Responsibility-based allocation of cartel damages

Stefan Napel
Dept. of Economics, University of Bayreuth
stefan.napel@uni-bayreuth.de

Dominik Welter
Dept. of Economics, University of Bayreuth
dominik.welter@uni-bayreuth.de

September 6, 2017

Abstract

Cartel members are liable jointly and severally: one may be forced to compensate victims on behalf of all. EU law stipulates that others must pay internal redress in proportion to "relative responsibility for the harm". We operationalize this responsibility by evaluating counterfactual damages had one or more cartelists rejected collaboration. Formalizing causality and other requirements calls for aggregation of counterfactual overcharges via the Shapley value. We characterize damage allocations for important benchmarks, propose approximations and evaluate ad hoc suggestions based on market shares or profits. A new decomposition of the Shapley value allows to establish non-trivial bounds on payment obligations.

Keywords: cartel damages; damage allocation; Shapley value; joint liability; relative responsibility; rule of contribution

JEL codes: L40; L13; D04; D43

We are grateful to Matthew Braham, Michael Kramm, Sascha Kurz, Nicola Maaser and Gunnar Oldehaver for helpful comments on earlier drafts, and to Nikolaus Bosch and Niels Frank for instructive discussions. We have also benefited from comments by audiences in Augsburg, Bayreuth, Paderborn, at the 5th GAMES World Congress, the 2016 EARIE Annual Conference, the 2016 Annual Meeting of the Verein für Socialpolitik, the 2017 Spring Meeting of Young Economists, the 4th Leipzig Workshop on Cooperative Game Theory in Business Practice, the 2017 GTM Conference in St. Petersburg and the 2017 Meeting of the Theoretical Committee of the Verein für Socialpolitik. The usual caveat applies.

1. Introduction

Cartels are illegal because they generally harm customers and suppliers of the involved firms and possibly others. Victims have for long had a right to compensation but the pertinent legal hurdles used to be high. Annually up to 23.3 billion euro of damages have remained unclaimed from EU-wide cartels according to the European Commission (SWD/2013/203/Final recital 67). This is about to change after 2014's *Directive on Antitrust Damages Actions* (2014/104/EU) has fully been implemented into national law. The position of plaintiffs is improving and some big cases are already pending – e.g., against the air cargo, elevator or truck cartels.

Two provisions for the compensation of cartel victims, which the Directive sought to harmonize in Europe, motivate this paper. First, the members of a cartel are liable *jointly and severally*. An injured party can sue *any* cartel member for the full amount of its damages; if courts confirm the claim, the defendant must compensate the plaintiff on behalf of the entire cartel. This is regardless of whether the plaintiff made its purchases from the sued firm or other ones. Similar provisions also apply in Australia, Japan and the US.

Second, the sued cartel member is entitled to *internal redress*. That is, in Europe, all co-infringers are obliged to contribute to the compensation which the unlucky one of them had to pay out.[1] The billion euro question is: how much? In other words, how are damages of a cartel to be allocated among its potentially heterogeneous members?

The goal of this paper is to operationalize the vague redress norm established by the European Union in its Directive 2014/104/EU in an economically sensible way.[2] According to the Directive, cartelists' internal obligations in compensating any external claimant must reflect *"... their relative responsibility for the harm caused by the infringement of competition law"* (Article 11(5)). The Directive is not specific on how "relative responsibility" should be quantified. It leaves doors wide open by stating

[1] In federal US antitrust cases, defendants are *not* entitled to compensation from other defendants. See Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981, Easterbrook et al. (1980) and Baker (2004). In Japan, a defendant may force other cartel members to join the initial lawsuit or demand reimbursement for excess payments later. In Australia, the question of contribution is open.

[2] The issue of how alternative norms (cf. fn. 1) affect incentives for cartel formation, whistleblowing, settlements, etc. is here left aside. See, for instance, Landes and Posner (1980), Polinsky and Shavell (1981), Goetz et al. (2006) or Hviid and Medvedev (2010).

that ". . . determination of that share [of compensation] as the relative responsibility of a given infringer, and the relevant criteria such as turnover, market share, or role in the cartel, is a matter for the applicable national law, while respecting the principles of effectiveness and equivalence" (recital 37).

Different quantification strategies have been used in the literature in order to shed light on how cartel damages accrue *to different injured parties* along the value chain. See, e.g., Baker and Rubinfeld (1999), Nieberding (2006), Friederiszick and Röller (2010), Inderst et al. (2013) and the European Commission (SWD/2013/205). We take individual damages as given and seek to disentangle contributions *by different injurers*.

Ad hoc allocations of harm suffered by a victim among the infringers – like a division by their sales or revenues – may yield a good approximation of causal contributions and responsibility by chance. In general, however, a systematic approach is warranted. A key reason is that asymmetry between the cartel's members can translate very differently into asymmetric market shares, relative profits, etc. Picking one ad hoc criterion rather than the other involves undue arbitrariness in view of Article 11's explicit reference to relative responsibility. One can and should do better, not just because big numbers are involved.

A systematic investigation involves a reflection on properties that suitable allocations should satisfy. For instance, in order to account for the Directive's responsibility criterion, a firm should contribute to compensating a given customer if and only if this customer's damages would have been lower had the firm refused to participate in the cartel. And if cartel membership of two firms had identical effects on a particular harm, then both should contribute the same to its remedy. The damage allocation should not depend on the currency of account nor on whether quantities refer to tons, kilograms, etc. We will formalize these and other desiderata. Classical results by Shapley (1953a) and Young (1985) then imply that the *Shapley value* of an appropriately defined game of transferable utility is the unique best way to split external compensation obligations among the offenders.

The Shapley value has long been established as a tool for allocating costs and profits in joint ventures[3] and the corresponding rationale applies to the division of

[3]See Shubik (1962), Roth and Verrecchia (1979), and Littlechild and Thompson (1977) for pioneering contributions, Young (1994) for a comparison to other methods of cost allocation, and Moretti and Patrone (2008) for an overview of more recent contributions.

compensation claims at least as well. This has recently been acknowledged by Dehez and Ferey (2013, 2016) and also Huettner and Karos (2017) for jointly and severally liable tortfeasers in *sequential liability games*. Such games model incremental harm caused by chronologically ordered acts of negligence. They are convex, hence have a non-empty core; the Shapley value then serves to select from the core. Use of the Shapley value for the allocation of cartel damages, which arise simultaneously, has been proposed and explained in qualitative terms by Schwalbe (2013) and Napel and Oldehaver (2015) to law audiences. This paper is the first to analyze the quantitative aspects of what its use would entail and how individual payments relate to industry structure.

We concentrate on *overcharge damages* caused by hardcore cartels that fixed quantities, sales areas or prices of differentiated goods. For the time being, this leaves aside other kinds of antitrust infringements and harm associated with deadweight losses, including forgone profits of suppliers of intermediary or complement goods (and even general equilibrium effects; see Eger and Weise 2015). Though compensation for the latter types of damage has so far played a negligible role, nothing in principle would preclude a generalization of the suggested approach. Changes in prices could, e.g., be replaced by those in downstream profits or indirect utility.

A Shapley damage allocation divides cartel markups over competitive product prices – hence overcharge damages when multiplied by sales – according to individual abilities of the participating firms to influence prices. We analyze how the resulting damage shares are linked to an industry's demand and cost parameters. We derive bounds on a given firm's responsibility for own overcharges and those by other cartel participants. For instance, responsibility for the former is always higher than for the latter if goods are differentiated symmetrically (Section 5.2). We also compare Shapley allocations to ad hoc ones based on market shares, profits or an equal per head assignment. Such divisions have been suggested by law practitioners and we propose binary approximations of damages as a more robust, potentially very useful tool for reaching settlements. At a technical level, we identify a new decomposition of the Shapley value. This could be of independent general interest. It combines the key features of decompositions discovered by Kleinberg and Weiss (1985) and Rothblum (1988): start with a flat per-head allocation as the baseline; then correct this according to whether damages in cartel scenarios with a fixed number of infringers are greater with or without the firm in question. The decomposition mathematically helps to obtain asymptotic results but may also facilitate educated guesses about relative responsibility when data are scarce.

The remainder of the paper is structured as follows. We start with a more detailed illustration of the problem in Section 2. Then, in Section 3, we discuss several intuitive requirements for responsibility-based damage allocations and conclude that the Shapley value should be invoked. We decompose it into two components in Section 4 and study important benchmark situations in Section 5. Section 6 specializes the analysis to linear market environments. For these, heuristic allocation rules are compared to the Shapley benchmark and to binary approximations in Section 7. Section 8 discusses extensions to leniency provisions. We conclude in Section 9.

2. Illustration

A cartel member i having responsibility[4] for damages of a given claimant k requires that k's damages are *causally linked* to i's cartel membership, i.e., their scale, scope or distribution would have been different without i's illegal action. Identifying the causal links between anticompetitive conduct and harm is generally fraught with difficulty (see, e.g., Lianos 2015). What makes economic analysis of responsibility for cartel damages especially interesting is that even symmetric cost and demand structures may generate asymmetric links to harm suffered by a specific victim. Namely, price effects of individual cartel membership in a but-for-test differ across cartel participants as long as own-price and cross-price elasticities of the respective demands differ.

For illustration, consider n otherwise identical firms on a Salop circle. Think of cement plants that are equally spaced on the shores of an unshippable lake. They sold their cement at inflated prices to local construction companies around the lake; their cartel was busted and a customer of firm i sues. Firm i's and another firm j's relative responsibilities for this customer's damages are tied to the counterfactual

[4]The canonical conception of legal and moral responsibility for damages includes three parts (see Feinberg 1970, p. 195f, for a classical discussion). Firstly, the defendant was at fault in acting. This clearly applies if, for instance, firm i's manager illegally coordinated its commodity production with competitors over dinner, violating antitrust laws. Secondly, the faulty act caused the harm: these conversations resulted in a price increase for the customer. Finally, the faulty aspects of the act were relevant to its causal connection to the harm: illegal coordination by the managers – not, perhaps, just the reaction of commodity speculators to observing them dine together – caused the increase.

price that the customer would have paid had i or respectively j refused to participate. Unless transportation costs are zero, and thus all products perfect substitutes, cartel membership of the northernmost vendor has smaller effect on overcharges faced by customers in the south than does membership of southern vendors, and vice versa (see, e.g., Levy and Reitzes 1992). The closer two firms are located and hence the more intensely they would have competed in the absence of the cartel, the greater the price effect of their collusion. So counterfactual prices that the suing customer would have paid if i or if j had not joined the cartel, but just best-responded to its illegal practices, vary according to i's and j's locations. Differential effects of cartel membership imply differential responsibilities for a specific customer's damage; hence different obligations for compensation.

Of course, a symmetric market structure entails that obligations which j and i respectively have in compensating each others' customers are the same. Mutual claims cancel out if all constructors sue, or if equal measures of them do everywhere. However, they do *not* cancel in almost all other situations – e.g., if just some construction companies in the south go to court. A general analysis hence requires that responsibility be allocated to the cartel members for the price overcharge on each single good in the cartel's portfolio. Such product-oriented analysis is all the more important if one wants to incorporate asymmetric market sizes, cross-price effects or cost structures – as we do.[5]

Ad hoc approaches can lead to very different damage allocations under heterogeneity. Consider an example with three producers of differentiated goods. Their respective costs be $C_1(q_1) = 30q_1$, $C_2(q_2) = 20q_2$ and $C_3(q_3) = 10q_3$ and they compete à la Bertrand. Let demands be $D_1(p) = 100 - 4p_1 + 3p_2 + 0.4p_3$, $D_2(p) = 100 - 4p_2 + 3p_1 + 0.4p_3$ and $D_3(p) = 150 - 3p_3 + 0.4(p_1 + p_2)$. So products 1 and 2 constitute closer substitutes than product 3.

The individual maximization of profits yields Bertrand equilibrium prices $p^B = (44.7; 41.0; 35.7)$ rounded to one decimal place. The corresponding equilibrium outputs are $q^B = (58.7; 84.2; 77.1)$, with revenues of $R^B = (2622.5; 3453.7; 2755.1)$ and profits of $\Pi^B = (861.5; 1770.6; 1983.7)$. If the firms form a cartel and maximize total industry

[5]We will assume equal roles in organizing the cartel for most part however. If a firm was the chief instigator or leader of the cartel, an elevated responsibility can be accommodated in analogy to the special treatment of immunity recipients. See Section 8.

profit, prices rise to $p^C = (82.2; 77.2; 47.9)$ while quantities fall to $q^C = (22; 57; 70)$. Ignoring potential side payments, individual profits in the cartelized market are $\Pi^C = (1147.6; 3258.4; 2653.7)$ from revenues of $R^C = (1807.6; 4398.4; 3353.7)$.

Profits increase but consumers suffer two types of damage. The first is the visible loss due to higher prices (*damnum emergens*): each unit of good i which was purchased involved an *overcharge damage* of $\Delta p_i = p_i^C - p_i^B$. Here, overcharges are $\Delta p = (37.5; 36.1; 12.2)$ per unit, resulting in product-specific total overcharge damages of $D = q^C \cdot \Delta p = (824.8; 2059.1; 853.7)$.

Further harm relates to deadweight losses: customers who would have made (additional) purchases, and thus would have enjoyed surplus had prices only been p^B, failed to do so. This is acknowledged as *lucrum cessans* in the legal literature but we are unaware of cases in which compensation for it has successfully been claimed. We will disregard those damages in what follows.[6]

Suppose now that a customer k who purchased $x_1^k = 10$ units from firm 1 at p_1^C, and nothing else, sues. The customer may take firm 2 to court because the plaintiff is free to choose; perhaps k perceives the best odds for enforcing his claim against the profit champion. Let k be granted compensation for his overcharges $O^k = 375$. Firm 2 must then pay out O^k but is entitled to reclaim some of this from firms 1 and 3.

The customer would have been less harmed had *any* one or two firms refused to participate in the cartel. Presuming equal roles in the cartel's practical operations, a default suggestion would be to divide O^k simply on an equal per head basis, i.e., to multiply O^k by $\rho^0 = (33.3\%; 33.3\%; 33.3\%)$.

Firm 3 would be right to reject this: its participation did not have the same effect on price p_1 as participation of firms 1 and 2. The latter are closer competitors; their anti-competitive conduct had a bigger effect on 1's customers, and hence greater part of k's damage is linked to misconduct by these firms. A division by cartel revenue shares, $\rho^1 = (18.9\%; 46.0\%; 35.1\%)$ does not reflect this either, nor does one by cartel sales $\rho^2 = (14.8\%; 38.3\%; 47.0\%)$. They would assign firm 1 smaller responsibility than 2 and 3, even though 1's participation affected p_1 more than that by firm 3. The same issue

[6]See the European Commission's Staff Working Document 205, 2013, on conceptual and practical difficulties in quantifying damages, and Basso and Ross (2010) on using price overcharges as a proxy for purchasers' harm. In the example here, about three quarters of total losses to customers are overcharges; one quarter derive from quantity effects.

applies to allocations based on $\rho^3 = (29.7\%; 39.1\%; 31.2\%)$ or $\rho^4 = (26.7\%; 38.4\%; 35.1\%)$ which are analogous market share figures in the competitive pricing regime. Certainly the firms' cartel or competitive profit shares $\rho^5 = (16.3\%; 46.2\%; 37.6\%)$ and $\rho^6 = (18.7\%; 38.3\%; 43.0\%)$ are not the right benchmark either; they would further raise 3's contribution relative to ρ^0 rather than lower it.

