (5). This is often impossible in other applications of the Shapley value. The parametric solutions allow to derive non-trivial upper and lower bounds for the responsibility-based contribution by a firm to harm of its own and of other firms’ customers, respectively. They also facilitate assessing the degrees to which, e.g., cartel-period revenue shares can serve as simple proxies of relative responsibility.

5.1. Linear Model

Let us focus on a cartel of $n \geq 3$ suppliers where each firm $i \in N = \{1, \ldots, n\}$ produces a single good. Firm $i$’s costs are given by

$$C_i(q_i) = \gamma_i q_i \text{ for } \gamma_i \geq 0.$$  

(13)

Demand at price vector $p = (p_1, \ldots, p_n)$ is described by

$$D_i(p) = a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j \text{ for } a_i > \gamma_i, \ d_i > 0, \text{ and } b_{ij} > 0 \text{ for all } j \neq i.$$  

(14)

---

14 Quadratic costs do not change findings much: Proposition 3 below then involves cost parameter $\gamma$ but remains independent of $a$. Later expressions get significantly more unwieldy, however.
We presume $D_i(\gamma) > 0$, i.e., demand is positive when all firms price at cost. The corresponding parameter restriction is $a + (n - 1)b\gamma > d\gamma$ in the symmetric case when $\gamma_i = \gamma, a_i = a, d_i = d$ and $b_{ij} = b$ for all $i \neq j \in N$. Firms set prices simultaneously à la Bertrand. If some group $S \subseteq N$ of them forms a cartel, outsiders $j \notin S$ best-respond to the anticipated decisions of insiders.

Members of $S \subseteq N$ maximize the sum of their profits

$$\Pi_S(p) = \sum_{i \in S} (p_i - \gamma_i)D_i(p)$$

(15)

with corresponding first-order conditions

$$\frac{\partial \Pi_S(p)}{\partial p_j} = D_j(p) + \sum_{i \in S} (p_i - \gamma_i) \frac{\partial D_i(p)}{\partial p_j}$$

for all $j \in S$. (16)

Analogous expressions hold if $j$ is a cartel outsider. It is sufficient for existence and uniqueness of a Nash equilibrium that a uniform increase of all prices and a unilateral increase of any single price respectively decrease individual and total demand. Formally, this requires $\sum_{j=1}^n \frac{\partial D_i}{\partial p_j} < 0$ and $\sum_{j=1}^n \frac{\partial D_j}{\partial p_i} < 0$, i.e., we will assume

$$\alpha_i := \frac{d_i}{\sum_{j \neq i} b_{ij}} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N.$$ (17)

This simplifies to $\alpha := \frac{d}{(n-1)b} > 1$ in the symmetric case.

Products are relatively good substitutes when $\alpha_i$ is small; then price increases by one firm significantly raise profits for other firms. The cartel internalizes this externality; the price $p_i$ set by cartel member $i$ will be the higher, the smaller $\alpha_i$.

For any given $S \subseteq N$, the (unique) Nash equilibrium $p^S$ summarizes equilibrium prices $p^S_i$ of all products $i \in N$ assuming firms in $S$ coordinate and the remaining ones act competitively. See, e.g., Davis and García (2009, ch. 8).

5.2. Symmetric Case

Own and cross price elasticities of demand for the considered differentiated goods vary even under symmetry ($\gamma_i = \gamma, a_i = a, d_i = d, b_{ij} = b$). We hence distinguish the “home” firm $h$ that produced the product for which a given customer suffered harm versus cartel members $j \neq h$ that were not part of the transaction, and consider

15See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6)
the per unit overcharge $\Delta p_h$ for good $h$. After solving for the Nash equilibria $p^S$ implied by $\{13\} - \{17\}$ for all $S \subseteq N$, the percentages of $\Delta p_h$ for which firms $h$ and $j$ are respectively responsible, $\rho^*_h$ and $\rho^*_j$, can be determined as an explicit function of the model parameters (see the Appendix):

**Proposition 3.** Suppose firms are symmetric in the linear market environment defined by equations $\{13\}$, $\{14\}$ and $\{17\}$. Let $h$ be the producer of the good for which overcharge damages are to be apportioned, and $j$ be one of $h$’s $n - 1$ competitors. The responsibility-based Shapley shares then are

$$
\rho^*_h = \frac{1}{n} + \frac{n - 1}{n} \sum_{s=2}^{n-1} \frac{(s - 1) \cdot (4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n - 1)^2 - 2(n + s - 3)(n - 1)\alpha + s(n - s) - 2(n - 1)}
$$

and $\rho^*_j = (1 - \rho^*_h)/(n - 1)$.

