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MONOTONICITY OF POWER AND POWER MEASURES

Abstract. Monotonicity is commonly considered an essential requirement for power measures; violation of local monotonicity or related postulates supposedly disqualifies an index as a valid yardstick for measuring power. This paper questions if such claims are really warranted. In the light of features of real-world collective decision making such as coalition formation processes, ideological affinities, a priori unions, and strategic interaction, standard notions of monotonicity are too narrowly defined. A power measure should be able to indicate that power is non-monotonic in a given dimension of players' resources if – given a decision environment and plausible assumptions about behaviour – it *is* non-monotonic.

1. INTRODUCTION

Consider a set of players who jointly take decisions under a given set of rules. For example, the rules may specify that any player $i = 1,\dots, n$ has a specific voting weight w_i and that a collective decision requires enough supporters such that their total weight exceeds a decision quota *d*. Power indices address the question of how much power collective decision rules, e.g. a weighted voting rule, award to individual players: Is player *i* more or less powerful than player *j*, and by how much?

Let p_i be the power value assigned to player *i* by a power index. A power distribution indicated by this index for a given rule involving voting weights is *locally monotonic* if $w_i > w_j$ implies $p_i \geq p_j$, i.e., a voter *i* who controls a larger share of vote does not have less power than a voter *j* with a smaller voting weight. An index which for any decision rule produces locally monotonic power distributions is said to satisfy *local monotonicity* or be a *locally monotonic index*. An index which violates local monotonicity exhibits a so-called *weighted voting paradox*, i.e. a player with greater weight having smaller power compared to another player, for at least some – though not necessarily many – weight assignments and quotas.

Monotonicity is commonly considered an essential requirement for power measures. Felsenthal and Machover (1998, p. 221ff), for instance, argue that any a priori measure of power that violates local monotonicity is 'pathological' and should be disqualified as serving as a valid yardstick for measuring power. However, several notions of monotonicity have been defined and no consensus has been reached about how devastating a violation of a given type of monotonicity is in itself and in comparison to violations of other types. In our view, this has a good reason. Namely, the correct notion of monotonicity and whether monotonicity of a power distribution – and hence an index – is meaningful at all is highly context-dependent.

Typically, there is much more to a decision rule than weights and quota. Players face particular opportunities (and restrictions) of coordinating their support for – or opposition to – a given decision proposal. They can to different extents and by many different ways influence which possible decisions are considered in the first place. These aspects of the environment in which an at first sight very narrowly defined decision rule is applied – viewed from the different perspectives of individual players – constitute *resources* in a wider sense. Players can have very different ways to employ them, e.g. in a utility-maximizing way or using a particular adaptive heuristic. We will argue that a power measure should be able to indicate that power is non-monotonic in certain resource dimensions if – in view of plausible behavioural assumptions – it *is* non-monotonic in these resources (also see Holler and Napel, 2004).

The remainder of the paper is organized as follows: Section 2 discusses several postulates that are closely related to local monotonicity. Section 3 deals with special cases of voting bodies in which power is (designed to be) proportional to voting weights, guaranteeing monotonicity. In contrast, section 4 considers several aspects of real-world collective decision situations and behavioural assumptions which can lead to violations of monotonicity. Section 5 discusses implications of a redistribution of votes and of new members joining a decision body, before section 6 concludes.

2. MONOTONICITY POSTULATES

There are at least three postulates in addition to the requirement of local monotonicity which try to capture intuitive notions of monotonicity in the context of power: The *dominance postulate* requires $p_i \geq p_j$ whenever i dominates j, i.e., if it is true that for every coalition S such that j is not in S and the union of S and $\{j\}$ is a *winning coalition*, i.e. is able to take a decision, the union of S and $\{i\}$ is also a winning coalition.[1]

The *transfer postulate* demands that the power of any voter i given a weighted voting rule should not increase if i donates a part of his or her voting rights to another voter j provided that i is the sole donor. If this postulate is violated, then the corresponding power measure suffers from the *donation paradox* for some – though not necessarily many – weighted voting rules: i gains power by giving away votes to j.