Normalization of the firms' relative profit increases of $\Delta\Pi = (33.2\%; 84.0\%; 33.8\%)$ compared to the competitive regime would yield relative 'cartel benefit shares' of $\rho^6 = (22.0\%; 55.6\%; 22.4\%)$. However, positive fixed costs would imply different shares – for instance, fixed costs of $K = (300, 100, 500)$ would yield $\hat{\rho}^6 = (27.5\%, 48.1\%, 24.4\%)$ – even though market outcomes and arguably responsibilities for damages are unchanged. More generally, profits-based responsibility ascription is flawed because benefiting from someone's harm is conceptually distinct from causing it. An unwitting cartel outsider may have been the one to profit the most from increased prices (as an inadvertent free-rider). Such an outsider is generally not ascribed any responsibility for harm nor obliged to compensate anyone.

3. The Shapley value as a tool for allocating damages

Adopting heuristics ρ^0, ρ^1, ρ^2, etc. or more sophisticated ones is purely ad hoc. See corresponding criticism by the US Supreme Court in Texas Industries, Inc. v. Radcliff Materials, Inc., 451 U.S. 630, 1981. We will instead take a systematic approach and review several properties that a rule for allocating damages in line with relative responsibilities ideally should satisfy. It turns out that all are verified by the Shapley value; while any other sharing suggestion would violate at least one desideratum.

3.1. Notation and setup

In order to formalize sensitivity to individual responsibility and other desirable properties of a damage allocation rule, we adopt some terminology from the theory of TU games. The latter describe situations in which *transferable utility* (TU), such as a surplus or cost, is to be divided among *players* from a given set $N = \{1, \ldots, n\}$. In our context, the players are the firms involved and N is the detected cartel. Of course, the relevant market may also comprise firms $j \notin N$ which did not partake in the cartel. However, they are not required to contribute to compensations. So only members of cartel N play a role for us.[7]

For every subset or *coalition* $S \subseteq N$ of players who might cooperate with one another in a TU game, a real number $v(S)$ captures the positive or negative *worth* which cooperation by S creates and which may be shared arbitrarily. In our context, $v(S)$ describes damage inflicted on a given customer or the overcharge on one unit of a given good.

Individual economic responsibilities of the cartelists are driven by the fact that overcharges would have differed from the observed damage $v(N)$ if players' conducts had differed, i.e., if some firms had not joined N. A given firm's role in bringing about a customer's harm generates the obligation to contribute to its remedy. Naturally, $v(S) = 0$ if the set S of collaborators is either empty ($S = \varnothing$) or comprises but a single firm, i.e., if $\#S = 1$. For other coalitions $S \subset N$, an estimate $v(S)$ is needed to describe the damage which would have accrued if only firms $i \in S$ had coordinated their actions, while firms $j \in N \setminus S$ had maximized their respective profits in a competitive fashion. We here take no stance on how sophisticated the estimates $v(S)$ ought to be in practice. For instance, the analysis of a hypothetical scenario with a sub-cartel $S \neq N$ may consider the question of whether S satisfies suitable stability conditions, and put $v(S) = 0$ if not. The computations below will keep things simple in this direction.

Directive 2014/104/EU highlights the role of counterfactual scenarios: "*. . . quantifying harm means assessing how the market in question would have evolved had there been no infringement. This assessment implies a comparison with a situation which is by definition hypothetical . . .*" (recital 46). Defining $v(S)$ for *every* set $S \subseteq N$ and not just for $S \in \{\varnothing, N\}$ extends this logic from *quantifying harm* to *quantifying contributions to harm*. Comparisons of the factual situation to intermediate counterfactuals in which some, but not all firms adhered to antitrust rules is key to disentangling differential individual effects. Each number $v(S)$ with $S \subseteq N \setminus \{i\}$ reflects a scenario for how the market might have evolved if there had been no infringement *by firm i*. That firm $j \neq i$ would then have joined the cartel anyhow ($j \in S$) or that under the changed circumstances it would have stayed legal too ($j \notin S$) is both plausible, though not necessarily equiprobable. In principle, all partial cartels $S \subseteq N \setminus \{i\}$ are relevant in

[7] In the EU, customers of firm $j \notin N$ can take an arbitrary cartelist $i \in N$ to court and establish that they suffered from so-called *umbrella effects* (see Bos and Harrington 2010, Inderst et al. 2014 and ECJ Case C-557/12). The analysis extends straightforwardly to them.

assessing i's contribution to the situation which called for compensation, hence i's relative responsibility.[8]

3.2. *Desirable properties of responsibility-based allocations*

With damages in the factual cartel scenario and the related counterfactuals described by (N, v), a general *damage allocation rule* corresponds to a mapping Φ from any conceivable cartel damage problem (N, v) to a vector $\Phi(N, v) \in \mathbb{R}^n$. Such a mapping is referred to as a *value* for general TU games.[9] The i-th component $\Phi_i(N, v)$ denotes the part of the compensation for damages $v(N)$ which cartel member $i \in N$ must contribute.

That an allocation rule reflects relative responsibilities can be translated into several formal properties of a value Φ. The first one is as follows. Suppose that participation or not of a particular firm i would never have made a difference to the damage in question, i.e., $v(S)$ is always unaffected by removing player i if $i \in S$ or, equivalently, by adding player i if $i \notin S$. Damage $v(S)$ thus is independent of i's conduct and the conditions are not met for i being 'responsible' under the term's usual conception (see fn. 4). Hence, no responsibility-based obligations to contribute follow. More technically speaking, a player i for whom $v(S) = v(S \setminus \{i\})$ for every $S \subseteq N$ is known as a *null player*. The first requirement for Φ to be based on relative responsibility then is the so-called *null player property*:

$$\Phi_i(N, v) = 0 \text{ whenever } i \text{ is a null player in } (N, v). \tag{NUL}$$

Presumably, the supply and demand structure in real markets is hardly compatible with a convicted cartel member being a null player. Still it is a valid thought experiment, and helps to formalize that responsibility requires causal links. The

[8] As we hypothesize about consequences of a refusal by i to join in, it is also conceivable that several partial cartels might have formed. This could be captured by considering a mapping V from the set of all *partitions* $\mathcal{P} = \{P_1, \ldots, P_r\}$ of set N (satisfying $\bigcup_l P_l = N$ and $P_l \cap P_k = \emptyset$ for any $l, k \in \{1, \ldots, r\}$) to estimated damages $V(\mathcal{P})$ instead of $v(S)$. This is left for future research. It is not clear to us if courts would find hypothetical situations admissible in which firms offend antitrust law in a different, strategically more complex way than was factually observed (cf. Ray and Vohra 1999).

[9] The respective mapping $v\colon 2^N \to \mathbb{R}$ is known as the *characteristic function* of the *TU game* (N, v). The main restriction imposed by the cartel context is $v(\{i\}) = 0$ for all $i \in N$. Provided a higher price of i's product is associated with a non-negative externality on profits of $j \neq i$, function v is monotonic but typically not superadditive nor convex. (N, v)'s core is empty, e.g., if $v(S) \approx v(N)$ for big $S \subset N$.

null player property also ensures some robustness to misspecification of the relevant market. A large cartel may have caused damage in several regions with independent costs and demand. If a firm is accidentally included as a 'player' in a market where it did not play a role, it will not need to contribute there if (NUL) is satisfied.

As i's responsibility derives from the causal link between its cartel membership and the accrued damages, another straightforward requirement is that i's damage share $\Phi_i(N, v)$ should be determined by this link – and this link alone. So, presuming that v correctly describes the factual damages that are to be shared as well as the relevant counterfactuals, $\Phi_i(N, v)$ shall be a function *only* of the differences $v(S) - v(S \setminus \{i\})$ that i makes to $S \subseteq N$. These differences are also called i's *marginal contributions* in (N, v). The corresponding formal property of *marginality*, introduced by Young (1985), demands that i's shares in two allocation problems (N, v) and (N, v') ought to coincide whenever i's marginal contributions do:[10]

$$\Phi_i(N, v) = \Phi_i(N, v') \text{ whenever } v(S) - v(S \setminus \{i\}) = v'(S) - v'(S \setminus \{i\}) \text{ holds for all } S \subseteq N. \tag{MRG}$$

Marginality does not pin down *how* $\Phi_i(N, v)$ should depend on the differences that i makes to various coalition S. For instance, imposing (MRG) does not imply (NUL). The properties formalize different aspects of requiring Φ to reflect firms' responsibilities.

A third such property refers to situations in which the roles of two firms i and j in determining damages $v(S)$ are perfectly symmetric to another. Formally, players i and j are called *symmetric* if $v(S \cup \{i\}) = v(S \cup \{j\})$ for every coalition $S \subseteq N \setminus \{i, j\}$. When adding i to a sub-cartel S has the same damage implications as adding j whenever S previously contained neither,[11] their responsibilities are the same. So Φ should also satisfy *symmetry*:

$$\Phi_i(N, v) = \Phi_j(N, v) \text{ whenever } i \text{ and } j \text{ are symmetric in } (N, v). \tag{SYM}$$

Irrespective of whether a damage allocation reflects responsibility of the involved

[10]Note that *criminal sanctions* follow different principles than civil law obligations to victims or co-offenders. We are concerned only with the latter. The former seek to punish and deter; they may well differ for (N, v) and (N, v') even if damages and individual contributions are identical. Repeat offenses, legal severity of the violations, possible obstruction of the investigation, etc. play a role there. See, e.g., Ginsburg and Wright's (2010) overview on fines and prison terms or the European Commission (2011).

[11]As pointed out with the Salop example in Section 2, two firms need not be *symmetric players* in relation to a specific damage $v(N)$ even though they have symmetric roles in the market at large.

players or alternative normative criteria, it is desirable that individual contributions of all firms $i \in N$ add up to $v(N)$. In the context of TU games, this condition is referred to as *efficiency* of a value:[12]

$$\sum_{i \in N} \Phi_i(N, v) = v(N). \qquad \text{(EFF)}$$

Another natural requirement is *scale invariance*: the factual distribution of damages should not depend on whether they are expressed in US dollars, euro, or any other unit of account. Multiplying all values $v(S)$ by some exchange rate $\lambda > 0$ should merely re-scale firms' contributions by the same factor, i.e., $\Phi(N, \lambda \cdot v) = \lambda \cdot \Phi(N, v)$.

Finally, if the same cartel N caused damages to suing customers in two or more markets – reflected by a characteristic function v^1 for market 1, by v^2 for market 2, etc. – then the total damage contribution of firm $i \in N$ should not depend on whether the allocation rule is applied to damages v^l in one market l at a time, or in one go to the total $v = v^1 + v^2 + \ldots$ (Different 'markets' could here refer to different plaintiffs or subsidiaries of the same plaintiff, to different products in the cartel's portfolio, or – below – distinct quantities of the same product.) This *additivity* requirement combines with scale invariance to the *linearity* condition:

$$\Phi(N, \lambda \cdot v + \lambda' \cdot v') = \lambda \cdot \Phi(N, v) + \lambda' \cdot \Phi(N, v') \qquad \text{(LIN)}$$

for any scalars $\lambda, \lambda' \in \mathbb{R}$ and any characteristic functions v, v'.[13]

3.3. *Shapley value*

The above desiderata are more than is needed in order to conclude that the adopted damage allocation rule Φ should have a particular form.

Theorem 1. *(Shapley-Young) The following statements about a damage allocation rule* $\Phi\colon (N, v) \mapsto \mathbb{R}^n$ *are equivalent:*

[12]Combining symmetry and efficiency in 2-player games $(\{1,2\}, v)$ with $v(\{1\}) = v(\{2\})$ implies $\Phi_1(\{1,2\}, v) = \Phi_2(\{1,2\}, v) = \frac{1}{2}v(\{1,2\})$. A value Φ that satisfies a mild generalization of this has been called *standard* by Hart and Mas-Colell (1989). They showed that a standard value Φ coincides with the Shapley value iff it satisfies a suitable consistency condition regarding sequential divisions of $v(N)$.

[13]Additivity applies also to different *types* of damages described by v and v'. (LIN) hence ensures that it would be possible to deal with overcharge damages and additional harm induced by deadweight losses separately or jointly, without affecting the total allocation. See Moulin (2002) for a general discussion of linearity in cost and surplus sharing.

S	$v(S)$	$v(S) - v(S \setminus \{1\})$	$v(S) - v(S \setminus \{2\})$	$v(S) - v(S \setminus \{3\})$
$\emptyset, \{1\}, \{2\}, \{3\}$	0	0	0	0
$\{1,2\}$	28.20	28.20	28.20	0
$\{1,3\}$	1.67	1.67	0	1.67
$\{2,3\}$	0.73	0	0.73	0.73
$\{1,2,3\}$	37.49	36.76	35.82	9.29

Table 1: Marginal contributions to Δp_1 for Section 2's example

(I) Φ *satisfies (NUL), (SYM), (EFF) and (LIN).*

(II) Φ *satisfies (MRG), (SYM) and (EFF).*

(III)

$$\Phi_i(N,v) = \varphi_i(N,v) := \sum_{S \subseteq N} \frac{(s-1)!(n-s)!}{n!} \cdot \Big[v(S) - v(S \setminus \{i\})\Big] \tag{1}$$

where $s = \#S$ denotes the cardinality of coalition S.

Equivalence of (I) and (III) was established by Shapley (1953a); equivalence of (II) and (III) by Young (1985).[14] $\varphi(N,v)$ is called the *Shapley value* of (N,v).

As illustration re-consider the overcharge damage which accrued to the exemplary purchaser of 10 units of good 1 in Section 2. If we want to allocate this to the three firms in line with the listed requirements, we need to check their marginal contributions to overcharge Δp_1 and weight them according to eq. (1). Table 1 collects the damages for all conceivable cartel scenarios S implied by the indicated market model, and lists the respective differences that participation by a given firm makes. The numbers confirm that cooperation by firm 1 and 2 is the main driver of overcharges on product 1. Firm 3's participation has an effect, too; but mainly when 1 and 2 are already collaborating. So, as economic intuition in Section 2 had it, firm 3's responsibility for k's damage is small. Those of firms 1 and 2 are similar to another, with a slightly bigger average contribution for 1. Aggregating the figures according to eq. (1) yields $\varphi(N,v) = (17.2; 16.8; 3.5)$; that

[14]See, e.g., Maschler et al. (2013, ch. 18) for an excellent exhibition. Applicability of Shapley's and Young's results to our setting rests on the observation that we can conceive of any TU game (N,v) as a damage allocation problem and vice versa. To see this, consider cartels in which firms $i \in T \subseteq N$ produce perfect substitutes with competitive price $p^* = 0$ and cartel price $p^C = 1$ while firms $j \notin T$ operate in distinct independent markets. Damages then are $v(S) = u_T(S)$ where $u_T(S) = 1$ if $T \subseteq S$ and 0 otherwise. (N, u_T) is the so-called *carrier game over T* and the collection of carrier games $\big\{(N, u_T)\big\}_{T \subseteq N}$ forms a basis of the vector space of all TU games (N,v). In other words, one can obtain any (N,v) by re-scaling and adding damages in markets with perfect substitutes.

is, overcharges to the considered customer of product 1 are to be shared proportionally to $\rho^* = (46.0\%; 44.7\%; 9.3\%)$.

Similar computations yield allocations of compensation owed to customers of firms 2 and 3. Conveniently, linearity of the Shapley value permits to focus on the damage associated with a single unit of the respective good i. Obligations of the three firms for compensating a plaintiff with purchases of $x = (x_1, x_2, x_3)$ then follow from the matrix multiplication

$$(x_1, x_2, x_3) \cdot \begin{pmatrix} \varphi(N, v^1) \\ \varphi(N, v^2) \\ \varphi(N, v^3) \end{pmatrix}$$

where characteristic function v^i reflects the overcharge on a *single unit* of good i. For the example at hand and the considered customer, suggestions $\rho^0 = (33.3\%, 33.3\%, 33.3\%)$, $\rho^1 = (18.9\%, 46.0\%, 35.1\%)$, etc. from Section 2 are all quite far off. Competitive revenue shares $\rho^3 = (29.7\%, 39.1\%, 31.2\%)$ have smallest $\|\cdot\|_1$-distance to ρ^*. We investigate in Section 7 whether this is a fluke.