The common unit cost $\gamma$ and demand intercept $a$ have no effect on $h$’s share. It is determined only by ratio $\alpha = d/(n - 1)b$ of own and cross-price parameters. Proposition 2 directly implies $\rho^*_h > 1/n$.

If the degree of differentiation $\alpha$ is low, discipline by all cartel members is especially important for maintaining an overcharge. In the limit, each firm’s participation is essential and affects damage equally:

$$
\lim_{\alpha \to 1} \rho^*_h = \frac{1}{n} \text{ and } \lim_{\alpha \to 1} \rho^*_j = \frac{1}{n} \text{ for } j \neq h.
$$

(18)

Note that equal responsibility is implied for firms that produce perfect substitutes with identical technology by (SYM) also in general non-linear settings.

If, in contrast, firms produce highly differentiated goods, $\rho^*_h$ simplifies to

$$
\lim_{\alpha \to \infty} \rho^*_h = \frac{1}{n} + \frac{1}{n(n - 1)} \sum_{s=2}^{n-1} (s - 1) = \frac{1}{2}.
$$

(19)

One can check that $\rho^*_h$ is strictly increasing in $\alpha$. So seller $h$ is responsible for at most half of the compensation due for own overcharges:

**Corollary 1.** If $\nu(N)$ is the damage compensation obtained by a customer of firm $h \in N$, then apportionment by relative responsibility calls for firm $i$ to contribute

$$
\varphi_i(N, \nu) \in \begin{cases} 
\left( \frac{\nu(N)}{n}, \frac{\nu(N)}{2} \right) & \text{if } i = h, \\
\left( \frac{\nu(N)}{2(n-1)}, \frac{\nu(N)}{n} \right) & \text{if } i \neq h.
\end{cases}
$$

(20)
Figure 1: Share $\rho^*_h$ of overcharge damages on product $h$ attributed to its vendor
$(d = 2, b = 2/[(n - 1)\alpha])$

Figure 1 illustrates the behavior of $\varphi_h(N, v)$ for intermediate degrees of differentiation.

5.3. **Asymmetric Case**

The bounds in Corollary 1 provide guidance for mildly asymmetric markets by continuity. When firms are sufficiently heterogeneous, it is however possible that the producer of a good $h$ has lower responsibility for harm and will be assigned a smaller share of compensation than its competitors, i.e., $\varphi_h(N, v) < v(N)/n$. This happens when the cross-price effects involving firm $h$ are sufficiently smaller than those between other cartel members. We can, e.g., have three firms such that demands of firm 1 and 2 involve high mutual cross-price reactions $b_{12}$ and $b_{21}$, while there are only small linkages $b_{i3}$ and $b_{3i}$ with firm 3 ($i \neq 3$). Firm 3’s cartel participation contributes to the overcharges on $p_1$, $p_2$ and $p_3$ if all parameters are positive. But a significant increase of $p_3$ would have occurred even if firm 3 had not been part of the cartel and had just best-responded. This part of $\Delta p_3$ is caused by price increases on goods 1 and 2, which are mostly driven by shutting down competition between firms 1 and 2. The latter hence bear greater responsibility for $\Delta p_3$ than firm 3 itself.\(^{16}\)

This implies that asymmetry in cross-price effects does not come with useful bounds.

By contrast, asymmetry in demand parameters $a_i$ or costs $\gamma_i$ can be dealt with, though calculations become tedious. For instance, supposing $\gamma = 0$ and that firm-

---

\(^{16}\)For instance, assume $a_i = 10, d_i = 3, \gamma_i = 0$ for $i \in \{1, 2, 3\}$, $b_{12} = b_{21} = 2, b_{13} = b_{23} = b_{31} = b_{32} = 0.5$ and consider $v(N) = \Delta p_3$. Then Shapley shares evaluate to $\rho^*_1 = \rho^*_2 \approx 35.1\% > \rho^*_3 \approx 29.8\%$. 

specific demand intercepts \(a_i\) are the only asymmetry, we have:

**Proposition 4.** Suppose firms are symmetric except for the demand intercepts \(a_1, \ldots, a_n\) in the linear market environment defined by equations (13), (14) and (17) with \(\gamma = 0\). Firm \(h\)’s Shapely share then is

\[
\rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} \left( 6\alpha(n-1) + (s+4-n) + \left( 4\alpha^2(n-1)^2 + \tau_s \right) \frac{a^n}{a_h} \right) (\alpha - 1)(2\alpha - 1)
\]

with \(\bar{a}_{-h} := \sum_{i \neq h} a_i/(n-1)\), \(\tau_s := (n-s-2)\) and \(\eta_s := s(n-s) - 2(n-1)\).

