Intergenerational talent transmission, inequality, and social mobility*

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Abstract

The paper investigates the effects of intra-family talent transmission when human capital exhibits indivisibilities and parental financing of education involves borrowing constraints. Positive talent correlation reduces social mobility but steady state inequality and macroeconomic history-dependence are not affected.

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1 Introduction

Intergenerational mobility matters for reasons of equity as well as efficiency if agents' abilities vary across generations.[1] It reflects dynamic inequality that may or may not be orthogonal to possible wealth differences within a generation. While Becker and Tomes (1979) and Loury (1981) originally studied both kinds of inequality as intertwined phenomena, more recent theoretical literature has inadvertently ruled out steady state mobility by presuming agents with homogenous abilities (Ljungqvist 1993; Galor and Zeira 1993; Freeman 1996; Mookherjee and Ray 2003).

Exceptions are Maoz and Moav (1999) and Mookherjee and Napel (2007). Both consider parental investment into children's education that must be financed by parents' wealth. Educated children earn a skill premium determined endogenously by aggregate investment when they join the workforce as parents of the next generation. Concave utility implies that richer parents find investment subjectively cheaper, inducing greater incentives to educate. This can prevent steady state mobility, but children's ability or talent and the associated cost of education are subject to shocks: some parents may have exceptionally talented children who require no training; education of others may be too costly even for the richest parents. For wide enough talent support, a positive fraction of skilled parents will not invest whilst some unskilled parents do. This can keep wages and the aggregate skill ratio constant, i.e., constitute a macroeconomic steady state with mobility.

In Maoz and Moav (1999) and Mookherjee and Napel (2007), talent is *independently and identically distributed* (i.i.d.) across families and occupations. This is at odds with reality regarding cognitive skills (see, e.g., Bouchard and McGue 1981; Devlin et al. 1997) and presumably other traits which influence occupational choice. In this paper, a child's ability depends on its parent's ability.[2] Apart from that, the model is a special case of Mookherjee and Napel (2007). We show that intergenerational talent correlation affects social mobility but leaves intragenerational inequality and the number of steady states unchanged.

[1] For recent empirical studies see Mazumder (2005) or Jäntti et al. (2006).

[2] See Couch and Morand (2005) for related growth analysis with exogenous educational returns.

2 Model

Consider a unit mass of families indexed by $i \in [0,1]$. In each generation $t = 0, 1, 2, \dots$ family i comprises a child and an adult; the latter supplies one unit of labor in occupation $o_t^i \in \{n, s\}$ (unskilled or skilled)[3]. The child has an observable ability which requires either a low financial investment $x^l > 0$ or a high one $x^h > x^l$ in order to become educated. Educated children can take skilled jobs when they become the family's adult in period $t+1$. Unskilled labor requires no investment. Parents cannot borrow against their children's income. There are no financial bequests and thus education must be financed by current income. The fraction of educated agents in period t is denoted by λ_t and coincides with employment in the skilled sector in equilibrium.

A single consumption good is produced competitively according to the production function

$$H(\lambda_t) = {\lambda_t}^{\gamma}(1-\lambda_t)^{(1-\gamma)} \tag{1}$$

with $\gamma \in (0,1)$. Agents invest in education only if skilled jobs pay a premium. So, in equilibrium $\lambda_t < \gamma$ and wages are given by

$$w_t^s \equiv w^s(\lambda_t) = \gamma\Big(\frac{1-\lambda_t}{\lambda_t}\Big)^{(1-\gamma)} \quad \text{and} \quad w_t^n \equiv w^n(\lambda_t) = (1-\gamma)\Big(\frac{\lambda_t}{1-\lambda_t}\Big)^{\gamma}. \tag{2}$$