4. A new decomposition

Shapley's formula for φ in eq. (1) looks unwieldy at a first glance. The weight $(s-1)!(n-s)!/n!$ on i's marginal contribution to a given coalition S first and foremost follows from the listed desiderata. Still, it can be interpreted in meaningful ways. The most prominent one, suggested by Shapley (1953a), is to think of these weights as reflecting a uniform probability distribution over all orderings of the players. These orderings can be viewed as arising in a sequential process of coalition formation starting from the empty set. Stipulating equal probability of each sequence presumes homogeneity of the players regarding the moral choice to stay legal or not, despite possible heterogeneity of the economic effects that are linked to these choices.

Insights into how a firm i's damage share φ_i comes about can also be obtained from writing Shapley's formula in different ways, which are not necessarily more elegant or concise. Kleinberg and Weiss (1985) have shown that the single summation of marginal contributions in eq. (1) can be decomposed to

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n-1}\sum_{s=1}^{n-1} \tau(k)^{-1} \sum_{\substack{S \ni i \\ \#S=s}} \Big[v(S) - \underbrace{\sum_{\#R=s} v(R) \binom{n}{s}^{-1}}_{=:\mathcal{B}(s)} \Big] \tag{2}$$

where $\tau(k) := \binom{n-2}{s-1}$. That is, we can also conceive of the Shapley damage allocation as follows: all firms start out with equal shares; these are then corrected for asymmetric damages caused by this or that firm behaving cooperatively. Firm i has to bear a higher share of the compensation than $v(N)/n$ if its membership of a cartel coalition S goes with damage above average, i.e., $v(S) - \mathcal{B}(s) > 0$, as we consider all conceivable coalition sizes s and coalitions S.

Rothblum (1988) has established another way of highlighting different-from-average effects as the determinant of the Shapley value. He suggested to re-write eq. (1) as

$$\varphi_i(N,v) = \frac{1}{n}\sum_{s=1}^{n} \binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} \Big[v(S) - \underbrace{\binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R)}_{=:C_i(s)} \Big] \tag{3}$$

where $C_i(n)$ is set to zero. So each firm i should pay the *average incremental damage* $v(S) - C_i(s)$ caused by i being a member of a coalition S with given size s instead of not being part when the size is s. While the decomposition in eq. (2) compares coalitions which include i to *all* coalitions of the same size s, eq. (3) just compares those which include i to those which exclude i.

A third and it seems yet unacknowledged decomposition of the Shapley value combines the perspectives of Kleinberg and Weiss (1985) and Rothblum (1988). One can start with equal shares, as in eq. (2), and then correct this by considering the average size-specific damage changes which stem from i's participation, just like in eq. (3). Specifically, the average damage $\bar{v}^{i}(s)$ caused by coalitions of size s which *include* firm i and the average damage $\bar{v}^{\not i}(s)$ caused by coalitions of size s which *exclude* firm i simply need to be added up and scaled by $1/n$:

Proposition 1. *An equivalent way of expressing the Shapley of a TU game* (N,v) *is*

$$\varphi_i(N,v) = \frac{v(N)}{n} + \frac{1}{n}\sum_{s=1}^{n-1} \Big[\bar{v}^{i}(s) - \bar{v}^{\not i}(s) \Big] \tag{4}$$

where

$$\bar{v}^{i}(s) := \binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} v(S) \quad \textit{and} \quad \bar{v}^{\not i}(s) := \binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R). \tag{5}$$

Proof. Starting from Rothblum's decomposition in eq. (3) we have

$$\begin{aligned}
n \cdot \varphi_i(v) &= \sum_{s=1}^{n} \binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} v(S) - \sum_{s=1}^{n} \binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} \binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R) \\
&= \sum_{s=1}^{n} \binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} v(S) - \sum_{s=1}^{n} \binom{n-1}{s-1}^{-1} \cdot \binom{n-1}{s-1} \cdot \binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R) \\
&= \sum_{s=1}^{n} \left[\binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} v(S) - \binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R) \right] \\
&= v(N) + \sum_{s=1}^{n-1} \left[\binom{n-1}{s-1}^{-1} \sum_{\substack{S \ni i \\ \#S=s}} v(S) - \binom{n-1}{s}^{-1} \sum_{\substack{R \not\ni i \\ \#R=s}} v(R) \right].
\end{aligned} \tag{6}$$

□

Many more ways of writing the Shapley value trivially result from conducting arbitrary term re-arrangements. The merit of those in eqs. (2)–(4) or ones which consider player orderings lies in providing a useful perspective on φ_i. A specific advantage of decomposition (4) is that we will be able to sign $[\bar{v}^{i}(s) - \bar{v}^{\not i}(s)]$ below.

Because any degenerate 'cartel' of size $s = 1$ leaves prices constant, overcharge damages are $\bar{v}^{i}(1) = \bar{v}^{\not i}(1) = 0$ for each $i \in N$. From Theorem 1 we can hence conclude:

Theorem 2. *We must use the damage allocation rule*

$$\Phi_i(N, v) = \varphi_i(N, v) = \frac{v(N)}{n} + \frac{1}{n} \sum_{s=2}^{n-1} \left[\bar{v}^{i}(s) - \bar{v}^{\not i}(s) \right] \tag{7}$$

if and only if Φ *is to have properties (EFF), (LIN), (SYM), (NUL) and (MRG).*

Equation (7) provides a succinct way of allocating an overcharge damage to cartel members according to their relative responsibilities for it: start out with *equal shares per head* of the applicable compensation payment $v(N)$. Then correct this for asymmetric individual effects on the overcharge in question. The correction term for firm i is proportional to the *difference between average damages which are associated with partial cartels including i and excluding i* as we imagine that respectively $1, 2, \ldots, n-2$ cartel members had refused to take part in the infringement.

5. Damage allocations in benchmark cases

Let us investigate the implications of Theorem 2 for some focal cases. We start with symmetric situations and then turn to specific asymmetric ones where damages are dichotomous. The latter situations may at first seem a bit artificial but can serve as useful approximations of many others.

5.1. *Perfect substitutes*

If all members of a cartel S and potential outsiders $j \in N \setminus S$ produce *perfect substitutes*, customers will pay the same price $p(S)$ independently of their specific vendor. The applicable overcharge relative to the competitive benchmark $p^* = p(\emptyset)$ is $v(S) = p(S) - p^*$. This depends only on the cardinality $s = \#S$ if we assume that firms operate with identical technology and focus on symmetric equilibria. Each firm's participation then contributes symmetrically to damage.

In standard Cournot settings, the overcharge rises when more firms cooperate, i.e., $v(S)$ is strictly increasing in s for $s > 1$. If firms compete à la Bertrand with constant returns, $v(S) = 0$ for all $s < n$. But no matter how damages precisely vary in s, symmetry of all firms as players in (N, v) implies an *equal per head allocation*:

Proposition 2. *If all cartel members $i \in N$ produce perfect substitutes with identical technology then $\varphi_i(N, v) = v(N)/n$.*

This follows from the fact that φ satisfies (SYM). Note that the latter is directly visible in decomposition (7) – since symmetry implies $\bar{v}^i(s) = \bar{v}^k(s)$ – but not so transparent in the standard formula for φ in eq. (1).

5.2. *Symmetric differentiated substitutes*

Things change if goods are differentiated. Customers pay a firm-specific price $p_i = p^i(S)$ for purchases from i when cartel S is formed and this generally depends on the composition of S rather than just its size. We will consider a particularly well-behaved environment with $n \geq 3$ firms[15] which highlights that even strong symmetry in the market can go with asymmetric individual responsibilities for damage.

[15]One trivially obtains $\varphi(N, v) = (v(N), v(N))/2$ if $N = \{1, 2\}$ because $v(\{1\}) = v(\{2\}) = 0$. Participants in 2-firm cartels are to share damages equally even if they are asymmetric in size or costs: refusal to join by either would have maintained competition and have avoided all overcharges relative to duopoly.

Let the profits Π_i of every firm $i \in N$ be a smooth, strictly concave function of a profile $y = (y_1, \ldots, y_n)$ of 'actions' of all firms with $\partial\Pi_i/\partial y_i\big|_{y=0} > 0$. These actions could be price choices, production levels, choices on the geographic radius of operation, etc. We presume that the associated prices $p = (p_1, \ldots, p_n)$ are smooth functions of y, too, and if $\partial p_i/\partial y_i$ is positive (negative) then the same should go for the sign of the externality $\partial\Pi_j/\partial y_i$ that firms exert on each other.[16] Specifically, we think of goods as differentiated substitutes and require

$$\frac{\partial \Pi_j}{\partial y_i} \cdot \frac{\partial p_i}{\partial y_i} > 0 \text{ for all } i \neq j \in N \tag{8}$$

for the relevant range of actions. For instance, if firm i's output choice y_i negatively affects its own price p_i, we assume it also has a negative effect on any competitor's profits Π_j. If i's action is its price, i.e., $p_i(y) \equiv y_i$, then Π_j increases in y_i.

A coalition $S \neq \emptyset$ chooses $(y_i)_{i \in S}$ to maximize $\Pi_S(y) = \sum_{i \in S} \Pi_i(y)$ for given actions $y_{-S} = (y_j)_{j \notin S}$ of outsiders. If S is a singleton, this corresponds to individual profit maximization by all, implying the competitive benchmark prices $p_1^*, \ldots, p_n^*$. We assume that a unique, interior profit maximizer exists for each non-empty $S \subseteq N$. So, for any fixed cartel S, reaction functions $R_S(y_{-S})$ and $(R_j(y_{-j}))_{j \notin S}$ are well-defined by the first-order conditions

$$\frac{d\Pi_i}{dy_i} = \frac{\partial \Pi_i}{\partial y_i} \qquad = 0 \;\text{ if } i \notin S, \tag{9}$$

$$\frac{d\Pi_S}{dy_i} = \sum_{j \in S} \frac{\partial \Pi_j}{\partial y_i} = 0 \;\text{ if } i \in S. \tag{10}$$

We further specialize this to *strongly symmetric* situations in which profits Π_i and prices p_i depend identically on i's own action y_i for each $i \in N$ and identically also on any respective action y_j by a firm $j \neq i$. Formally, for each $i \neq j$ and every permutation $\varrho\colon N \to N$ with $\varrho(i) = j$ and $\varrho(j) = i$

$$p_i(y_1, \ldots, y_n) \equiv p_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}) \;\text{ and }\; \Pi_i(y_1, \ldots, y_n) \equiv \Pi_j(y_{\varrho(1)}, \ldots, y_{\varrho(n)}). \tag{11}$$

One can, e.g., think of equal measures of customers with a favorite product i to whom all varieties $j \neq i$ are identically imperfect substitutes. This assumes greater symmetry than the Salop model.[17] In particular, cross effects on prices and profits are identical

[16]Without an externality, competitive and cartel behavior would not differ and no harm arise.

[17]There, some permutation ϱ with $\varrho(i) = j$ and $\varrho(j) = i$ satisfies (11), not every such permutation.

for all firms. The first-order condition (10) for a cartel member $i \in S$ then simplifies to

$$\frac{d\Pi_S}{dy_i} = \frac{\partial \Pi_i}{\partial y_i} + (s-1)\frac{\partial \Pi_j}{\partial y_i} = 0. \tag{12}$$

The only asymmetry is that i's own actions may affect p_i and Π_i differently from the actions of $j \neq i$. We will suppose own actions have bigger effects and therefore

$$\left|\frac{\partial p_i}{\partial y_i}\right| > \left|\frac{\partial p_i}{\partial y_j}\right|. \tag{13}$$

The inequality is trivially satisfied for price competition. Otherwise it formalizes that inverse demand responds more to changes of the quantity, delivery range, etc. of the product in question than that of others.

We assume that for any fixed cartel S, the simultaneous best-response behavior by it and any outsiders $j \in N \setminus S$ determine a unique type-symmetric Nash equilibrium profile $y^*(S) = (y_1^*(S), \dots, y_n^*(S))$ where $y_i^*(S) \equiv y^C(S)$ if $i \in S$, and $y_i^*(S) \equiv y^O(S)$ if $i \notin S$. We will drop the argument S below when the reference is clear. Sufficient conditions for such an equilibrium to exist can be found in Section 6.

The first-order conditions (9) and (12) cannot simultaneously be satisfied for $s > 1$ if $y^C = y^O$: $\partial \Pi_j / \partial y_i \neq 0$ implies either $y^C > y^O$ or $y^C < y^O$ in equilibrium. The former holds if the externality is positive, the latter if it is negative.

For specificity, suppose quantity competition with a negative externality $\partial \Pi_j / \partial y_i < 0$ and $\partial p_i / \partial y_i < 0$ for a moment. The key observation then will be that $y^C < y^O$ translates into higher prices for the goods sold by cartel members. This implies that for a cartel S of a fixed size s, firm i's prices – and hence its customers' damages – depend on whether i is an element of S or not. In particular, if v describes the damages of a customer of good i then $\bar{v}^i(s) > \bar{v}^{\not i}(s)$.

To see this formally, let $S = \{1, \dots, s\}$ w.l.o.g. and consider the straight line L which connects profile $\hat{y} = (y^O, y^C, \dots, y^C, y^O, \dots, y^O, y^C)$ to $\hat{\hat{y}} = (y^C, y^C, \dots, y^C, y^O, \dots, y^O, y^O)$ in the space of output choices. L can be parameterized by

$$r(t) = (\underbrace{y^O - t, y^C, \dots, y^C}_{s \text{ terms}}, \underbrace{y^O, \dots, y^O, y^C + t}_{n-s \text{ terms}}) \tag{14}$$

with $t \in [0, y^O - y^C]$, i.e., we simultaneously decrease firm 1's action and increase firm n's action by identical amounts as we move along L. The gradient $\nabla p_n = \left(\frac{\partial p_n}{\partial y_1}, \dots, \frac{\partial p_n}{\partial y_n}\right)$ of function p_n can be used in order to evaluate the price change caused by switching from $\hat{y}$ to $\hat{\hat{y}}$. In particular, the gradient theorem for line integrals (see, e.g., Protter and Morrey 1991, Thm. 16.15) yields

$$p_n(\hat{\hat{y}}) - p_n(\hat{y}) = \int_L \nabla p_n \, dr = \int_0^{y^O - y^C} \nabla p_n(r(t)) \cdot r'(t) \, dt \tag{15}$$

$$= \int_0^{y^O - y^C} \Big(\frac{\partial p_n}{\partial y_1}, \ldots, \frac{\partial p_n}{\partial y_n}\Big)\Big|_{y=r(t)} \cdot \Big(-1, 0, \ldots, 0, 1\Big) \, dt \tag{16}$$

$$= \int_0^{y^O - y^C} \left[\frac{\partial p_n(r(t))}{\partial y_n} - \frac{\partial p_n(r(t))}{\partial y_1}\right] dt < 0. \tag{17}$$

The inequality follows from own actions having bigger effects than a competitor's actions: (13) entails $\frac{\partial p_n}{\partial y_n} < \frac{\partial p_n}{\partial y_1}$ when $\partial p_n / \partial y_n < 0$. The strong symmetry of the considered setting (see condition (11)) then implies

$$p^1(s) := p_1(y^C, y^C, \ldots, y^C, y^O, \ldots, y^O, y^O) = p_n(y^O, y^C, \ldots, y^C, y^O, \ldots, y^O, y^C) \tag{18}$$
$$= p_n(\hat{y}) > p_n(\hat{\hat{y}}) = p_n(y^C, y^C, \ldots, y^C, y^O, \ldots, y^O, y^O) := p^{\not n}(s).$$

That is, the price $p^1(s)$ of good 1 when its producer is one of s symmetric cartel members exceeds the price $p^{\not n}(s)$ of good n when firm n is *not* part of a cartel with s members.