Ratio \(\bar{a}_{-h}/a_h\) relates the market sizes of firm \(h\) and its competitors: a large ratio means firm \(h\) is comparatively small, a ratio close to zero that \(h\)’s market is big. It can be checked that \(\rho_h^*\) is maximized when (i) firms produce highly differentiated goods, i.e., \(\alpha \to \infty\), and (ii) when firm \(h\)’s market size is massive, that is, \(\lim a_h/\bar{a}_{-h} \to 0\). Then firm \(h\) is responsible for half of the damage to its customers. Contrary, \(\rho_h^*\) is minimized when goods are close substitutes and firm \(h\)’s market size is small. Then, firm \(h\) is responsible for around \(1/n\) of the customer’s damage.

So if only demand parameters \(a_i\) vary and we focus on firm \(h\)’s share then the same bounds obtain as in Corollary 1. Things differ for firms \(j \neq h\), however. The key determinant of \(j\)’s share in \(\Delta p_h\) is \(\bar{a}_{-h,j} := \sum_{i \in N \setminus \{h,j\}} a_i/(n-2)\), the average demand intercept of firms other than \(h\) and \(j\). If \(a_j \gg \bar{a}_{-h,j}\) then \(j\) is the only large competitor of \(h\) and both end up splitting \(\Delta p_h\) about 50:50. If conversely the market size of \(j\) is negligible compared to that of \(h\)’s other competitors (i.e., \(a_j \ll \bar{a}_{-h,j}\)) then \(j\) is approximately a null player. This can be summarized by

**Corollary 2.** Suppose firms are symmetric except for the demand intercepts \(a_1, \ldots, a_n\) in the linear market environment defined by equations (13), (14) and (17) with \(\gamma = 0\). If \(v(N)\) reflects damages to a customer of firm \(h \in N\), then

\[
\varphi_i(N, v) \in \begin{cases} 
\left( \frac{v(N)}{n}, \frac{v(N)}{2} \right) & \text{if } i = h, \\
\left( 0, \frac{v(N)}{2} \right) & \text{if } i \neq h.
\end{cases}
\]  

(21)

The same bounds apply to firms which are symmetric in all but technology. The case is illustrated in Figure 2. It considers responsibility for per unit overcharges of \(\Delta p_1\) and \(\Delta p_3\) for two low-cost firms (1 and 2) and two high-cost producers (3 and 4) with common parameters \(a = 10\), \(d = 2\), and \(b = \frac{1}{3a}\). No matter whether the selling
firm has (a) low costs $\gamma_1 = \gamma_2 = 1$ or (b) high costs $\gamma_3 = \gamma_4 = 5$, it bears economic responsibility for between 25% and 50% of overcharges on its product, and always the greatest share.

5.4. Comparison to Heuristic Apportioning

A reliable apportioning heuristic could save the effort of above calculations. Perhaps market shares, which are comparatively easy to obtain, are a good proxy for whose cartel participation is responsible for which proportion of damages, at least under some identifiable circumstances? If yes, should we use sales or revenues? From the cartel or competitive regime? Or perhaps better use a profit measure?

Before addressing these questions in numerical simulations, let us review a specific symmetric situation depicted in Figure 1. Namely, for four firms with $a = 10$, $d = 2$, $\gamma = 1$, $b = 1/3$ (so $\alpha = 2$), the competitive Bertrand equilibrium involves prices $p_i^\circ = 4$ and per firm output $D_i(4) = 6$, revenues $4 \cdot 6 = 24$, and profits $3 \cdot 6 = 18$ for each firm. If they form a cartel $N = \{1, 2, 3, 4\}$ and maximize industry profit, prices rise to $p_i^N = 5.5$ and individual sales fall to $D_i(4) = 4.5$. Ignoring potential side payments, individual revenues of 24.75 generate profits of 20.25 in the cartel regime. Customers of any firm suffer an overcharge damage $\Delta p_i = 1.5$ per unit, resulting in total overcharge damages of $D = 4 \cdot 4.5 \cdot \Delta p_i = 27$.