Finally, the *bloc postulate* stipulates that the power of a merged entity $\{i, j\}$ is strictly larger than the power of player i before the merger provided that j is not a dummy, i.e. is a crucial member of at least some feasible coalition in the sense that he or she can turn it from winning to losing by leaving.

The definition of local monotonicity makes it particularly clear that the vote distribution is supposedly the measure rod with which a given power distribution must compare well. This is inherent also to the three mentioned principles. Thus, more or less explicitly, the monotonicity discussion is based on the premise that, in the case of weighted voting, there is a close relationship between power and votes, where votes represent resources. At the same time it is widely agreed that the vote distribution is a poor proxy for the distribution of voting power – this is, in fact, the main motivation behind the study of power indices.

It is well-known that the *normalized* (relative) Banzhaf index (Banzhaf, 1965) violates the transfer postulate and the bloc postulate, but satisfies the dominance postulate and local monotonicity. As a consequence, Felsenthal and Machover (1995, p. 225) conclude that the normalized Banzhaf index "must, at best, be regarded as seriously flawed." The non-normalized (absolute) Banzhaf index and the Shapley-Shubik index (1954) obey all four principles, while the Deegan-Packel (1978) index and the Public Good Index (Holler and Packel, 1983) violate all four principles.

If voting weights are identified with power values then the above postulates are trivially satisfied, and none of the paradoxes related to their violations is possible. For example, if we simply equate power with voting weights then, by definition, giving away voting weights reduces power and the donation paradox can never be observed.

If any violation of local monotonicity or one of the postulates is regarded as an indicator of a serious flaw in the definition of an index, one may ask: Why do we not simply take the voting weights (or their ratio to the decision quota) as power measures instead of more sophisticated measures that may suffer from various paradoxes? Of course, many different weight configurations can yield exactly the same sets of winning coalitions (and losing coalitions) and what matters to a coalition in the context of weighted voting is not by how much it is above or below the quota – just if it is. Taking any *arbitrary* element of the equivalence class of weights yielding a given set of winning coalitions is therefore clearly misleading and must produce inconsistencies. One could avoid this, for example, by using the minimal representation of a voting game (see Ostmann, 1987). This will always satisfy local monotonicity. Does the distance between values of an index, which always respects the weight ordering, really contribute to the understanding of power?

Perhaps violations of local monotonicity and the various paradoxes tell us a more substantial story about the properties of power in voting bodies? Brams and Affuso (1976, p. 52f) observe: "Given the widespread use and acceptance of power indices, we believe that an aberration they show up must be taken seriously. Instead of thinking of the paradox of new members as 'aberrant', however, we prefer to view it as an aspect of voting power whose existence would have been difficult to ascertain in the absence of precise quantitative concepts… It is a limitation in our thinking and models, not an aberration in the phenomenon, that has heretofore led us to equate power and size."

The *paradox of new members* they refer to occurs if the power value of an incumbent voter i increases when a new voter j enters the voting body and the votes of all incumbent voters and the percentage quota remain unaltered (implying, however, that the relative share of votes of the incumbent voter decrease).

Examples show that different power indices may not agree concerning the paradox of new members (see Brams and Affuso, 1976). For instance, for a specific voting game, the Shapley-Shubik index may indicate a paradox of new members while the (normalized) Banzhaf index does not. In principle, however, both measures can point out the existence of this paradox. This is, in our view, more an advantage than a flaw. A similar thing can be argued for violations of local monotonicity, which cannot happen for the Shapley-Shubik and Banzhaf indices but for the Deegan-Packel and Public Good indices.

Holler and Napel (2004) discuss in more detail whether local monotonicity is a property of the power distribution of a specific voting game, as represented by the chosen power measure, or whether it is a property of power *per se*. In the latter case, any reasonable power measure has to satisfy the local monotonicity axiom. If we accept the former perspective, then a measure which does not allow for a violation of local monotonicity is inappropriate to express this dimension of power. Only if the measure is able to indicate non-monotonicity it can serve to answer how the voting game has to be designed such that power values are monotonic in voting weights. Measures which produce values close to voting weights, irrespective of the vote distribution and the decision rule, and do not indicate any possible reversal of order (i.e. non-monotonicity) are rather useless instruments from this perspective.