By symmetry, we have $p^{\not 1}(s) = p^{\not n}(s)$ and $p^1(s) = p^n(s)$. So we can conclude $p^1(s) > p^{\not 1}(s)$ from (18) for $1 < s < n$.[18] The same applies to any other firm, too – for instance, the plaintiff's 'home' firm $h \in N$ from which its disputed purchases were made:

$$p^h(s) > p^{\not h}(s) \text{ for any } s = 2, \ldots, n-1. \tag{19}$$

The average per-unit damage to h's customer in scenarios where h behaves anti-competitively is

$$\bar{v}^h(s) = p^h(s) - p_h^* \tag{20}$$

where p_h^* is h's price in the competitive benchmark (identical across firms). The price of firm h does not depend on the specific $s-1$ firms with which h colludes, and neither does the damage. Analogously, the per-unit damage when firm h behaves competitively but s others collude is

$$\bar{v}^{\not h}(s) = p^{\not h}(s) - p_h^*. \tag{21}$$

[18] Recall that there is no well-defined partial cartel for $s = 1$ or n.

Inequality (19) then yields

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) > 0 \;\text{ for any }\; s = 2, \ldots, n-1. \tag{22}$$

So all summands in the Shapley value's correction term of Theorem 2 are positive. It follows that the 'home' firm's share in compensating overcharges on its own sales must *strictly exceed* $1/n$; that of others must consequently be less than $1/n$.

This extends to other interpretations of variables $y_1, \ldots, y_n$, notably price competition: inequalities (19) and hence (22) also follow when positive externalities $\partial \Pi_j / \partial y_i > 0$ and $\partial p_i / \partial y_i > 0$ are concerned. The cartel members choose $y^C(S) > y^O(S)$ for any fixed S; the reversed orientation as we integrate from $t = 0$ to $y^O - y^C < 0$ in (17) and the reversed sign of integrand $\partial p_n / \partial y_n - \partial p_n / \partial y_1$ cancel. In summary, we have:

Proposition 3. *Let $n \geq 3$ firms be strongly symmetric in the sense of (11) and let assumptions (8) and (13) be satisfied by smooth own and cross-effects of firms' actions. If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N, v) \begin{cases} > \frac{v(N)}{n} & \text{if } i = h, \\ < \frac{v(N)}{n} & \text{if } i \neq h. \end{cases} \tag{23}$$

Simple rules of thumb like distributing damages on a per-head basis or according to market shares, profits, etc. will allocate exactly $1/n$-th of compensation payments to all producers if they are symmetric. Proposition 3 shows that this generally clashes with a responsibility-based allocation. Only if identical numbers of customers of all firms act against the cartel, each $h \in N$ is the 'home' producer equally often and asymmetric responsibilities for overcharges Δp_h perfectly net out. Otherwise, responsibility of vendor h is underestimated and that of its collaborators $j \neq h$ overestimated.

5.3. Dichotomous damage scenarios

To get a feel for the Shapley allocation of overcharges between asymmetric firms, consider damages with a dichotomous nature. That is, let the applicable overcharge either be 0 or have a fixed positive level which we normalize to 1. One can think of such situations as approximations of ones in which there is a first-order difference between 'small' and 'big' damages, with minor differences within each category.

For instance, exact damages $v(S) = 1.67$ and 0.74 for coalitions $\{1, 3\}$ and $\{2, 3\}$ in

Section 2's example are small compared to the damage values of 28.20 and 37.49 for coalitions $\{1,2\}$ and $\{1,2,3\}$ (see Table 1 on p. 12). Dividing by $v(N)$ yields normalized values of 0.04, 0.02, 0.75 and 1.00, respectively. A corresponding binary approximation of v by $\tilde{v}$ with

$$\tilde{v}(S) = \begin{cases} 1 & \text{if } \{1,2\} \subseteq S, \\ 0 & \text{otherwise} \end{cases} \tag{24}$$

is not far off. It could in practice be derived from a qualitative assessment which finds firms 1 and 2 competing a lot more closely with each other than firm 3 – without full estimates of cost and demand functions. The corresponding Shapley value $\varphi(N,\tilde{v}) = (50\%; 50\%; 0)$ is pretty close to the Shapley vector $\rho^* = (46.0\%; 44.7\%; 9.3\%)$ computed for (N, v) in Section 3.3. It is much closer than Section 2's heuristic suggestions $\rho^0, \rho^1, \ldots$ derived from market or profit shares.

More generally, consider the allocation of overcharge damages described by (N, v) with $v(S) \in \{0,1\}$. The monotonicity condition $S \subseteq T \Rightarrow v(T) \geq v(S)$ and the restrictions $v(\emptyset) = 0$ and $v(N) = 1$ define a *(monotonic) simple game* (N, v) in cooperative game theory. Such games have been studied extensively (see, e.g., Taylor and Zwicker 1999) and often arise in the context of voting and election rules. Hence a coalition S such that $v(S) = 1$ is typically referred to as *winning* and one with $v(S) = 0$ as *losing*. In our application, a winning coalition $S \subseteq N$ corresponds to a (partial) cartel which could profitably impose a big overcharge.

The assumption that if a given partial cartel S can 'win', so does a larger cartel T which contains S, could be restrictive in very specific setups[19] but generally is innocuous. It allows to fully define the mapping v by the list $\mathcal{M}(v) = \{S \subseteq N : v(S) = 1 \text{ and } T \subset S \Rightarrow v(T) = 0\}$ of *minimal winning coalitions* (MWC). Any coalition $S \in \mathcal{M}(v)$ and all its supersets cause damage; collaboration by a strict subset of S does not.

As illustration, consider a market with $N = \{\text{A, B, C, D}\}$ where collaboration by firm A with at least one other firm implies a unit damage. The corresponding set of MWC is

$$\mathcal{M}(v) = \{\text{AB, AC, AD}\}.$$

Here we write AB as shorthand for {A, B}, etc. Firm A's participation is essential

[19] Think of some cartel members producing complements rather than substitutes, which generates non-monotonicities. Such cases seem rare but possible (e.g., the 1992–2004 bathroom fittings cartel).

$\mathcal{M}(v)$	$\varphi(N,v)$	$\mathcal{M}(v)$	$\varphi(N,v)$
1. AB	(50%, 50%, 0%, 0%)	11. AB, ACD, BCD	$(33.\bar{3}\%, 33.\bar{3}\%, 16.\bar{6}\%, 16.\bar{6}\%)$
2. AB, AC	$(66.\bar{6}\%, 16.\bar{6}\%, 16.\bar{6}\%, 0\%)$	12. AB, AC, AD, BC, BD	$(33.\bar{3}\%, 33.\bar{3}\%, 16.\bar{6}\%, 16.\bar{6}\%)$
3. AB, AC, BC	$(33.\bar{3}\%, 33.\bar{3}\%, 33.\bar{3}\%, 0\%)$	13. AB, BC, CD	$(16.\bar{6}\%, 33.\bar{3}\%, 33.\bar{3}\%, 16.\bar{6}\%)$
4. ABC	$(33.\bar{3}\%, 33.\bar{3}\%, 33.\bar{3}\%, 0\%)$	14. AB, AC, AD, BC	$(41.\bar{6}\%, 25.0\%, 25.0\%, 8.\bar{3}\%)$
5. ABC, ABD	$(41.\bar{6}\%, 41.\bar{6}\%, 8.\bar{3}\%, 8.\bar{3}\%)$	15. ABC, ABD, ACD, BCD	(25%, 25%, 25%, 25%)
6. ABCD	(25%, 25%, 25%, 25%)	16. AB, AC, AD, BCD	$(50\%, 16.\bar{6}\%, 16.\bar{6}\%, 16.\bar{6}\%)$
7. AB, AC, BCD	$(41.\bar{6}\%, 25.0\%, 25.0\%, 8.\bar{3}\%)$	17. AB, AC, AD, BC, BD, CD	(25%, 25%, 25%, 25%)
8. AB, AC, AD	$(75.0\%, 8.\bar{3}\%, 8.\bar{3}\%, 8.\bar{3}\%)$	18. AC, AD, BC, BD	(25%, 25%, 25%, 25%)
9. AB, CD	(25%, 25%, 25%, 25%)	19. ABC, ABD, ACD	$(50\%, 16.\bar{6}\%, 16.\bar{6}\%, 16.\bar{6}\%)$
10. AB, ACD	$(58.\bar{3}\%, 25\%, 8.\bar{3}\%, 8.\bar{3}\%)$	continued for $n = 5$ in Appendix A	

Table 2: Shapley allocations for all dichotomous damage scenarios with $n \leq 4$ firms

for overcharges; non-participation by up to two other cartelists would not noticeably change things. Many people's intuition is probably that the singular importance of A – with a veto position – entails greater responsibility for compensating victims. But how much greater? Operationalizing responsibility in a systematic way yields the answer. Presuming one deems the properties discussed in Section 3 desirable, the allocation should be

$$\varphi(N,v) = (75\%, 8.\bar{3}\%, 8.\bar{3}\%, 8.\bar{3}\%)\,.$$

One can easily generalize the idea of dichotomous approximation to bigger scenarios. With one large firm A and $n-1$ small ones such that a unit damage accrues if and only if A and at least one more firm cooperate,[20] one obtains $\bar{v}^{\mathrm{A}}(s) = 1$ and $\bar{v}^{\not\mathrm{A}}(s) = 0$ for all $1 < s < n$. Proposition 1 then directly implies

$$\varphi_{\mathrm{A}}(N,v) = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1}(1-0) = \frac{n-1}{n} \quad \text{and} \quad \varphi_i(N,v) = \frac{1}{n(n-1)} \text{ for } i \neq \mathrm{A}. \tag{25}$$

For small cartels, it is possible to enumerate *all* dichotomous damage scenarios which can arise. They correspond to simple games (N,v) with n players such that $v(S) = 1$ implies $\#S \geq 2$. Exactly 19 such scenarios exist for $n \leq 4$ firms, up to relabeling. They are listed in Table 2 with the corresponding Shapley values.

For instance, scenario 1 approximates situations in which only cooperation by

[20]A related scenario would have all *pairs* $\{\mathrm{A}, i\} \subseteq S$ with $i \neq \mathrm{A}$ cause *incremental* unit damages, independently of each other. The corresponding mapping v with $v(S) = s-1$ if $\mathrm{A} \in S$ and $v(S) = 0$ otherwise, assumes more than two values and is no simple game. Still, it is not hard to conclude from $\bar{v}^{\mathrm{A}}(s) = s-1$ and $\bar{v}^{\not\mathrm{A}}(s) = 0$ that $\varphi_{\mathrm{A}}(N,v) = \frac{1}{2}v(N)$ and $\varphi_i(N,v) = \frac{1}{2(n-1)}v(N)$ for $i \neq \mathrm{A}$.

firms A and B is critical for the overcharges in question; then A and B share responsibility for the damage 50 : 50. We saw that this is a reasonable approximation for the example in Section 2. Scenario 2 corresponds to the big firm-small firm situation in eq. (25) with $n = 3$. Here, firm D is – with the caveat that we may deal with a binary approximation of the real market – a null player; hence it bears no responsibility for damages. In scenario 3, cooperation by any two firms from {A, B, C} causes damage; while that of all three is necessary and sufficient for damage in scenario 4; etc.

The number of distinct scenarios involving n firms is related to the *Dedekind numbers* in discrete mathematics. These grow at a doubly exponential rate. A list of all dichotomous damage scenarios with $n = 5$ non-null players already involves 160 entries. They are collected in Appendix A.[21] A comprehensive categorization may be useful for ballpark assessments of responsibility in contribution settlements. The key practical advantage is that binary approximations just require a big-or-small classification of damages, not a full-blown market simulation. Even if some approximation error cannot be avoided the corresponding Shapley allocations reflect marginal contributions and hence responsibility, in contrast to profits or market shares.

6. Linear market environments

Sometimes binary evaluations of the counterfactual damages will be considered as too inaccurate. Defining (N, v) then calls for a parametric specification of the market. Price and damage estimates for partial cartels can be obtained from equilibrium analysis in close analogy to merger simulation (see Davis and Garcés 2009, ch. 8, or Budzinski and Ruhmer 2010). We illustrate this for a linear price competition model with differentiated goods. It gives an upper bound on the responsibility of a firm h for its own overcharge Δp_h in symmetric markets. We can also see that bounds on damage shares are affected differently by different types of asymmetry.

[21]See Straffin (1983) for $n \leq 4$ and Baldan (1992) for $n = 5$. We have fewer games because $v(S) = 1$ requires $\#S \geq 2$ in a cartel context. Appendix A corrects several hidden typos in Baldan's list. Note that some games in the list, such as scenario 9, would be considered as *improper* in the context of voting: they involve disjoint winning coalitions. If we think of A and B as two producers and of C and D as their retailers, damage may plausibly arise already if the producers or the retailers cooperate. Presuming little scope for further marginalization by vertical coordination, $\mathcal{M}(v) = \{AB, CD\}$ makes good sense.

6.1. Model

We continue to focus on a cartel by $n \geq 3$ suppliers where each firm $i \in N = \{1, \ldots, n\}$ produces a single good.[22] Firm i's costs are given by

$$C_i(q_i) = \gamma_i q_i \text{ for } \gamma_i \geq 0. \tag{26}$$

Demand at price vector $p = (p_1, \ldots, p_n)$ is described by

$$D_i(p) = \max\Big\{a_i - d_i \cdot p_i + \sum_{j \in N \setminus \{i\}} b_{ij} \cdot p_j,\ 0\Big\} \text{ for } a_i > \gamma_i, d_i > 0, b_{ij} > 0 \text{ for all } j \neq i. \tag{27}$$

Imposing *symmetry* by setting $\gamma_i = \gamma$, $a_i = a$, $d_i = d$ and $b_{ij} = b$ for all $i \neq j \in N$ implies strong symmetry in the sense of condition (11).

Firms set prices simultaneously à la Bertrand and we suppose that this continues to hold if some group $S \subseteq N$ of them forms a cartel. In particular, cartel outsiders $j \notin S$ choose prices without observing the coordinated decisions of insiders.

Members of $S \subseteq N$ maximize the sum of their profits

$$\Pi_S(p) = \sum_{i \in S} (p_i - \gamma_i) D_i(p) \tag{28}$$

with corresponding first-order conditions

$$\frac{\partial \Pi_S(p)}{\partial p_j} = D_j(p) + \sum_{i \in S} (p_i - \gamma_i) \frac{\partial D_i(p)}{\partial p_j} \text{ for all } j \in S. \tag{29}$$

Analogous expressions hold if j is a cartel outsider.

It is sufficient for existence and uniqueness of a Nash equilibrium that a uniform increase of all prices as well as a unilateral increase of any single price would always decrease the industry's aggregated demand.[23] Formally, this requires $\sum_{j=1}^{n} \partial D_i / \partial p_j < 0$ and $\sum_{j=1}^{n} \partial D_j / \partial p_i < 0$, i.e., we will assume that the dominant diagonal condition

$$\alpha_i := d_i / \sum_{j \neq i} b_{ij} > 1 \text{ and } d_i > \sum_{j \neq i} b_{ji} \text{ for all } i \in N \tag{30}$$

is satisfied. In the symmetric case, the condition simplifies to $\alpha := d/(n-1)b > 1$.