In this symmetric case, damage shares $\rho$ derived from firms’ sales, revenues, or
profits all coincide with an equal per head attribution \( \rho^0 = (25\%, 25\%, 25\%, 25\%) \). So, if all cartel victims receive full compensation, each firm would need to contribute \( H_i^{\rho^0} := \mathcal{D}/n = 27/4 = 6.75 \) in total. The relative responsibilities for the damage of an individual customer that bought, say, one unit of product 1 differ from \( \rho^0 \). Namely, the Shapley shares in \( \Delta p_1 \) are \( \rho^* \approx (38.9\%, 20.4\%, 20.4\%, 20.4\%) \), reflecting that cartel participation of firm 1 had the greatest influence on price \( p_1 \).

So first assume only this individual customer claims compensation for harm of \( \Delta p_1 \). Then under heuristic \( \rho^0 \), firm 1 would under-contribute 0.21 (or 14\% of the per-unit damage) and firms 2 – 4 would each over-contribute 0.07 (or 4.7\% of the per-unit damage). Alternatively assume all customers claim compensation. Then the higher product-specific Shapley share of 38.9\% for harm caused by own overcharges and a lower share of 20.4\% in other cartel members’ overcharges cancel out: total payments that result from first apportioning total damages \( \mathcal{D} \) at the product level à la Shapley, and then aggregating these for all sales, amounts to a total Shapley payment \( \Phi_i = 6.75 = H_i^{\rho^0} \) per firm. Not surprisingly, a heuristic division of compensation payments by heads, or any measure of aggregate market shares, is perfectly in line with relative responsibilities if firms are symmetric and every harmed customer sues the cartel and wins compensation.

Let us keep the latter assumption but now move to asymmetric configurations. We compare aggregate payments under Shapley apportionment (as implied by respective shares \( \rho^*_i \) in each \( \Delta p_j \), determined at the product level) and several heuristic alternatives. So total overcharge damage

\[
\mathcal{D} = \sum_{i \in N} q_i^C \cdot \Delta p_i
\]  

will either be allocated according to the Shapley value \( \varphi(N, v^j) \) for each individual product \( j \) with aggregate Shapley payments for firm \( i \) of

\[
\Phi_i = \sum_{j \in N} \varphi_i(N, v^j) = \sum_{j \in N} q_j^C \cdot \Delta p_j \cdot \rho^*_i(N, v^j)
\]  

or \( \mathcal{D} \) will be allocated according to some market shares \( \rho \). Absolute values of over and under-payments implied by \( \rho \) relative to \( \Phi_i \) are summed across firms and normalized
by $D$ to give an index of aggregate mis-allocation of damages

$$M^\rho := \sum_{i \in N} | \Phi_i - H_i^\rho | / D$$

(24)

where $H_i^\rho$ denotes aggregate payments by firm $i$ according to heuristic shares $\rho$. $M^\rho$ is proportional to the expected mis-allocation of compensation for a unit purchase by a randomly drawn customer, for a customer who made purchases from all firms in proportion to their cartel sales, or when all customers go after the cartel with identical positive probability.\(^{17}\)

In Figure 3, we start from the baseline scenario $a = 10$, $\gamma = 1$, $d = 2$, $b = d/(3\alpha)$ and break symmetry for one parameter at a time. The two top panels consider heterogeneity in firm-specific market sizes $a_i$. Panel (a) involves two large and two small firms; in panel (b) all firms differ. An equal per head allocation $\rho^0$ non-surprisingly performs well when differentiation is very low. It soon loses out to allocating damages in proportion to *market shares based on competitive sales* $\rho^4$ and to *market shares based on cartel sales* $\rho^2$. Market shares determined by *cartel revenues* $\rho^1$ or *competitive revenues* $\rho^3$ produce high mis-allocations at all levels of differentiation. Only apportioning in proportion to *cartel profits* $\rho^5$ is worse.

Panels (c) and (d) assume an intermediate and a big cost asymmetry between firms 1 and 2 vs. firms 3 and 4. The deviations from the Shapley payments, aggregated for each firm across all four overcharges, are significantly higher for the big asymmetry in (d) than the smaller one in (c).\(^{18}\) Revenue-based market shares $\rho^1$ or $\rho^3$ and sales-based competitive market shares $\rho^4$ all perform well.