Parents care about their own consumption and the future wealth of their child. Specifically, let agents maximize

$$U(c_t^i, w_{t+1}^{o_{t+1}^i}) = \ln c_t^i + \delta \ln w_{t+1}^{o_{t+1}^i}, \tag{3}$$

where c_t^i denotes parental consumption ($= w_t^{o_t^i} - x$ in case of investment), $w_{t+1}^{o_{t+1}^i}$ is the child's income ($= w_{t+1}^s$ in case of investment) and parameter $\delta > 0$ scales parental altruism. If the subjective benefit from investment

$$B(\lambda_{t+1}) \equiv \delta\big[\ln w^s(\lambda_{t+1}) - \ln w^n(\lambda_{t+1})\big] \tag{4}$$

is strictly greater (smaller) than the subjective cost

$$C^k(\lambda_t, x) \equiv \ln w^k(\lambda_t) - \ln\big[w^k(\lambda_t) - x\big], \tag{5}$$

[3]See Mookherjee and Ray (2003) on the role played by the number of occupations.

then a parent who faces pecuniary cost x and has occupation $k \in \{n, s\}$ will invest (not invest) given λ_t and λ_{t+1}; if $B(\lambda_{t+1}) = C^k(\lambda_t, x)$ he may invest with arbitrary probability.

The selected specification of technology and preferences guarantees that unskilled's net benefit from investment, $B(\lambda) - C^n(\lambda, x)$, changes sign in λ at most twice: rising w^n first makes investment affordable, but then non-lucrative as $w^s - w^n$ diminishes.[4] This will be required for Proposition 1. All other results only use the standard monotonicity and curvature properties which are exhibited by $U(\cdot)$ and $H(\cdot)$ (ensuring a unique competitive equilibrium ratio λ_{t+1} for any given $\lambda_t \in (0, \gamma)$ and treating consumption and investment as substitutes).

A child's ability depends on that of his parent in a Markov way. The conditional probability that a parent with education cost x^j has a son with cost $x^{j'}$ is given by

$$p_{j \to j'} \equiv \Pr(x^i_{t+1} = x^{j'} \mid x^i_t = x^j) \tag{6}$$

for $j, j' \in \{l, h\}$. The *degree of dependence* is captured by[5]

$$p_{l \to l} - p_{h \to l} = p_{h \to h} - p_{l \to h} \equiv \kappa \in (-1, 1). \tag{7}$$

A dynamic competitive equilibrium corresponds to a sequence $\{\lambda_t\}_{t=0,1,\ldots}$ such that for every $t = 0, 1, \ldots$ the current skill ratio λ_t and expectations $\lambda^e_{t+1} = \lambda_{t+1}$ about next period induce a total measure λ_{t+1} of unskilled and skilled investors (all those with strict preference and market-clearing shares $\alpha_t \in [0, 1]$ and $\beta_t \in [0, 1]$ of indifferent unskilled and skilled parents).[6] We focus on equilibria with a stationary skill ratio, i.e., aggregate *steady states (SS)* where $\lambda_t = \lambda_{t+1} = \lambda^*$, and amongst these on *steady states with mobility (SSM)*, i.e., stationary equilibria in which the measure of unskilled investors is positive and equals the measure of skilled non-investors.

In t, family $i \in [0, 1]$ is in a state $r^i_t \in \{s^l, s^h, n^l, n^h\}$ where k^j indicates parental occupation k and parental cost level j. This gives rise to an aggregate parental occupation and cost distribution

$$\pi(t) \equiv (\pi_{s^l}(t), \pi_{s^h}(t), \pi_{n^l}(t), \pi_{n^h}(t)) \tag{8}$$

[4] A more general sufficient condition for this is a Cobb-Douglas technology coupled with constant relative risk aversion of one or more (Mookherjee and Napel 2007, Lemma 3).

[5] $\kappa > 0$ seems most relevant, but we do not rule out negative correlation.