[22]If we have a *multi-product* firm and all its prices are determined either competitively or cooperatively then we would need to consider overcharges $\Delta p_l = p_l^C - p_l^B$, $l \in P$, for a set of products P which no longer coincides with the set of players N. If, in contrast, the conduct decision is made autonomously for each l by distinct departments of the firm, then each of these should be included as a player in N.

[23]See Vives (1999, Sec. 6.2) and Federgruen and Pierson (2011, Cor. 4.6).

Products are relatively good substitutes when α_i is small; then a price increase by one firm significantly raises profits of the other firms. The cartel internalizes this externality and the price p_i set by cartel member i will be the higher, the smaller α_i.

6.2. *Equilibrium prices*

We adopt the concise notation by Davis and Garcés (2009, ch. 8) and let $b_{ii} := -d_i$. If we focus on price vectors with positive demand we can write the demand system as

$$D(p) = A + B \cdot p \quad \text{with} \quad A = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \text{ and } B = \begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & \cdots & \vdots \\ b_{n1} & \dots & b_{nn} \end{pmatrix}. \tag{31}$$

For any given coalition $S \subseteq N$, define the *cartel participation matrix* $P^S = ({P^S}_{ij})_{n \times n}$ by

$$P^S_{ij} = \begin{cases} 1 & \text{if } i, j \in S \text{ or } i = j, \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

and let

$$(P^S \circ B)' = \begin{pmatrix} {P^S}_{11} \cdot b_{11} & \cdots & {P^S}_{n1} \cdot b_{n1} \\ \vdots & \cdots & \vdots \\ {P^S}_{1n} \cdot b_{1n} & \cdots & {P^S}_{nn} \cdot b_{nn} \end{pmatrix} \tag{33}$$

denote the transpose of the (*Hadamard* or *Schur* or) *entrywise product* of P^S and B. Then the first-order conditions (29) for joint profit maximization by S's members and by all competitive firms $j \notin S$ can be written compactly as

$$A + B \cdot p + (P^S \circ B)' \cdot (p - \Gamma) = 0 \quad \text{with} \quad \Gamma = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{pmatrix}. \tag{34}$$

Under assumption (30) these conditions are necessary and sufficient for the respective profit maximums. The unique solution

$$p^S = [(P^S \circ B)' \cdot \Gamma - A] \cdot [B + (P^S \circ B)']^{-1} \tag{35}$$

summarizes equilibrium prices p_i^S of all products $i \in N$ if firms in S coordinate prices and the remaining ones act competitively.

6.3. Symmetric case

In a symmetric environment with

$$A = \begin{pmatrix} a \\ \vdots \\ a \end{pmatrix}, \quad B = \begin{pmatrix} -d & b & \cdots & b \\ b & -d & \cdots & b \\ \vdots & \vdots & \ddots & \vdots \\ b & b & \dots & -d \end{pmatrix} \text{ and } \Gamma = \begin{pmatrix} \gamma \\ \vdots \\ \gamma \end{pmatrix}, \tag{36}$$

the cartel price evaluates to

$$p^C := p_i^N = \left(\frac{a}{d-(n-1)b} + \gamma \right) \Big/ 2 \tag{37}$$

for each (symmetrically differentiated) product $i \in N$. Corresponding competitive prices are

$$p^B := p_i^{\varnothing} = \frac{a + d\gamma}{2d - (n-1)b} \text{ for all } i \in N. \tag{38}$$

This implies cartel overcharges of

$$\Delta p = p^C - p^B = \frac{a/d - \gamma(1 - \frac{1}{\alpha})}{4\alpha - 6 + 2/\alpha} \quad \text{with} \quad \alpha = \frac{d}{(n-1)b} > 1 \tag{39}$$

for each product $i \in N$. They are homogeneous of degree one in (a, γ) and strictly decreasing in differentiation parameter α as well as in unit costs γ. Note that $\lim_{\alpha \to \infty} \Delta p = 0$: collaboration by firms with independent demands entails no overcharges.

If there is a partial cartel S of size $s = 2, \dots, n-1$, equilibrium prices are

$$p_i^S = \begin{cases} \dfrac{a(2d+b) + \gamma\Big(2d^2 + bd(3-2s) + b^2(ns - n - s^2 + 1)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \in S, \\[2ex] \dfrac{a(2d - sb + 2b) + \gamma\Big(2d^2 - bd(s-2) - b^2(s^2 - s)\Big)}{4d^2 - 2(n+s-3)bd + b^2\eta_s} & \text{if } i \notin S \end{cases} \tag{40}$$

with $\eta_s = s(n-s) - 2(n-1)$ and $s = \#S$.[24] Comparing the price p_h^S of the home product $h \in N$ of a suing customer in case that the respective producer h is part of a cartel with s members, i.e., for $h \in S$, to the respective price p_h^S if h is not, i.e., for $h \notin S$,

[24]Static stability of the industry-wide cartel requires that the degree of differentiation is not too large for $n > 3$. This is no concern, however, in the derivation of bounds on contributions. See Deneckere and Davidson (1985), Weikard (2009) and Federgruen and Pierson (2011) on cartel profits under price competition and their relation to internal vs. external stability.

yields

$$\bar{v}^h(s) - \bar{v}^{\not h}(s) = p^h(s) - p^{\not h}(s) = \frac{b(s-1)\big(a - \gamma(b+d-bn)\big)}{4d^2 - 2(n+s-3)bd + \eta_s b^2}. \tag{41}$$

Inserting this into eq. (7) gives the Shapley allocation in absolute terms. Division by $v(N) = \Delta p$ yields h's share as an explicit function of the model's parameters:

$$\rho_h^* = \frac{\varphi_h(N,v)}{v(N)} = \frac{1}{n} + \frac{n-1}{n} \sum_{s=2}^{n-1} \frac{(s-1)\cdot(4\alpha^2 - 6\alpha + 2)}{4\alpha^2(n-1)^2 - 2(n+s-3)(n-1)\alpha + \eta_s}. \tag{42}$$

One can see that, in the symmetric case, the common unit cost γ or demand intercept a have no effect on h's share. It is determined only by the degree of differentiation, i.e., ratio $\alpha = d/(n-1)b$ of own and cross-price parameters. ρ_h^* is strictly increasing in α.

Discipline by all cartel members is the more important for maintaining an overcharge on product h, the lower the degree of differentiation. In the limit, each firm's participation is essential and contributes equally to Δp:

$$\lim_{\alpha\to 1} \rho_h^* = \frac{1}{n} \quad \text{and} \quad \lim_{\alpha \to 1} \rho_j^* = \frac{1}{n} \text{ for } j \neq h. \tag{43}$$

This coincides with the allocation in case of perfect substitutes (see Section 5.3). If, in contrast, products are highly differentiated, eq. (42) yields

$$\lim_{\alpha\to\infty} \rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)} \sum_{s=2}^{n-1} (s-1) = \frac{1}{2}. \tag{44}$$

So seller h must cover up to half of the compensation for its overcharges,[25] and we can complement the general lower bound in Proposition 3 by an upper bound:

Proposition 4. *Suppose $n \geq 3$ firms are symmetric in the linear market environment defined by equations (26), (27) and (30). If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N,v) \in \begin{cases} \left(\frac{v(N)}{n}, \frac{v(N)}{2}\right) & \text{if } i = h, \\ \left(\frac{v(N)}{2(n-1)}, \frac{v(N)}{n}\right) & \text{if } i \neq h. \end{cases} \tag{45}$$

Figure 1 illustrates the behavior of ρ_h^* for intermediate degrees of differentiation.

[25] Recall however that Δp vanishes as $\alpha \to \infty$.

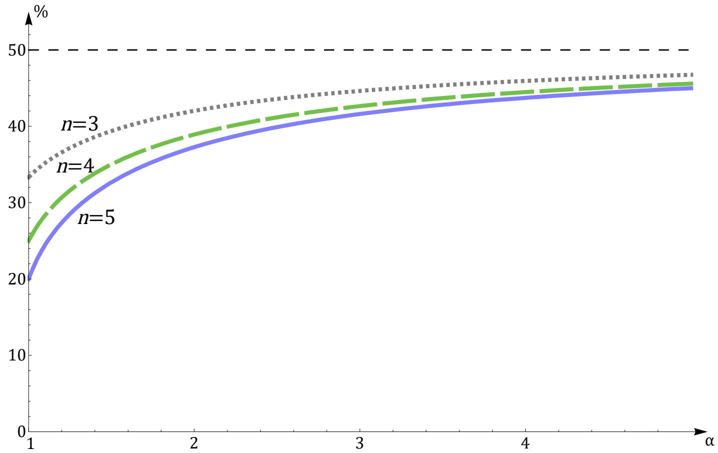

Figure 1: Share ρ_h^* of overcharge damages on product h attributed to its vendor ($a = 10$, $d = 2$, $\gamma = 0$, $b = 2/(n-1)\alpha$)

6.4. Asymmetric case

Bounds for the symmetric case provide guidance for mildly asymmetric markets by continuity. But things change when firms are truly heterogeneous. It is then possible that the producer h of a good for which compensation is sought will be assigned a *smaller* share than its competitors, i.e., $\varphi_h(N, v) < v(N)/n$. Such cases arise if the cross-price effects involving firm h are sufficiently smaller than those between other cartel members. We can, e.g., have three firms such that demands of firm 1 and 2 involve high mutual cross-price reactions b_{12} and b_{21}, while there are only small linkages b_{i3} and b_{3i} with firm 3 ($i \neq 3$). Firm 3's cartel participation contributes to the overcharges on p_1, p_2 and p_3 if all parameters are positive. But a significant increase of p_3 would have occurred even if firm 3 had not been part of the cartel and had just best-responded. This part of Δp_3 is caused by price increases on goods 1 and 2, which are mostly driven by shutting down competition between firms 1 and 2, not firm 3. Hence the former bear greater responsibility for Δp_3 than the latter.[26]

Asymmetry in cross-price effects does not come with useful bounds. One either needs to do full quantitative analysis based on specific parameter estimates. Or one uses a binary approximation as discussed in Section 5.3. (For the 3-firm example, approximations by $\mathcal{M}(v) = \{\{1, 2\}\}$ suggest themselves for Δp_1 and Δp_2; that by $\mathcal{M}(v) = \{\{1, 2, 3\}\}$ for Δp_3.)

[26]For numerical illustration, consider Section 2's example again: the Shapley value $\varphi(N, v^3)$ corresponds to shares $\rho^* = (35.5\%; 37.2\%; 27.3\%)$ for overcharge Δp_3.

Bounds for asymmetry in the demand parameters a_i or costs γ_i can be derived. But the computations become very tedious. Supposing $\gamma = 0$ and that firm-specific intercepts a_i are the only asymmetry, one can for instance compute

$$\Delta p_h = p_h^C - p_h^B = \frac{b(n-1)[b(3d+2b-bn)a_h + (2d^2+b^2n-b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(d+b-bn)(2d+b-bn)} \tag{46}$$

as the cartel's price increase for product h. It rises in the saturation level a_h of firm h's demand as well as in the average saturation quantity $\bar{a}_{-h} := \sum_{i \neq h} a_i/(n-1)$ of firms $i \neq h$.

The corresponding Shapley value of firm h in the allocation of Δp_h is

$$\varphi_h = \frac{\Delta p_h}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{b(s-1)[b(6d+b(s+4-n))a_h + (4d^2+\tau_s b^2)\bar{a}_{-h}]}{2(d+b)(2d+b)(4d^2-(2n-6+2s)db+\eta_s b^2)} \tag{47}$$

with $\tau_s := (n-s-2)$ and $\eta_s := s(n-s) - 2(n-1)$. The implied damage share of firm h can, after a good dose of algebraic manipulations, be written as a function of $\alpha = \frac{d}{b(n-1)}$ and $\bar{a}_{-h}/a_h$ as follows

$$\rho_h^* = \frac{1}{n} + \frac{1}{n(n-1)}\sum_{s=2}^{n-1} \frac{(s-1)\Big[6\alpha(n-1) + (s+4-n) + \Big(4\alpha^2(n-1)^2 + \tau_s\Big)\frac{\bar{a}_{-h}}{a_h}\Big] \cdot (\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big) \cdot \Big[(3\alpha + \frac{2-n}{n-1}) + (2\alpha^2(n-1)+1)\frac{\bar{a}_{-h}}{a_h}\Big]}. \tag{48}$$

Ratio $\bar{a}_{-h}/a_h$ relates the market sizes of firm h and its competitors: a large ratio means firm h is comparatively small, a ratio close to zero that h's market is big.

It can be checked that ρ_h^* is strictly decreasing in $\bar{a}_{-h}/a_h$. From that follows

$$\rho_h^* \leq \lim_{\bar{a}_{-h}/a_h \to 0} \rho_h^* = \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{(s-1)}{(n-1)} \cdot \frac{\Big[6\alpha(n-1) + (s+4-n)\Big] \cdot (\alpha-1)(2\alpha-1)}{\Big(4\alpha^2(n-1) - (2n-6+2s)\alpha + \frac{\eta_s}{n-1}\Big) \cdot \Big(3\alpha + \frac{2-n}{n-1}\Big)}. \tag{49}$$

The right-most fraction, with terms involving α, is maximal for $s = n-1$. This maximum can be shown to be strictly increasing in α. It is hence bounded by its limit as $\alpha \to \infty$, which evaluates to 1. This gives

$$\rho_h^* \leq \frac{1}{n} + \frac{1}{n}\sum_{s=2}^{n-1} \frac{s-1}{n-1} = \frac{1}{2} \tag{50}$$

as an upper bound. A lower bound $1/n \leq \rho_h^*$ follows from considering $\bar{a}_{-h}/a_h \to \infty$ and $\alpha \to 1$. (Details are available upon request.)

Hence the same bounds as for symmetric firms obtain if only demand parameters a_i vary *and* we focus on firm h's share in allocating harm of its customers. Things

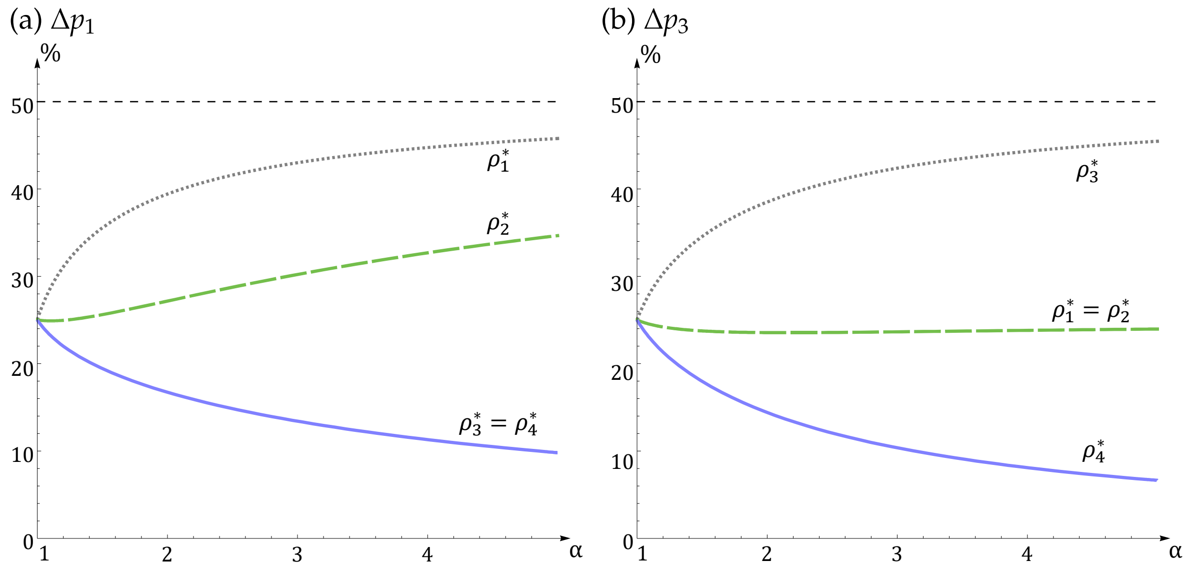

Figure 2: Shares ρ^* for cost leaders $i = 1, 2$ and laggards $j = 3, 4$

differ for a firm $j \neq h$, however. The key determinant of j's share in Δp_h is $\tilde{a}_{-h,j} := \sum_{i \in N \setminus \{h,j\}} a_i/(n-2)$, the average demand intercept of firms other than h and j. If $a_j \gg \tilde{a}_{-h,j}$ then j is the only large competitor of firm h and both end up splitting Δp_h about 50 : 50. If conversely the market size of firm j is negligible compared to that of h's other competitors (i.e., $a_j \ll \tilde{a}_{-h,j}$) then j is basically a null player.