Panel (e) assumes firms 3 and 4 face bigger own-price elasticities than firms 1 and 2. Market shares based on cartel sales or competitive revenues are close to the Shapley value, as far as aggregate payments to all victims are concerned. The final panel (f) assumes heterogeneity in cross-price effects: firms 1 and 2 face a fixed cross-price parameter of $1/4$, competition between firms 3 and 4 is more intense by some factor $\beta$. Somewhat unexpectedly, after investigating five environments in which its ranking

\(^{17}\)The latter may be a plausible a priori assumption. Cartel members might then pool their obligations in a trust to save transaction costs. Symmetry would call for equal shares in the trust. The simulations show, however, that no simple market or profit share rule applies to funding the trust under asymmetry.

\(^{18}\)The kink that is visible in panel (c) for $\rho^3$ – or $\rho^2$ in (e) – results from cancellation of product-specific deviations at the firm level when these initially have opposite signs but switch to same sign.
Figure 3: Mis-allocation $M^\rho$ by different heuristics considering $i = 1, 2$ and $j = 3, 4$

was consistently low, apportioning by cartel profits $\rho^5$ comes closest to representing a short-cut to the exact Shapley payments.

Overall, there is no heuristic which always outperforms the others in terms of approximating Shapley apportionments. Those based on market shares – preferably sales-based for heterogeneity in $a_i$, otherwise revenue-based – tend to score better than a profits-based division; but panel (f) provides an exception to the rule. Generally, when firms produce close substitutes and hence ratio $\alpha = d/[((n-1)b]$ is close to 1, an
equal division by heads is close to the Shapley benchmark. The good performance, however, comes with the warning that Figure 3 considers aggregate mis-allocation. If few customers only of specific firms should seek compensation for their harm, as in the stylized cement cartel of Section 2, the picture looks significantly worse: equal division tends to over and understate relative responsibilities.

6. Modification for Ringleaders and Leniency Applicants

Regarding the conduct of a firm, we have so far discriminated only between being a member of the cartel or competing with it. There are at least two cases where the specific roles of firms require more attention in applications.

First, some members may have acted as ringleaders of the cartel and therefore bear greater responsibility for inflicted harm. The legal literature emphasizes the role of leader in an infringement, i.e., in organizing the operation of an existing cartel, and of instigator of an infringement by particularly furthering the establishment or enlargement of a cartel (e.g., EC Case T-15/02 (14)). Empirical analysis has shown how a cartel’s success – and thus harm caused – is sensitive to its organizational characteristics (see, e.g., Davies and De 2013). Attributions based only on a model of cost and demand likely understate a ringleader’s due share.

Second, former offenders that cooperate with the authorities often enjoy reduced obligations. More than 60 jurisdictions have leniency programs aimed at deterring cartel operations. Their applicants are first to admit wrong and this can attract the major damage cases later; liability exemptions partly offset this disadvantage and raise the attractiveness of coming clean, just like immunity from criminal charges and fines. This is appreciated in the EU Damages Directive. Article 11(4) specifies “... an immunity recipient is jointly and severally liable ... (a) to its direct or indirect purchasers or providers; and (b) to other parties only where full compensation cannot be obtained from the other undertakings that were involved in the same infringement ...”

Such a restricted role in compensating victims can be combined with otherwise apportioning compensation by relative responsibility. The key step is to modify condition (SYM). Kalai and Samet (1987) have relaxed it to requiring merely that $v(S)$ is distributed in a consistent way whenever $S$ is a “partnership”; that is, when members
of $S$ make contributions to coalitions of other players only if they are together, i.e., $v(R \cup T) = v(R)$ for any strict subset $T \subset S$ and any $R \subseteq N \setminus S$. The members of partnerships have identical marginal contributions and responsibilities. But if surplus or, here, damages shall be split asymmetrically for reasons not reflected by $v$, at least there should be no inconsistency between a two-step allocation – first to partnerships in their entirety, then internally – or one directly to individual members.

Including this requirement, while dropping symmetry, leads to the use of weighted Shapley values, which were introduced by Shapley (1953b). They impose a non-negative vector $\omega = (\omega_1, \ldots, \omega_n)$ of weights and modify Shapley value $\varphi$ to some $\varphi^\omega$ such that the shares of players $i \in T$ in any game $(N, u_T)$ where $u_T(S) = 1$ if $T \subseteq S$, and 0 otherwise, are proportional to their weights, i.e., $\varphi^\omega_i(N, u_T) = \omega_i / \sum_{j \in T} \omega_j$ if $i \in T$, and 0 otherwise.