[6] Existence and uniqueness follows from Mookherjee and Napel (2007, Lemma 1).

with $\pi_{s^l}(t) + \pi_{s^h}(t) = \lambda_t$. The transition from $\pi(t)$ to $\pi(t+1)$ is governed by endogenous investment choices of parents given their respective child's realized cost type. One obtains a time-heterogenous Markov chain whose period-t transition matrix is determined by λ_t (and respective market-clearing levels of α_t and β_t). If, for example, λ_t implies that all parents invest if their child has cost x^l and skilled parents are indifferent for x^h, then we would have

$$\pi(t+1) := (\pi_{s^l}(t), \pi_{s^h}(t), \pi_{n^l}(t), \pi_{n^h}(t)) \underbrace{\begin{pmatrix} p_{l\to l} & \beta_t p_{l\to h} & 0 & (1-\beta_t)p_{l\to h} \\ p_{h\to l} & \beta_t p_{h\to h} & 0 & (1-\beta_t)p_{h\to h} \\ p_{l\to l} & 0 & 0 & p_{l\to h} \\ p_{h\to l} & 0 & 0 & p_{h\to h} \end{pmatrix}}_{P(\lambda_t)}. \tag{9}$$

Any stationary λ^* implies a particular stationary transition matrix $P(\lambda^*)$. If λ^* is a SSM, then each Markov chain $\{r_t^i\}_{t=0,1,\dots}$ is time-homogenous, irreducible and aperiodic (recall $|\kappa| < 1$). There is hence a unique *invariant measure* π^* such that

$$\pi^* \, P(\lambda^*) = \pi^*. \tag{10}$$

We will investigate such invariant measures for given technology, preference, and cost parameters.

3 Analysis of steady states with mobility

For fixed cost type x, $w^s > w^n$ implies that an unskilled parent will only invest if a skilled one does, too. And for a given wage w, a parent will only invest in a child with cost x^h if he would invest in one with x^l, too. So only SSM with the following investment incentives may arise:

Type I	$C^s(\lambda^*, x^l) < C^n(\lambda^*, x^l) < B(\lambda^*) = C^s(\lambda^*, x^h) < C^n(\lambda^*, x^h)$
Type II	$C^s(\lambda^*, x^l) < C^n(\lambda^*, x^l) = B(\lambda^*) = C^s(\lambda^*, x^h) < C^n(\lambda^*, x^h)$
Type III	$C^s(\lambda^*, x^l) < C^n(\lambda^*, x^l) < B(\lambda^*) < C^s(\lambda^*, x^h) < C^n(\lambda^*, x^h)$
Type IV	$C^s(\lambda^*, x^l) < C^n(\lambda^*, x^l) = B(\lambda^*) < C^s(\lambda^*, x^h) < C^n(\lambda^*, x^h)$

One can check that SS without mobility generically appear in entire intervals. But there seems to exist at least some mobility in every society. Macroeconomic history dependence then becomes very limited:

PROPOSITION **1**

(a) There are never more than two SSM; two SSM exist only if x^h is high enough such that $B(\lambda) < C^n(\lambda, x^h)$ for all λ.

(b) If $B(\lambda) \geq C^n(\lambda, x^h)$ for some λ and investment incentives are of type III for some λ', then there exists a unique SSM.

Proof: (a) Assume that x^h is high enough so that unskilled parents only invest for cost x^l. We consider investment incentives if agents expect λ to prevail. Presuming that fractions $\alpha, \beta \in [0, 1]$ of indifferent agents invest, one can obtain a transition matrix $Q(\lambda, \alpha, \beta)$, which coincides with $P(\lambda)$ for appropriate α and β if λ is in fact a SS.