We can state the following analogue to Proposition 4 on the one hand:

Proposition 5. *Suppose $n \geq 3$ firms are symmetric except for the demand intercepts $a_1, \ldots, a_n$ in the linear market environment defined by equations (26), (27) and (30) with $\gamma = 0$. If v reflects damages to a customer of firm $h \in N$, then*

$$\varphi_i(N, v) \in \begin{cases} \left(\frac{v(N)}{n}, \frac{v(N)}{2}\right) & \text{if } i = h, \\ \left(0, \frac{v(N)}{2}\right) & \text{if } i \neq h. \end{cases} \quad (51)$$

On the other hand, bounds concerning competitors $i \neq h$ of the suing customer's seller are now so wide that they are unlikely to be of practical help.

The same applies to firms which are symmetric in all but technology. This is illustrated in Figure 2. It considers two low-cost and two high-cost producers with common parameters $a = 10$, $d = 2$, and $b = \frac{2}{3\alpha}$. No matter whether the selling firm has (a) low costs $\gamma_i = 1$ or (b) high costs $\gamma_j = 5$, it bears between 25% and 50% of overcharges on its product, and always the greatest share. In the former case, the share of the other low-cost firm increases in differentiation and approaches 50% for

$\alpha \to \infty$. The share of a high-cost firm in overcharges on the product of a competitor falls in α and eventually vanishes.

7. Comparison to heuristics

The data requirements for the quantification of a real damage allocation problem (N, v) are the same as for merger simulation analysis. The latter has established itself in the toolkit of competition policy,[27] but a reliable heuristic could save the high costs of calibrating a suitable model. Perhaps market shares, which are much easier to obtain than demand and cost estimates, inadvertently are a good proxy for whose cartel participation contributed how much to damages, at least under some identifiable circumstances? If yes, should we use sales or revenues? From the cartel or competitive regime? Or perhaps better use some profit measure after all?

We can address these issues by some *in vitro* comparisons. Specifically, we consider Shapley allocations under a range of parameter choices for the linear model of Section 6 and numerically compare deviations from this benchmark for some of the heuristics discussed in Section 2. We also contrast the Shapley allocation of the full model with that of its product-specific best binary approximations (see Section 5.3).

We adopt an aggregate perspective here and suppose that *every* harmed customer goes after the cartel. Then the total overcharge damage

$$D := \sum_{i \in N} q_i^C \cdot \Delta p_i \tag{52}$$

will either be allocated according to the Shapley allocation $\varphi(N, v^j)$ for each individual product j, or according to some heuristic. Firm i's aggregate Shapley payments are

$$\Phi_i := \sum_{j \in N} \varphi_i(N, v^j) = \sum_{j \in N} q_j^C \cdot \Delta p_j \cdot \rho_i^*(N, v^j). \tag{53}$$

Absolute values of over or under-payments relative to Φ_i are summed across firms

[27] See Budzinski and Ruhmer (2010) for a survey, Weinberg (2011) or Knittel and Metaxoglou (2011) for critical discussions and, e.g., COMP/M.5644–Kraft Foods/Cadbury or COMP/M.5658–Unilever/Sara Lee for applications to cases in Europe. In the US, prominent cases include U.S. District Court for the Northern District of California, U.S. v. Oracle Corp., 2004.

and normalized to give an index of *aggregate mis-allocation of damages*

$$M^{\rho} = \sum_{i \in N} \mid \Phi_i - H_i^{\rho} \mid \Big/ D \qquad (54)$$

where H_i^{ρ} denotes aggregate payments by firm i according to heuristic shares ρ. M^{ρ} is proportional to the expected mis-allocation of compensation for a unit purchase by a randomly drawn customer, for a customer who made purchases from all firms in proportion to their cartel sales, or when all customers go after the cartel with identical positive probabilities.[28] Considering M^{ρ} rather than over and under-payments at the product-specific level gives heuristics a good shot: differential responsibilities for own and other firms' customers (cf. Proposition 3) can net out across products. In particular, an equal distribution per head, by market shares, or by profits all yield zero aggregate mis-allocation for symmetric environments.

We hence focus on asymmetric configurations and report on six distinct variations of the example underlying Figure 2. The baseline parameters are $\gamma = 1$, $a = 10$, $d = 2$ and $b = d/3\alpha$; we break symmetry for one parameter at a time. Several other variations which we tried, e.g., with six firms instead of four, yielded similar patterns.

The two top panels of Figure 3 consider heterogeneity in firm-specific market sizes a_i. Panel (a) involves two large and two small firms; in panel (b) all differ. An *equal per head* allocation ρ^0 non-surprisingly performs well when differentiation is very low. But, among the simple heuristics discussed in Section 2, it soon loses out to allocating damages in proportion to *market shares based on competitive sales* ρ^4 and to *market shares based on cartel sales* ρ^2. Market shares determined by *cartel revenues* ρ^1 or *competitive revenues* ρ^3 produce high mis-allocations at all levels of differentiation. Only allocation in proportion to *cartel profits* ρ^5 is worse. The smallest mis-allocations follow from aggregating the Shapley values of product-specific *dichotomous damage scenarios*, denoted as ρ^D.

We grant it would be heroic to presume that one could always identify the first-best binary approximation $(N, \tilde{v}^i)$ of an unknown true damage allocation problem (N, v^i) (based here on $\| \cdot \|_1$-distances between induced Shapley allocations, with switches of

[28]The latter may be a plausible a priori assumption. It suggests that cartel members should pool their contribution obligations in a kind of trust in order to save on transaction costs. Symmetry of firms would then directly call for symmetric shares in the trust. The analysis in this section shows, however, that no simple market or profit share rule applies to funding the trust under asymmetry.

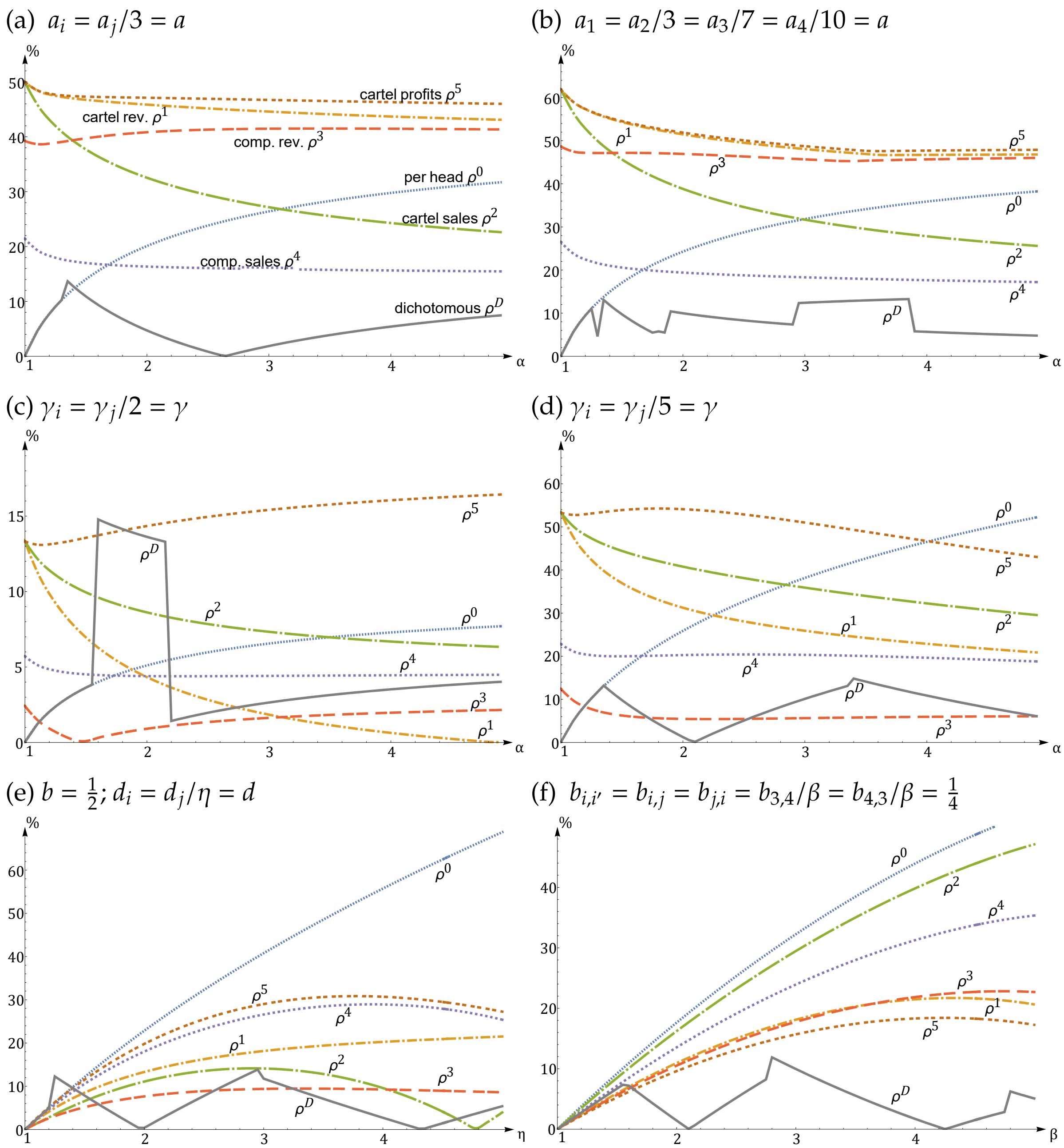

Figure 3: Mis-allocation M^ρ by different heuristics considering $i = 1, 2$ and $j = 3, 4$

the minimizers generating the kinks). We have checked, however, that – apart for panel (c), where mis-allocations are small across the board – performance is quite similar if the respective second best-possible approximations $\tilde{\tilde{v}}^i$ are invoked.[29]

Panels (c) and (d) assume an intermediate and a big cost asymmetry between firms 1 and 2 vs. firms 3 and 4. The deviations from the Shapley payments, aggregated for each

[29] In fact, using the second-best approximation of each (N, v^i) with $i \in \{1, 2, 3, 4\}$ performs better than ρ^D for some intervals of α, η or β: there, bigger mis-allocations at the product level happen to cancel more of each other at the firm level than they do for the best approximations.

firm across all four overcharges, is significantly higher with the bigger asymmetry in (d). The kink which is visible in panel (c) for ρ^3 – or ρ^2 in (e) – result from cancellation of product-specific deviations at the firm level when these initially have opposite signs but switch to same sign. Revenue-based market shares ρ^1 or ρ^3 and sales-based competitive market shares ρ^4 all perform well. For several parameter constellations, they beat the discretization heuristic ρ^D by a small absolute margin.

The latter also holds in panel (e), which assumes firms 3 and 4 to face bigger own-price elasticities than firms 1 and 2. Market shares based on cartel sales or competitive revenues are close to the Shapley value, as far as aggregate payments to all victims are concerned. The final panel (f) assumes heterogeneity in cross-price effects: firms 1 and 2 face a fixed cross-price parameter of $1/4$, competition between firms 3 and 4 is more intense by some factor β. Somewhat unexpectedly, after investigating five environments in which its ranking was consistently low, the allocation ρ^5 by cartel profits comes closest to representing a short-cut to the exact Shapley payments among the simple heuristics. The heuristic ρ^D of using binary damage approximations at the product level is still much better here.

Overall, as one might have suspected, there is *no* heuristic which always out-performs the others. Those based on market shares – preferably sales-based for heterogeneity in a_i, otherwise revenue-based – tend to score better than a profits-based division; but panel (f) provides an exception to the rule. Generally, when firms produce close substitutes and hence ratio $\alpha = d/(n-1)b$ is close to 1, an equal division by heads performs well. This comes with the warning that all figures consider *aggregate* mis-allocation. If different fractions of customers across firms seek compensation for their harm, the picture looks much worse for heuristics based on aggregate market statistics. In contrast, discretizations at the product level robustly perform well for compensation of customers at the individual level as well as in total.

8. Extension to leniency rules

Regarding the conduct of a firm, we have so far discriminated only between being a member of the cartel or competing with it. There are at least two cases where the specific roles of firms require more attention.

First, some members may have acted as *ringleaders* of the cartel and therefore bear greater responsibility for inflicted harm than otherwise. The legal literature points to the role of *leader in an infringement*, i.e., in organizing the operations of an existing cartel, and of *instigator of an infringement* by particularly furthering the establishment or enlargement of a cartel (see EC Case T-15/02 (14)). A cartel's success and therefore also the harm it causes are known to vary in its organizational characteristics. See, e.g., Harrington (2006), Levenstein and Suslow (2006), Connor and Bolotova (2006), Davies and De (2013) or Awaya and Krishna (2016). Responsibility allocations based only on a model of cost and demand structure likely understate a ringleader's due share in compensating victims.

Second, firms which are granted immunity by the authorities can often ask for special treatment also regarding the civil law consequences of their offenses. Liability exemptions raise the attractiveness of coming clean just like immunity from fines and criminal charges. Many cartels are brought down not by external investigators but insiders who fear others take advantage of immunity before they do. Suitably structured leniency rules can foster distrust and help to deter cartels (see, e.g., Leslie 2006, Harrington and Chang 2009, Miller 2009, and Bigoni et al. 2012). This is appreciated in Directive 2014/104/EU (see recital 38) and Article 11(4) makes the leniency provision that "*... an immunity recipient is jointly and severally liable as follows: (a) to its direct or indirect purchasers or providers; and (b) to other parties only where full compensation cannot be obtained from the other undertakings that were involved in the same infringement of competition law*."

The restricted role of an immunity recipient in compensating victims can be incorporated into the proposed systematic approach rather easily. The key modification is to replace the symmetry requirement (SYM) in Section 3's identification of a responsibility-based damage allocation rule by something more flexible. This leads to the use of *weighted Shapley values*.

These were first suggested by Shapley (1953b). Their prominent axiomatic characterization by Kalai and Samet (1987) takes up the result of Shapley (1953a) and relaxes symmetry to requiring merely that $v(S)$ is distributed in a 'consistent' way whenever S is a 'partnership'. The latter entails that the members of S make contributions to coalitions of other players only when they are together, i.e., $v(R \cup T) = v(R)$ for any strict subset $T \subset S$ and any $R \subseteq N \setminus S$. The members of a partnership are symmetric to another in terms of their marginal contributions. If surplus or costs must be split asymmetrically, for reasons not reflected by v, at least there should be no inconsistency between a two-step allocation – first to partnerships in their entirety, then internally – or one directly to individual members.