Leniency as in Article 11(4) can therewith be accommodated as follows: (i) use $\varphi^\omega$ with $\omega = (1, \ldots, 1)$, i.e., the standard Shapley value $\varphi$, for allocating any overcharge damages $(N, v')$ for goods produced by leniency recipient $l \in N$; (ii) by contrast use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega} = (1, \ldots, 1, 0, 1, \ldots, 1)$ where $\tilde{\omega}_l = 0$ when overcharges by $l$’s competitors are concerned. This can be generalized to the case of multiple immunity recipients $L \subseteq N$: use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_i = 1$ if $i \notin L$ or overcharges $\Delta p_i$ are concerned, and $\tilde{\omega}_i = 0$ otherwise.

The same kind of modification can account for elevated responsibility of ringleaders: use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_r = \kappa > 1$ for any ringleader $r \in R \subset N$, and $\tilde{\omega}_i = 1$ for conventional cartel members $i \notin R$. The appropriate value for $\kappa$ – or possibly different levels $\kappa_1 \geq \kappa_2 \geq \ldots > 1$ when there existed multiple ringleaders – depends on how pronounced the respective leading or instigating role was. This is outside the scope of our setup but criminal rulings and fines, which typically precede civil actions against cartels, should provide an appropriate reference point.

### 7. Concluding Remarks

The key conclusion of our analysis is that a well-founded way of attributing relative responsibility for cartel damages and dividing compensation exists: start out with equal shares; then these shares are to be corrected in a transparent way for greater

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19 We simplify here. Potential divisions by zero are avoided by actually working with a lexicographic weight system, consisting of strictly positive weights and an ordered partition of $N$ into classes $N_1, \ldots, N_m$. See [Kalai and Samet (1987)] and [Nowak and Radzik (1995)]. Moreover, convexity of $v$ is required to guarantee monotonicity of $\varphi^{\tilde{\omega}}(N, v)$ in $\omega$ when $|L| > 1$ (cf. [Monderer et al. 1992]).
or smaller-than-average effects on overcharges. Doing so, i.e., application of the Shapley value, follows from translating the norm that contributions reflect relative responsibility for harm into the marginality property of Young (1985) and other natural requirements. We have investigated what the Shapley value’s use entails for cartel damages and how the identified relative responsibilities relate to ad hoc ascriptions based on market shares.

We should end by acknowledging that, of course, merits of Shapley apportionings depend on the quality of their input, i.e., the description of counterfactual damage scenarios by characteristic function $v$. Reaching a reasonable level of agreement on it among former cartel members, either in private or before court, is bound to be difficult. But it is unlikely to be an unsurmountable hurdle. Estimating $v$ essentially means calibrating and simulating a structural model of the relevant market. The pertinent trade-offs between tractability and temporal, spatial, or other details are known, e.g., from merger simulation and also arise when quantifying a cartel victim’s harm in the first place.
A.  Appendix – Proofs of Propositions 3 and 4

Proof of Proposition 3  Suppose \( n \geq 3 \) firms are symmetric in the linear market environment defined by equations (13), (14) and (17). The cartel price then evaluates to

\[
p^c := p^N = \left( \frac{a}{d - (n-1)b} + \gamma \right)/2 \tag{A.1}
\]

for each differentiated product \( i \in N \). Corresponding competitive Bertrand prices are

\[
p^B := p^B_i = \frac{a + dy}{2d - (n-1)b} \text{ for all } i \in N. \tag{A.2}
\]

This implies per unit cartel overcharges of

\[
v(N) = \Delta p = p^c - p^B = \frac{a/d - \gamma(1 - \frac{1}{a})}{4a - 6 + 2/a} \text{ with } \alpha = \frac{d}{n-1} > 1 \tag{A.3}
\]

for each product \( i \in N \). They are homogeneous of degree one in \((a, \gamma)\) and strictly decreasing in differentiation parameter \( \alpha \) as well as in unit costs \( \gamma \).