3. STRICT PROPORTIONAL REPRESENTATION

Lionel Penrose conjectured already in the 1940s that in a weighted voting game with simple majority decision rule the ratio between the power values, measured by what later became known as the Banzhaf index, of any two voters converges to the ratio between their voting weights if the total number of voters goes to infinity. In a recent paper, Lindner and Machover (2002) provide sufficient conditions for this to be true. Strict proportionality between power and voting weights – or actually $p = w$ after rescaling – is the strongest form of monotonicity. Obviously, local monotonicity and the three monotonicity postulates or principles described above are immediately satisfied if strict proportionality holds.

In most real-world applications, however, we deal with a finite number of players which is often too small for these convergence results to matter. Then, the power distribution generically differs from the seat distribution. There is a substantial literature which discusses the problem of *equating* power with votes under the label of 'fair allocation of votes' (see, e.g., Laruelle and Widgrén, 1998, Laruelle, 2001, Leech, 2003, and Sutter, 2000). The starting point of this literature is that a (substantial) deviation of power from votes may be regarded as both an undesirable intransparency of the decision rule given the common misperception of power as proportional to weight, and – in a loose sense – unfair to players who get less out of their vote share than others. One can therefore consider redistributing votes such that $p = w$ is approximated "as closely as possible."

Even for large n, there are only finitely many power allocations p that can result from applying any of the established indices. In particular for small numbers of players the scope for achieving $p = w$ or, generally, for choosing voting weights $w = (w_1,...,w_n)$ such that the vector $p = (p_1,...,p_n)$ which expresses the power values assigned to the n agents of the voting game $v = (d; w)$ equals a design vector $k = (k_1,...,k_n)$ which describes an exogenously given – perhaps particularly 'fair' – allocation of voting power to the n agents, is very limited because of this discreteness.

Shapley (1962) proved that *strict proportionality* is obtained, i.e. $k = p = w$, if for any given w, quota d is uniformly distributed over the interval (0,1) and power is measured by the Shapley-Shubik index. Dubey and Shapley (1979) show that this is also true when power is measured by the Banzhaf index. Holler (1985) and Berg and Holler (1986) apply the randomized decision rule principle to discrete probability distributions and small n, in order to achieve strict proportionality.

There are a few real-world applications of randomised collective decision making. For example, the recently proposed compromise concerning the composition of the Governing Council of the European Central Bank after expansion of the euro zone is that member countries get a seat on a rotating basis for a length of time corresponding to pre-specified *time shares*. From an a priori perspective these time shares can be interpreted as weights which correspond to the probability of having a vote on the issue arising at a random point of time. Decisions in the ECB Governing Council would according to the proposal be taken by simple majority among present Council members. This ensures that each member of the euro zone has power in strict proportion to its time weight.

However, to randomize not only in case of ties but as an essential part of a decision rule invites a number of objections – in particular from those unlucky players who happen to have no say on a given issue. If the principle of randomised decision rules were accepted in general, it would even be possible to randomly choose a dictator, with probabilities corresponding to the desired a priori power allocation – arguably the most straightforward way of guaranteeing monotonicity and even strict proportionality. This illustrates the high price that monotonicity can have. It might be one reason why we often find that strict proportionality is not satisfied in reality and why power indices are needed to illustrate corresponding distortions.

4. PERSPECTIVES ON LOCAL MONOTONICITY

Given the multitude of power measures that have been proposed in the literature, a possible strategy is to choose a favourite power measure and to try to convince others to share this choice. An alternative is to accept the multitude of measures and their interpretations and select an appropriate measure in concurrence with the intuition possibly based on the accompanying stories. A third alternative is to define discriminating properties, possibly in form of postulates or axioms, which a power measure has to satisfy in order to qualify as 'appropriate'. Local monotonicity has been proposed to be such a property.