Including this requirement in the list of desirable properties, while dropping symmetry, turns out to impose a non-negative vector $\omega = (\omega_1, \ldots, \omega_n)$ of weights. This modifies the symmetric Shapley value φ to φ^ω such that the shares of players $i \in T$ in any carrier game (N, u_T) over $T \subseteq N$ (where $u_T(S) = 1$ if $T \subseteq S$ and 0 otherwise; cf. fn. 14) are proportional to their weights, i.e., $\varphi_i^\omega(N, u_T) = \omega_i / \sum_{j \in T} \omega_j$ if $i \in T$ and 0 otherwise.

The leniency rules in Article 11(4) can therewith be accommodated as follows:[30] (i) use φ^ω with $\omega = (1, \ldots, 1)$, i.e., the standard Shapley value φ, for allocating any overcharge damages (N, v^l) which have accrued to direct or indirect purchasers of the goods produced by leniency recipient $l \in N$; (ii) by contrast use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega} = (1, \ldots, 1, 0, 1, \ldots, 1)$ where $\tilde{\omega}_l = 0$ when overcharges by l's competitors are concerned. This can be generalized to the case of multiple immunity recipients $L \subset N$: use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_i = 1$ if $i \notin L$ or overcharges Δp_i are concerned, and $\tilde{\omega}_i = 0$ otherwise.[31]

The same kind of extension can account for elevated responsibilities that derive from ringleader positions. Namely, use $\varphi^{\tilde{\omega}}$ with $\tilde{\omega}_r = \varepsilon > 1$ for any ringleader $r \in R \subset N$, and $\tilde{\omega}_i = 1$ for conventional cartel members $i \notin R$. The appropriate value for ε – or possibly different levels $\varepsilon_1 \geq \varepsilon_2 \geq \ldots > 1$ when there existed multiple ringleaders – depends on how pronounced the respective leadership or instigating role was. This is outside the scope of our setup. Criminal rulings and fines against the cartel members, which usually precede civil-law proceedings by victims in the EU, may provide a reasonable reference point.

9. Concluding remarks

Big part of the damages caused by cartels will probably continue to go uncompensated, despite lower legal hurdles in the EU. Overcharges associated with, e.g., the European

[30] Same applies to the liability restriction in Article 11(2) for *small or medium-sized enterprises*.

[31] We simplify here. Potential divisions by zero are avoided by actually working with a lexicographic *weight system*, consisting of strictly positive weights and an ordered partition of N into classes $N_1, \ldots, N_m$. Members of N_p receive zero when a carrier T also involves members of N_q with $p < q$. See Kalai and Samet (1987). Also see Nowak and Radzik (1995) on axiomatizing φ^ω based on (MRG) and (EFF).

truck cartel of 1997 to 2011 have been claimed to reach more than €10 000 per vehicle, with sales of up to 10 mio. trucks (Bentham Europe Ltd 2016). It is unlikely that the various victims – from the federation of the German logistics and trucking industry (BGL), bundling claims for up to 100 000 trucks, to small firms or municipalities with a single lorry procurement – will all press for redress with success before their claims are barred. Still, compensation payments and the need to assign contributions to individual members of a cartel are gaining importance.

Our analysis has derived relative responsibility for a given overcharge damage from the systematic consideration of all conceivable but-for scenarios regarding cartel membership. This is not necessarily congruent with the legal interpretations that national legislatures and courts in Europe will give 2014's EU Directive on Antitrust Damages Actions. There is substantial inherent difficulty in allocating individual responsibilities to joint tortfeasors.[32] One might conclude that this unavoidably generates arbitrariness and unresolvable disagreement; which could simply make it too complex to apportion contributions based on individual responsibility. Such suspicion prevailed when the US Supreme Court opted for no contributions in its ruling on Texas Industries, Inc. v. Radcliff Materials, Inc. in 1981 (cf. 451 U.S. 637–38).

We have tried to argue in this paper that a non-arbitrary way of allocating contributions in line with relative responsibilities exists. The co-conspirators are to start out with equal shares of any compensation payment; these shares then are to be corrected, in a well-defined way, for greater or smaller-than-average effects on the overcharge in question. This follows (Theorem 2) from the transparent translation of the requirement that contributions are determined by responsibility for harm into the marginality property of Young (1985) and other natural requirements.

The requirements' simultaneous satisfaction makes the Shapley value the right instrument for allocating cartel damages. This paper investigated what its use entails and how it relates to ad hoc approaches. For a start, we focused on particularly simple cost and demand structures and adopted a static perspective. Future research can hopefully extend some of the findings to more general settings. A first worthwhile robustness check would, e.g., be to replace the linear demand structure in Section 6 by a log-linear one or the popular almost ideal demand system (AIDS) of Deaton and Muellbauer (1980).

[32] See Braham and van Hees (2012) for a comprehensive philosophical account.

Of course, the merits of the damage allocations which result from applying the Shapley value depend on the quality of its input, i.e., the description of counterfactual overcharge scenarios by a characteristic function v. Reaching a reasonable level of agreement on this in court is bound to be difficult. This, however, is no conceptual flaw of the approach. It is a practical challenge which involves the modeling of a market and estimations. The same already arises when harm is quantified and the compensation that is to be paid out to the plaintiff is defined.

Once defined, it is tempting to divide damage payments by a quick-and-easy criterion. Baker (2004), for instance, "would recommend to a foreign parliament that it provide for contribution based on a single variable, such as sales of the product during the conspiracy" (p. 388). He holds that some contributions are fairer than none (as in US antitrust law) but reasons that "a multi-factor formula" could be hard to administer. We would reply that a binary description of which firms' cartel participation had big price effects compared to other constellations could probably be given by a clerk based on just a few phone calls. As we have seen in Section 7, using Shapley's formula even with coarse approximations tends to reflect responsibility better than a fixed indicator of market shares.

The usual trade-offs exist between tractability and reflecting temporal, spatial or other details of specific cartel operations. The truck and elevator businesses, air cargo, markets for automotive parts, sugar, etc. clearly differ. But the close analogy between investigating overcharges and merger simulation analysis suggests that viable balances can be struck. We prefer to endorse the implicit assumption in Article 11 of Directive 2014/104/EU rather than that of the US Supreme Court: it is not prohibitively complex but feasible to allocate cartel damages in line with responsibility.

A. Appendix: All dichotomous damage scenarios with n = 5 firms

$\mathcal{M}(v)$	$60 \cdot \varphi(N,v)$	$\mathcal{M}(v)$	$60 \cdot \varphi(N,v)$
1.–19. see Table 2 on p. 22		71. AB, AC, ADE, BCDE	(30, 10, 10, 5, 5)
20. AB, AC, AD, AE	(48, 3, 3, 3, 3)	72. AB, AC, ADE, BDE, CDE	(24, 9, 9, 9, 9)
21. AB, AC, AD, AE, BC	(28, 13, 13, 3, 3)	73. AB, AC, BC, ADE	(22, 17, 17, 2, 2)
22. AB, AC, AD, AE, BC, BD	(23, 18, 8, 8, 3)	74. AB, AC, BC, ADE, BDE	(19, 19, 14, 4, 4)
23. AB, AC, AD, AE, BC, BD, BE	(21, 21, 6, 6, 6)	75. AB, AC, BC, ADE, BDE, CDE	(16, 16, 16, 6, 6)
24. AB, AC, AD, AE, BC, BD, BE, CD	(16, 16, 11, 11, 6)	76. AB, AC, BC, DE	(14, 14, 14, 9, 9)
25. AB, AC, AD, AE, BC, BD, BE, CD, CE	(14, 14, 14, 9, 9)	77. AB, AC, BCD, BCE	(22, 17, 17, 2, 2)
26. AB,AC,AD,AE,BC,BD,BE,CD,CE,DE	(12, 12, 12, 12, 12)	78. AB, AC, BCD, BCE, BDE	(19, 19, 14, 4, 4)
27. AB, AC, AD, AE, BC, BD, BE, CDE	(18, 18, 8, 8, 8)	79. AB, AC, BCD, BCE, BDE, CDE	(16, 16, 16, 6, 6)
28. AB, AC, AD, AE, BC, BD, CD	(18, 13, 13, 13, 3)	80. AB, AC, BCD, BDE	(22, 17, 12, 7, 2)
29. AB, AC, AD, AE, BC, BD, CE	(18, 13, 13, 8, 8)	81. AB, AC, BCD, BDE, CDE	(19, 14, 14, 9, 4)
30. AB, AC, AD, AE, BC, BD, CE, DE	(16, 11, 11, 11, 11)	82. AB, AC, BCDE	(28, 13, 13, 3, 3)
31. AB, AC, AD, AE, BC, BD, CDE	(20, 15, 10, 10, 5)	83. AB, AC, BD, ADE	(22, 17, 7, 12, 2)
32. AB, AC, AD, AE, BC, BDE	(25, 15, 10, 5, 5)	84. AB, AC, BD, ADE, BCE	(19, 19, 9, 9, 4)
33. AB, AC, AD, AE, BC, BDE, CDE	(22, 12, 12, 7, 7)	85. AB, AC, BD, ADE, BCE, CDE	(16, 16, 11, 11, 6)
34. AB, AC, AD, AE, BC, DE	(20, 10, 10, 10, 10)	86. AB, AC, BD, ADE, CDE	(19, 14, 9, 14, 4)
35. AB, AC, AD, AE, BCD	(33, 8, 8, 8, 3)	87. AB, AC, BD, CD, ADE	(17, 12, 12, 17, 2)
36. AB, AC, AD, AE, BCD, BCE	(30, 10, 10, 5, 5)	88. AB, AC, BD, CD, ADE, BCE	(14, 14, 14, 14, 4)
37. AB, AC, AD, AE, BCD, BCE, BDE	(27, 12, 7, 7, 7)	89. AB, AC, BD, CDE	(17, 17, 12, 12, 2)
38. AB,AC,AD,AE,BCD,BCE,BDE,CDE	(24, 9, 9, 9, 9)	90. AB, AC, BD, CE	(12, 17, 17, 7, 7)
39. AB, AC, AD, AE, BCDE	(36, 6, 6, 6, 6)	91. AB, AC, BD, CE, ADE	(14, 14, 14, 9, 9)
40. AB, AC, AD, BC, BD, CDE	(17, 17, 12, 12, 2)	92. AB, AC, BD, CE, DE	(12, 12, 12, 12, 12)
41. AB, AC, AD, BC, BD, CE	(15, 15, 15, 10, 5)	93. AB, AC, BDE	(25, 15, 10, 5, 5)
42. AB, AC, AD, BC, BD, CE, DE	(13, 13, 13, 13, 8)	94. AB, AC, BDE, CDE	(22, 12, 12, 7, 7)
43. AB, AC, AD, BC, BDE	(22, 17, 12, 7, 2)	95. AB, AC, DE	(22, 7, 7, 12, 12)
44. AB, AC, AD, BC, BDE, CDE	(19, 14, 14, 9, 4)	96. AB, AC, DE, BCD	(19, 9, 9, 14, 9)
45. AB, AC, AD, BC, BE	(20, 20, 10, 5, 5)	97. AB, AC, DE, BCD, BCE	(16, 11, 11, 11, 11)
46. AB, AC, AD, BC, BE, CDE	(17, 17, 12, 7, 7)	98. AB, ACD, ACE	(37, 12, 7, 2, 2)
47. AB, AC, AD, BC, BE, DE	(15, 15, 10, 10, 10)	99. AB, ACD, ACE, ADE	(39, 9, 4, 4, 4)
48. AB, AC, AD, BC, DE	(17, 12, 12, 12, 7)	100. AB, ACD, ACE, ADE, BCD	(24, 14, 9, 9, 4)
49. AB, AC, AD, BCD, BCE	(27, 12, 12, 7, 2)	101. AB, ACD, ACE, ADE, BCD, BCE	(21, 16, 11, 6, 6)
50. AB, AC, AD, BCD, BCE, BDE	(24, 14, 9, 9, 4)	102. AB, ACD, ACE, ADE, BCD, BCE, BDE	(18, 18, 8, 8, 8)
51. AB, AC, AD, BCD, BCE, BDE, CDE	(21, 11, 11, 11, 6)	103. AB,ACD,ACE,ADE,BCD,BCE,BDE,CDE	(15, 15, 10, 10, 10)
52. AB, AC, AD, BCDE	(33, 8, 8, 8, 3)	104. AB, ACD, ACE, ADE, BCD, BCE, CDE	(18, 13, 13, 8, 8)
53. AB, AC, AD, BCE	(30, 10, 10, 5, 5)	105. AB, ACD, ACE, ADE, BCD, CDE	(21, 11, 11, 11, 6)
54. AB, AC, AD, BCE, BDE	(27, 12, 7, 7, 7)	106. AB, ACD, ACE, ADE, BCDE	(27, 12, 7, 7, 7)
55. AB, AC, AD, BCE, BDE, CDE	(24, 9, 9, 9, 9)	107. AB, ACD, ACE, ADE, CDE	(24, 9, 9, 9, 9)
56. AB, AC, AD, BE	(25, 15, 5, 5, 10)	108. AB, ACD, ACE, BCD	(22, 17, 12, 7, 2)
57. AB, AC, AD, BE, BCD	(22, 17, 7, 7, 7)	109. AB, ACD, ACE, BCD, BCE	(19, 19, 14, 4, 4)
58. AB, AC, AD, BE, BCD, CDE	(19, 14, 9, 9, 9)	110. AB, ACD, ACE, BCD, BCE, CDE	(16, 16, 16, 6, 6)
59. AB, AC, AD, BE, CDE	(22, 12, 7, 7, 12)	111. AB, ACD, ACE, BCD, BDE	(19, 19, 9, 9, 4)
60. AB, AC, AD, BE, CE	(20, 10, 10, 5, 15)	112. AB, ACD, ACE, BCD, BDE, CDE	(16, 16, 11, 11, 6)
61. AB, AC, AD, BE, CE, BCD	(17, 12, 12, 7, 12)	113. AB, ACD, ACE, BCD, CDE	(19, 14, 14, 9, 4)
62. AB, AC, AD, BE, CE, DE	(18, 8, 8, 8, 18)	114. AB, ACD, ACE, BCDE	(25, 15, 10, 5, 5)
63. AB, AC, AD, BE, CE, DE, BCD	(15, 10, 10, 10, 15)	115. AB, ACD, ACE, BDE	(22, 17, 7, 7, 7)
64. AB, AC, ADE	(42, 7, 7, 2, 2)	116. AB, ACD, ACE, BDE, CDE	(19, 14, 9, 9, 9)
65. AB, AC, ADE, BCD	(27, 12, 12, 7, 2)	117. AB, ACD, ACE, CDE	(22, 12, 12, 7, 7)
66. AB, AC, ADE, BCD, BCE	(24, 14, 14, 4, 4)	118. AB, ACD, BCD, CDE	(17, 17, 12, 12, 2)
67. AB, AC, ADE, BCD, BCE, BDE	(21, 16, 11, 6, 6)	119. AB, ACD, BCDE	(23, 18, 8, 8, 3)
68. AB, AC, ADE, BCD, BCE, BDE, CDE	(18, 13, 13, 8, 8)	120. AB, ACD, BCE	(20, 20, 10, 5, 5)
69. AB, AC, ADE, BCD, BDE	(24, 14, 9, 9, 4)	121. AB, ACD, BCE, CDE	(17, 17, 12, 7, 7)
70. AB, AC, ADE, BCD, BDE, CDE	(21, 11, 11, 11, 6)	122. AB, ACD, CDE	(20, 15, 10, 10, 5)