If there is a partial cartel \( S \) of size \( s = 2, \ldots, n - 1 \), equilibrium prices are

\[
p^S_i = \begin{cases} 
    \frac{a(2d + b) + \gamma(2d^2 + bd(3 - 2s) + b^2(ns - n - s^2 + 1))}{4d^2 - 2(n + s - 3)bd + b^2\eta_s} & \text{if } i \in S, \\
    \frac{a(2d - sb + 2b) + \gamma(2d^2 - bd(s - 2) - b^2(s^2 - s))}{4d^2 - 2(n + s - 3)bd + b^2\eta_s} & \text{if } i \notin S
\end{cases} \tag{A.4}
\]

with \( \eta_s = s(n - s) - 2(n - 1) \geq -(n - 1) \).

Comparing the price \( p^S_h \) of the home product \( h \in N \) paid by a suing customer in case that the respective producer \( h \) is part of a cartel with \( s \) members, i.e., for \( h \in S \), to the respective price \( p^S_h \) if \( h \) is not, i.e., for \( h \notin S \), yields\(^{21}\)

\[
\varphi^h(s) - \varphi^k(s) = p^h(s) - p^k(s) = \frac{b(s - 1)(a + (n - 1)b\gamma - d\gamma)}{4d^2 - 2(n + s - 3)bd + b^2\eta_s} > 0. \tag{A.5}
\]

Inserting this into eq. (5) gives the Shapley apportionment \( \varphi(N, \Delta) \) in absolute terms. Dividing the latter by \( v(N) = \Delta p \) yields \( h \)'s claimed Shapley share \( p_h^\varphi \). \( \Box \)

\(^{20}\)The detailed algebraic manipulations omitted here are available upon request.

\(^{21}\)The three factors in the numerator are strictly positive. Invoking \( s \leq n - 1 \) and \( \eta_s \geq -(n - 1) \) first, and \( d > (n - 1)b \) next, the denominator can be bounded below by \( 2d[2d - 2(n - 2)b] - b^2(n - 1) > 2d[2(n - 1)b - 2(n - 2)b] - bd = 3bd > 0. \) Hence \( p^h(s) - p^k(s) > 0. \)
Proof of Proposition 4. Suppose $n \geq 3$ firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (13), (14) and (17) with $\gamma = 0$. Then, firm $h$’s cartel price is

$$p^c_h = \frac{a_h d - (n - 2)a_h b + b(n - 1)\bar{a}_{-h}}{2(b + d)(d + b - bn)}$$

(A.6)

with $\bar{a}_{-h} = \sum_{l=1, l \neq h}^n a_l/(n - 1)$. Firm $h$’s corresponding competitive price is

$$p^B_h = \frac{2a_h d - (n - 2)a_h b + b(n - 1)\bar{a}_{-h}}{(2d + b)(2d + b - bn)}.$$  

(A.7)

A customer’s per unit cartel overcharge by the product $h$ then is

$$\Delta p_h = p^c_h - p^B_h = \frac{b(n - 1)[b(3d + 2b - bn)a_h + (2d^2 + b^2n - b^2)\bar{a}_{-h}]}{2(d + b)(2d + b)(d + b - bn)(2d + b - bn)}.$$  

(A.8)

It rises in the saturation level $a_h$ of firm $h$’s demand as well as in the average saturation quantity $\bar{a}_{-h}$ of firms $l \neq h$. The corresponding Shapley value of firm $h$ in apportioning $v(N) = \Delta p_h$ is

$$\varphi_h = \frac{\Delta p_h}{n} + \frac{1}{n} \sum_{s=2}^{n-1} \frac{b(s - 1)[b(6d + b(s + 4 - n))a_h + (4d^2 + \tau_s b^2)\bar{a}_{-h}]}{2(d + b)(2d + b)(4d^2 - (2n - 6 + 2s)db + \eta_s b^2)}$$

(A.9)

with $\tau_s := (n - s - 2)$ and $\eta_s := s(n - s) - 2(n - 1)$. Dividing $\varphi_h$ by $\Delta p_h$ and substituting $\alpha = d/(b(n-1))$ gives

$$\rho^*_h = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} \frac{(s - 1)[6\alpha(n - 1) + (s + 4 - n) + (4\alpha^2(n - 1)^2 + \tau_s)\frac{\bar{a}_{-h}}{a_h}](\alpha - 1)(2\alpha - 1)}{(4\alpha^2(n - 1) - (2n - 6 + 2s)\alpha + \frac{\eta_s}{n-1})(3\alpha + \frac{2-n}{n-1} + (2\alpha^2(n - 1) + 1)\frac{\bar{a}_{-h}}{a_h})}$$

(A.10)

as claimed. □
References