Restrict λ to the interval in which investment incentives are of type I–IV (the location of possible SSM). There, each $Q(\lambda, \alpha, \beta)$ has a unique invariant measure $\mu^*(\lambda, \alpha, \beta)$ ($= \pi^*$ if λ is a SSM). With

$$M(\lambda) \equiv \{\mu^*(\lambda, \alpha, \beta) \colon \alpha, \beta \in [0, 1]\} \tag{11}$$

we can define

$$u(\lambda) \equiv \left\{(1 - \lambda) \cdot \left[\frac{\mu_{n^l}}{\mu_{n^l} + \mu_{n^h}} p_{l \to l} + \frac{\mu_{n^h}}{\mu_{n^l} + \mu_{n^h}} p_{h \to l}\right] \,:\, \mu \in M(\lambda)\right\} \tag{12}$$

as the set of all possible measures of unskilled who would invest if λ held constant and if the composition amongst unskilled were as in a SSM with the same investment incentives as in λ. This upward flow correspondence $u(\lambda)$ is convex-valued and upper-semicontinuous (u.s.c.). So too is the analogous downward flow

$$d(\lambda) \equiv \left\{\lambda \cdot \left[\frac{\mu_{s^l}}{\mu_{s^l} + \mu_{s^h}} p_{l \to h} + \frac{\mu_{s^h}}{\mu_{s^l} + \mu_{s^h}} p_{h \to h}\right] \,:\, \mu \in M(\lambda)\right\}. \tag{13}$$

By construction, λ is a SSM iff $u(\lambda) \cap d(\lambda) \neq \varnothing$ (with a nonzero element).

Because *all* unskilled parents with x^l-children weakly prefer to invest on the whole λ-interval for which $u(\cdot)$ is defined[7] and there are fewer such

[7] We exclude cases in which unskilled parents with x^l-children never invest or only for a single λ: they cannot yield multiple SSM.

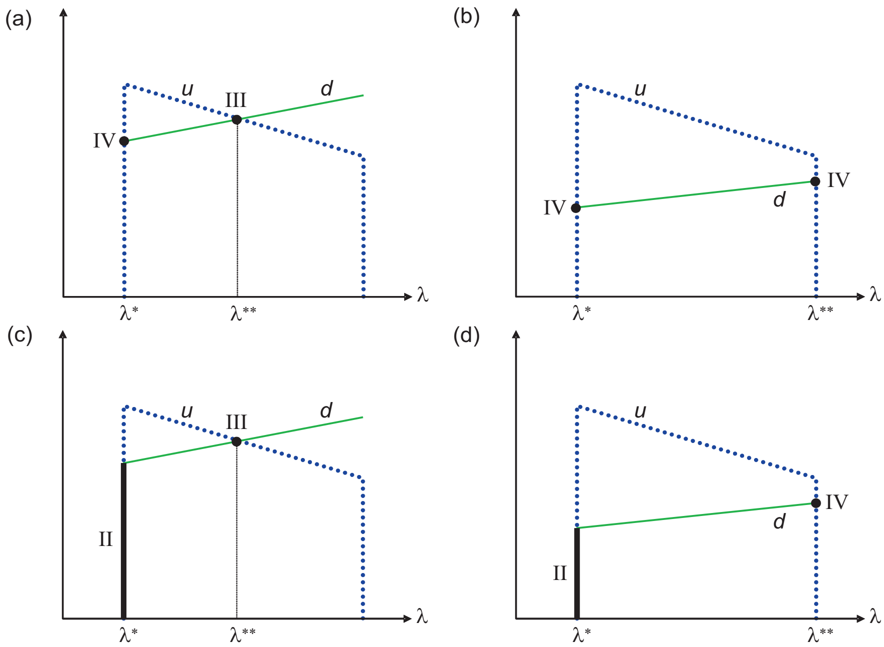

Figure 1: Upward and downward mobility flows for the case of two SSM

parents as λ increases, $u(\lambda)$ is decreasing in a 'saw-tooth' fashion. Similarly, $d(\lambda)$ is increasing. This allows for one or at most two nonempty intersections, with all possibilities of the latter illustrated in Figure 1. One can construct examples for each case.