	$\mathcal{M}(v)$	$60 \cdot \varphi(N,v)$
123.	AB, ACDE	(33, 18, 3, 3, 3)
124.	AB, AC, ADE, BDE	(27, 12, 7, 7, 7)
125.	AB, ACDE, BCDE	(21, 21, 6, 6, 6)
126.	AB, CD, ACE	(17, 12, 17, 12, 2)
127.	AB, CD, ACE, ADE	(19, 9, 14, 14, 4)
128.	AB, CD, ACE, ADE, BCE	(16, 11, 16, 11, 6)
129.	AB, CD, ACE, ADE, BCE, BDE	(13, 13, 13, 13, 8)
130.	AB, CE, ACE, BDE	(14, 14, 14, 14, 4)
131.	AB, CDE	(18, 18, 8, 8, 8)
132.	ABC, ABD, ABE	(27, 27, 2, 2, 2)
133.	ABC, ABD, ABE, ACD	(32, 12, 7, 7, 2)
134.	ABC, ABD, ABE, ACD, ACE	(34, 9, 9, 4, 4)
135.	ABC, ABD, ABE, ACD, ACE, ADE	(36, 6, 6, 6, 6)
136.	ABC, ABD, ABE, ACD, ACE, ADE, BCD	(21, 11, 11, 11, 6)
137.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE	(18, 13, 13, 8, 8)
138.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE	(15, 15, 10, 10, 10)
139.	ABC, ABD, ABE, ACD, ACE, ADE, BCD, BCE, BDE, CDE	(12, 12, 12, 12, 12)
140.	ABC, ABD, ABE, ACD, ACE, ADE, BCDE	(24, 9, 9, 9, 9)
141.	ABC, ABD, ABE, ACD, ACE, BCD	(19, 14, 14, 9, 4)
142.	ABC, ABD, ABE, ACD, ACE, BCD, BCE	(16, 16, 16, 6, 6)
143.	ABC, ABD, ABE, ACD, ACE, BCD, BDE	(16, 16, 11, 11, 6)
144.	ABC, ABD, ABE, ACD, ACE, BCD, BDE, CDE	(13, 13, 13, 13, 8)
145.	ABC, ABD, ABE, ACD, ACE, BCDE	(22, 12, 12, 7, 7)
146.	ABC, ABD, ABE, ACD, ACE, BDE	(19, 14, 9, 9, 9)
147.	ABC, ABD, ABE, ACD, ACE, BDE, CDE	(16, 11, 11, 11, 11)
148.	ABC, ABD, ABE, ACD, BCD	(17, 17, 12, 12, 2)
149.	ABC, ABD, ABE, ACD, BCD, CDE	(14, 14, 14, 14, 4)
150.	ABC, ABD, ABE, ACD, BCDE	(20, 15, 10, 10, 5)
151.	ABC, ABD, ABE, ACD, BCE	(17, 17, 12, 7, 7)
152.	ABC, ABD, ABE, ACD, BCE, CDE	(14, 14, 14, 9, 9)
153.	ABC, ABD, ABE, ACD, CDE	(17, 12, 12, 12, 7)
154.	ABC, ABD, ABE, ACDE	(30, 15, 5, 5, 5)
155.	ABC, ABD, ABE, ACDE, BCDE	(18, 18, 8, 8, 8)
156.	ABC, ABD, BCE	(30, 10, 10, 5, 5)
157.	ABC, ABD, ABE, CDE	(15, 15, 10, 10, 10)
158.	ABC, ABD, ACD, BCE	(15, 15, 15, 10, 5)
159.	ABC, ABD, ACD, BCE, BDE	(12, 17, 12, 12, 7)
160.	ABC, ABD, ACD, BCE, BDE, CDE	(9, 14, 14, 14, 9)
161.	ABC, ABD, ACD, BCDE	(18, 13, 13, 13, 3)
162.	ABC, ABD, ACE, ADE	(32, 7, 7, 7, 7)
163.	ABC, ABD, ACE, ADE, BCDE	(20, 10, 10, 10, 10)
164.	ABC, ABD, ACE, BCDE	(18, 13, 13, 8, 8)
165.	ABC, ABD, ACE, BDE	(15, 15, 10, 10, 10)
166.	ABC, ABD, ACE, BDE, CDE	(12, 12, 12, 12, 12)
167.	ABC, ABD, ACDE	(28, 13, 8, 8, 3)
168.	ABC, ABD, ACDE, BCDE	(16, 16, 11, 11, 6)
169.	ABC, ABD, CDE	(13, 13, 13, 13, 8)
170.	ABC, ABDE	(23, 23, 8, 3, 3)
171.	ABC, ABDE, ACDE	(26, 11, 11, 6, 6)
172.	ABC, ABDE, ACDE, BCDE	(14, 14, 14, 9, 9)
173.	ABC, ADE	(28, 8, 8, 8, 8)
174.	ABC, ADE, BCDE	(16, 11, 11, 11, 11)
175.	ABCD, ABCE	(18, 18, 18, 3, 3)
176.	ABCD, ABCE, ABDE	(21, 21, 6, 6, 6)
177.	ABCD, ABCE, ABDE, ACDE	(24, 9, 9, 9, 9)
178.	ABCD, ABCE, ABDE, ACDE, BCDE	(12, 12, 12, 12, 12)
179.	ABCDE	(12, 12, 12, 12, 12)

References

Awaya, Y. and V. Krishna (2016). On communication and collusion. *American Economic Review 106*(2), 285–315.

Baker, D. I. (2004). Revisiting history: What have we learned about private antitrust enforcement that we would recommend to others? *Loyola Consumer Law Review 16*(4), 379–408.

Baker, J. B. and D. L. Rubinfeld (1999). Empirical methods in antitrust litigation: Review and critique. *American Law and Economics Review 1*(1/2), 386–435.

Baldan, J. (1992). Properties of simple games. Mimeo, Drew University.

Basso, L. J. and T. W. Ross (2010). Measuring the true harm from price-fixing to both direct and indirect purchasers. *Journal of Industrial Economics 58*(4), 895–927.

Bentham Europe Ltd (2016, accessed March 7, 2017). *Proposed Truck Cartel Claim – Media Release*. `http://www.benthameurope.com/en/proposed-truck-cartel-claim-media-release`.

Bigoni, M., S.-O. Fridolfsson, C. Le Coq, and G. Spagnolo (2012). Fines, leniency, and rewards in antitrust. *RAND Journal of Economics 43*(2), 368–390.

Bos, I. and J. E. Harrington (2010). Endogenous cartel formation with heterogeneous firms. *RAND Journal of Economics 41*(1), 92–117.

Braham, M. and M. van Hees (2012). An anatomy of moral responsibility. *Mind 121*(483), 601–634.

Budzinski, O. and I. Ruhmer (2010). Merger simulation in competition policy: A survey. *Journal of Competition Law and Economics 6*(2), 277–319.

Connor, J. M. and Y. Bolotova (2006). Cartel overcharges: Survey and meta-analysis. *International Journal of Industrial Organization 24*(6), 1109–1137.

Davies, S. and O. De (2013). Ringleaders in larger number asymmetric cartels. *Economic Journal 123*(572), F524–F544.

Davis, P. and E. Garcés (2009). *Quantitative Techniques for Competition and Antitrust Analysis*. Princeton, NJ: Princeton University Press.

Deaton, A. and J. Muellbauer (1980). An almost ideal demand system. *American Economic Review 70*(3), 312–326.

Dehez, P. and S. Ferey (2013). How to share joint liability: A cooperative game approach. *Mathematical Social Sciences 66*(1), 44–50.

Dehez, P. and S. Ferey (2016). Multiple causation, apportionment, and the Shapley value. *Journal of Legal Studies 45*(1), 143–171.

Deneckere, R. and C. Davidson (1985). Incentives to form coalitions with Bertrand competition. *RAND Journal of Economics 16*(4), 473–486.

Easterbrook, F. H., W. M. Landes, and R. A. Posner (1980). Contribution among antitrust defendants: A legal and economic analysis. *Journal of Law and Economics 23*(2), 331–370.

Eger, T. and P. Weise (2015). Are consumers really the main victims of price cartels? *Homo Oeconomicus 32*(3/4), 375–386.

European Commission (2006). BASF AG v Commission of the European Communities. Case T-15/02.

European Commission (2011, accessed March 7, 2017). *Fines for breaking EU Competition Law*. `http://ec.europa.eu/competition/cartels/overview/factsheet_fines_en.pdf`.

European Commission (2013a). Impact Assessment Report. Damages actions for breach of the EU antitrust rules. Commission Staff Working Document 203.

European Commission (2013b). Practical Guide. Quantifying harm in actions for damages based on breaches of Article 101 or 102 of the Treaty on the Functioning of The European Union. Commission Staff Working Document 205.

European Commission (2014). On certain rules governing actions for damages under national law for infringements of the competition law provisions of the Member States and of the European Union. Directive 104/EU.

European Court of Justice (2014). Kone AG and others v ÖBB-Infrastruktur AG. Case C-557/12.

Federgruen, A. and M. Pierson (2011). The impact of mergers and acquisitions in price competition models. Working paper, Columbia Business School.

Feinberg, J. (1970). *Doing and Deserving*. Princeton, NJ: Princeton University Press.

Friederiszick, H. W. and L.-H. Röller (2010). Quantification of harm in damages actions for antitrust infringements: Insights from German cartel cases. *Journal of Competition Law and Economics 6*(3), 595–618.

Ginsburg, D. H. and J. D. Wright (2010). Antitrust sanctions. *Competition Policy International 6*(2), 3–39.

Goetz, C. J., R. S. Higgins, and F. S. McChesney (2006). More can be better: Contribution and setoffs in antitrust. *Antitrust Bulletin 51*(3), 631–660.

Harrington, J. E. (2006). *How Do Cartels Operate?* Boston, MA: Now Publishers.

Harrington, J. E. and M.-H. Chang (2009). Modelling the birth and death of cartels with an application to evaluating competition policy. *Journal of the European Economic Association 7*(6), 1400–1435.

Hart, S. and A. Mas-Colell (1989). Potential, value, and consistency. *Econometrica 57*(3), 589–614.

Huettner, F. and D. Karos (2017). Liability situations with joint tortfeasors. Working paper, ESMT Berlin and Maastricht University.

Hviid, M. and A. Medvedev (2010). The role of contribution among defendants in private antitrust litigation. *International Review of Law and Economics 30*(4), 306–316.

Inderst, R., F. P. Maier-Rigaud, and U. Schwalbe (2013). Quantifizierung von Schäden durch Wettbewerbsverstöße (Quantification of damages in EU and German competition law cases). In *Handbuch der Privaten Kartellrechtsdurchsetzung*. München: C.H. Beck.

Inderst, R., F. P. Maier-Rigaud, and U. Schwalbe (2014). Umbrella effects. *Journal of Competition Law and Economics 10*(3), 739–763.

Kalai, E. and D. Samet (1987). On weighted Shapley values. *International Journal of Game Theory 16*(3), 205–222.

Kleinberg, N. L. and J. H. Weiss (1985). A new formula for the Shapley value. *Economic Letters 17*(4), 311–315.

Knittel, C. R. and K. Metaxoglou (2011). Challenges in merger simulation analysis. *American Economic Review 101*(3), 56–59.

Landes, W. M. and R. A. Posner (1980). Joint and multiple tortfeasors: An economic analysis. *Journal of Legal Studies 9*(3), 517–555.

Leslie, C. R. (2006). Antitrust amnesty, game theory, and cartel stability. *Journal of Corporation Law 31*, 453–488.

Levenstein, M. C. and V. Y. Suslow (2006). What determines cartel success? *Journal of Economic Literature 44*(1), 43–95.

Levy, D. T. and J. D. Reitzes (1992). Anticompetitive effects of mergers in markets with localized competition. *Journal of Law, Economics, and Organization 8*(2), 427–440.

Lianos, I. (2015). Causal uncertainty and damages claims for the infringement of competition law in Europe. *Yearbook of European Law 34*(1), 170–231.

Littlechild, S. C. and G. Thompson (1977). Aircraft landing fees: a game theory approach. *Bell Journal of Economics 8*(1), 186–204.

Maschler, M., E. Solan, and S. Zamir (2013). *Game Theory*. Cambridge: Cambridge University Press.

Miller, N. H. (2009). Strategic leniency and cartel enforcement. *American Economic Review 99*(3), 750–768.

Moretti, S. and F. Patrone (2008). Transversality of the Shapley value. *TOP 16*(1), 1–41.

Moulin, H. (2002). Axiomatic cost and surplus sharing. In K. Arrow, A. Sen, K. Suzumura, and A. Villar (Eds.), *Handbook of Social Choice and Welfare*, Volume 1, Chapter 6, pp. 289–357. Amsterdam: Elsevier.

Napel, S. and G. Oldehaver (2015). Kartellschadensersatz und Gesamtschuldnerausgleich – Ökomisch faire Schadensaufteilung mit dem Shapley-Wert. *Neue Zeitschrift für Kartellrecht 3*, 135–140.

Nieberding, J. F. (2006). Estimating overcharges in antitrust cases using a reduced-form approach: Methods and issues. *Journal of Applied Economics 9*(2), 361–380.

Nowak, A. S. and T. Radzik (1995). On axiomatizations of the weighted shapley values. *Games and Economic Behavior 8*(2), 389–405.

Polinsky, A. M. and S. Shavell (1981). Contribution and claim reduction among antitrust defendants: An economic analysis. *Stanford Law Review 33*(3), 447–471.

Protter, M. H. and C. B. Morrey (1991). *A First Course in Real Analysis* (2nd ed.). New York, NY: Springer.

Ray, D. and R. Vohra (1999). A theory of endogenous coalition structures. *Games and Economic Behavior 26*(2), 286–336.

Roth, A. E. and R. E. Verrecchia (1979). The Shapley value as applied to cost allocation: a reinterpretation. *Journal of Accounting Research 17*(1), 295–303.

Rothblum, U. G. (1988). Combinatorial representations of the Shapley value based on average relative payoffs. In A. E. Roth (Ed.), *The Shapley Value – Essays in Honor of Lloyd S. Shapley*, pp. 121–126. Cambridge: Cambridge University Press.

Schwalbe, U. (2013). Haftungsquotierung bei Kartellschäden – Ein Ansatz aus der Theorie kooperativer Spiele. Discussion Paper, University of Hohenheim.

Shapley, L. S. (1953a). A value for *n*-person games. In H. Kuhn and A. Tucker (Eds.), *Contributions to the Theory of Games*, Volume II of *Ann. Math. Studies 28*, pp. 307–17. Princeton, NJ: Princton University Press.

Shapley, L. S. (1953b). *Additive and Non-Additive Set Functions*. Ph.D thesis, Princeton University.

Shubik, M. (1962). Incentives, decentralized control, the assignment of joint costs and internal pricing. *Management Science 8*(3), 325–343.

Straffin, P. D. (1983). Power indices in politics. In S. J. Brams, W. F. Lucas, and J. Straffin, Philip D (Eds.), *Political and Related Models*, pp. 256–321. New York, NY: Springer.

Taylor, A. D. and W. S. Zwicker (1999). *Simple Games*. Princeton, NJ: Princeton University Press.

Vives, X. (1999). *Oligopoly Pricing: Old Ideas and New Tools*. Cambridge, MA: MIT Press.

Weikard, H.-P. (2009). Cartel stability under an optimal sharing rule. *Manchester School 77*(5), 575–593.

Weinberg, M. C. (2011). More evidence on the performance of merger simulations. *American Economic Review 101*(3), 51–55.

Young, H. P. (1985). Monotonic solutions of cooperative games. *International Journal of Game Theory 14*(2), 65–72.

Young, H. P. (1994). Cost allocation. In R. Aumann and S. Hart (Eds.), *Handbook of Game Theory with Economic Applications*, Volume 2, Chapter 34, pp. 1193–1235. Amsterdam: Elsevier.