Finally, consider the case with some λ' such that $B(\lambda') \geq C^n(\lambda', x^h)$, i.e., all unskilled might invest if λ' held constant. Since incentives are monotonic in income, every *skilled* family must invest with strict preference for all $\lambda \leq \lambda'$, excluding a SSM there. For $\lambda > \lambda'$, the total measure of unskilled wanting to invest is decreasing in λ (strictly when it is positive). Likewise the measure of skilled people preferring not to invest is increasing. So there can be at most one SSM at which both measures are positive and equal.

(b) $B(\lambda) \geq C^n(\lambda, x^h)$ at λ implies that all skilled parents invest at λ. The ratio $\hat{\lambda} > \lambda$ at which skilled parents with x^h-children become indifferent (assuming $\hat{\lambda}$ held constant) must be such that unskilled parents with x^l-children still invest with strict preference (otherwise there could be no λ', necessarily to the right of $\hat{\lambda}$, with incentives of type III). So $u(\cdot)$ is initially positive and then strictly decreasing to zero on an interval $[\hat{\lambda}, \tilde{\lambda}]$. Since $0 \in d(\hat{\lambda})$ and $d(\cdot)$ is strictly increasing on $[\hat{\lambda}, \tilde{\lambda}]$, both u.s.c. correspondences must intersect at a unique λ^*. □

SSM multiplicity (or lack thereof) is driven by the levels of education costs, not their allocation. It is therefore unaffected by partial ability correlation. This extends to the precise location of SSM and hence wage inequality $w^s(\lambda) - w^n(\lambda)$, which is strictly decreasing in λ, provided we compare economies with the same aggregate steady state talent shares. For all variations of $p_{j\to j'}$ which leave the stationary measure of x^l-agents, σ, unchanged,[8] we have:

Proposition 2 *The location of SSM and hence wage inequality are unaffected by κ.*

Proof: SSM which involve indifference (types I, II, and IV) are characterized by equality of $B(\lambda)$ and $C^s(\lambda, x^h)$ or $C^n(\lambda, x^l)$, respectively. Neither is affected by κ, i.e., the corresponding SSM cannot shift in response to a κ-variation. For type III-incentives parents invest iff they face cost x^l. So $\lambda^* = \sigma$ independently of κ. □

In contrast, intergenerational social mobility in a SSM is unequivocally reduced by increased intra-family correlation of ability:

Proposition 3 *In any given SSM λ^*, social mobility is strictly decreasing in κ.*

Proof: It suffices to consider the effect of a κ-variation on $u(\lambda)$ and $d(\lambda)$ when incentives are of type III because both correspondences are u.s.c. Then agents invest iff they face cost x^l, implying

$$u(\lambda) = \{(1-\lambda)p_{h\to l}\} \text{ and } d(\lambda) = \{\lambda p_{l\to h}\}. \tag{14}$$

Substituting $p_{l\to l} = \sigma(1-\kappa)+\kappa$ in order to keep the total x^l-share constant, we have

$$u(\lambda) = \{(1-\lambda)(1-\kappa)\sigma\} \text{ and } d(\lambda) = \{\lambda(1-\kappa)(1-\sigma)\}, \tag{15}$$

i.e., both $u(\lambda)$ and $d(\lambda)$ strictly decrease in κ.[9] □

[8]Because the stationary measure is characterized by $\sigma = \sigma p_{l\to l} + (1-\sigma)p_{h\to l}$, this amounts to endogenously setting $p_{l\to l} = \sigma(1-\kappa)+\kappa$ for any reference levels of σ and κ.

[9]Vertical distance between $u(\lambda)$ and $d(\lambda)$ is $(\kappa-1)(\lambda-\sigma)$. Its sign is determined entirely by $(\lambda-\sigma)$, implying that SSM cannot disappear or change type as a result of a κ-variation.

4 Concluding remarks

Above investigation has been confined to the simplest case of two distinct abilities. The generalization of Proposition 1 is straightforward: for r discrete cost levels, up to $2(r-1)$ steady states with mobility can co-exist;[10] the sufficient condition for uniqueness stays as it is with x^h referring to the maximal cost type. Extensions of Propositions 2 and 3 are harder because it is not generally possible to capture intergenerational talent dependence adequately by a single parameter. Comparative statics – varying the degree of intra-family talent transmission while holding the aggregate talent distribution constant – may, however, still be derived for special cases. One can, e.g., compare a talent process at the family level which is described by the transition matrix $T = (p_{j \to j'})_{r \times r}$ and one which results from the latter's convex combination with talent constancy, i.e., the process described by transition matrix $(1-\kappa)T + \kappa I$ with $\kappa \in (0,1)$ and I denoting the identity matrix: greater κ does not affect the aggregate talent distribution but formalizes more inertia. Propositions 2 and 3 then extend very naturally: the upward and downward flows are scaled down by factor $(1-\kappa)$ and all SSM are preserved at their original location. This admittedly concerns only a particular parameterization of dependence,[11] but illustrates that above results are not driven by the assumed type dichotomy.

The key qualitative feature of our model is that persistence of non-market determinants of occupational choice – here referred to as educational talent – is compounded by market forces: the equilibrium wage gap needed to induce investment implies that unskilled parents require more beneficial ability draws than skilled ones in order to invest. This applies also if 'ability' or 'talent' is transmitted via cultural or social channels rather than biology: for example, well-connected parents may get their children into a well-paying job more cheaply than others, or children of alumni benefit from preferential admission to top colleges which imply a smaller total cost of getting a

[10] The upward flow exhibits $r-1$ 'saw-tooth'-like increases if unskilled parents invest in all cost types $x < x^h$, before it decreases monotonically. The downward flow can then cut through each 'saw-tooth' twice (as in Figure 1(a) with $r-2$ more upward jumps to the right of λ^{**}).

[11] Convex combinations with other transition matrices may shift SSM of type III: the conditional stationary distributions of talent amongst the skilled and unskilled can change even if the aggregate distribution stays constant. Flow changes prompted by κ-variations may then differ across occupations.

highly paid job. Such children stand a good chance of respectively being well-connected themselves or becoming an alumnus, too.

Societies differ substantially in the degree to which ability in this wider sense is transmitted across generations. The non-market determinants of social permeability are therefore an important policy issue: intergenerational mobility and equality of opportunity are greater, the less easily the relevant 'traits' can be passed on. A range of corresponding policy interventions exist – most notably legislation on (non-)discriminatory recruitment and affirmative action. Our model suggests that such programs have an effect on social mobility,[12] but not automatically on total output or cross-sectional inequality.

If, say, preferential admission of alumni's children was banned, hitherto unprivileged families would benefit and upward mobility would rise. But a fixed number of slots in top schools is being allocated – corresponding to a fixed stationary proportion of low cost draws in our comparative static analysis. Increased downward mobility must then keep the total number of skilled families and hence their wage premium constant. The situation would be different if individual productivity in the skilled sector rose with innate talent, assuming the latter determines total educational costs jointly with admission rules, availability of scholarships, and other policy-sensitive variables. Banning preferential admissions might then raise total output and, more speculatively, lower wage inequality by educating children who will be more productive. This cannot arise in a model that assumes a homogeneous skilled input factor, and represents a promising direction for future research. Redistributive policy interventions are another worthwhile topic: a social planner who can observe children's talents and make lump sum transfers can rather easily achieve a Pareto improvement; but this seems much harder – and perhaps is impossible – with a more realistic set of policy instruments.

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[12]So do the empirical studies that we are aware of – see, e.g., Chan and Eyster (2003) or Arcidiacono (2005). Interestingly, the latter only finds a weak effect of affirmative action in higher education on individual earnings – despite noticeable changes in the composition of students admitted to colleges in general and top colleges in particular.

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