Local monotonicity is an implication of *desirability* as proposed by Isbell (1958). This property formalizes that a voter i is at least as desirable as a voter j if for any coalition S, such that the union of S and $\{j\}$ is a winning coalition, the union of S and $\{i\}$ is also winning. Freixas and Gambarelli (1997) use desirability to define reasonable power measures. Since both the Deegan-Packel index and Public Good Index violate local monotonicity (see Holler, 1982, Holler and Packel, 1983), they also violate the desirability property. For example, given the vote distribution $w = (35, 20, 15, 15, 15)$ and a decision rule $d = 51$, the corresponding values of the Public Good Index are equal to: $h(d,w) = (16/60, 8/60, 12/60, 12/60, 12/60)$. A comparison of w and $h(d,w)$ shows a violation of local monotonicity. Because of the basic principles underlying the Public Good Index, which derive from the notion of a pure public good, (i.e. non-rivalry in consumption and exclusion of free-riding), only *decisive sets* (i.e. strict minimum winning coalitions) are considered when it comes to measuring power. All other coalitions are either losing or contain at least one member which does not contribute to winning. If coalitions of the second type are formed, then it is by luck, similarity of preferences, tradition, etc. – *but not because of power*.

It should be emphasized that the Public Good Index does not claim that only decisive sets will be formed but it suggests that that only decisive sets should be taken into consideration when it comes to measuring power and the outcome of collective decision making is a public good. As a consequence, each decisive set stands for a different kind of public good and thus alternative public goods can be characterized by the decisive sets which support them.

Some authors have explicitly specified coalition formation processes that can motivate non-monotonic indices. These processes naturally imply particular weights or probabilities for the ex post power enjoyed by a given player once a particular winning coalition has been formed. For example, Brams et al. (2003) consider simple majority voting with players who each have a linear preference ordering over possible coalition partners, and study two related coalition formation processes with significant empirical support. Specifically, they investigate a *fallback process* in which players seek coalition partners in descending order until a winning coalition of mutually 'acceptable' players is established, and a *build-up process* which augments the fallback version by the requirement that no player outside the established coalition is strictly preferred by some coalition member to one of the insiders.

The probability that a coalition of size s is formed, assuming that all strict preference ordering are equally likely, turns out to be bimodal with peaks at the simple majority and unanimity. For given preference ordering, only particular coalitions of a given size s will be formed. It can, e.g., be the case that a comparatively small winning coalition is stable while a larger is not. Similar observations could be made for weighted majority voting. Power, under non-trivial coalition formation processes, therefore cannot be expected to be always monotonic.

Alonso-Meijide and Bowles (2003) make similar observations in their detailed analysis of voting power in the International Monetary Fund (IMF). They take an important institutional feature into account that has been neglected in other studies.

The large number of 184 IMF member countries necessitates the a priori formation of 24 groups, each represented by a single director on the IMF's decision-making Executive Board. Member state's total power – resulting from power inside the group and the group's power in the Executive Board – can be derived using Owen's (1977, 1982) framework for power measurement with *a priori unions*. Alonso-Meijide and Bowles (2003) use sophisticated methods for the efficient computation of a priori union power indices. They evaluate three different measures of total power in the IMF – corresponding to alternative measures at the intra- and inter-group levels. The *Banzhaf-Owen index* applies the reasoning behind the Banzhaf index at both levels. It produces non-monotonic power indications, e.g. for Belgium and India when a European constituency that aggregates weights of European Union members and a quota of $d = 85\%$ are considered. Moreover, it has the additional 'drawback' of failing to satisfy symmetry for a priori unions of equal weight, i.e. they may have unequal aggregate power.

The *Owen index*, which follows the weighting of marginal contributions underlying the Shapley-Shubik index, is symmetric but it fails to be monotonic nevertheless. This is also true for a new index introduced by Alonso-Meijide and Bowles, which distributes intra-group power according to the Shapley-Shubik index but inter-group power according to the Banzhaf index. The alternative *v-composition framework* for measuring total power given hierarchical decision levels, modelling decision-making in the groups as separate simple games and taking the Executive Board to be their composition, produces non-monotonic power distributions, too. In other words: Non-monotonicity becomes a very persistent feature of power as soon as the assumption of completely independent random yes-no decisions is replaced by features of real-world institutions.

If yes-no decisions concern particular proposals in a possibly multi-dimensional policy space, the probability of a given player being pivotal and thus having ex post power depends on his or her own position or most-preferred alternative in the policy space as well as those of the other players. Suppose that player i's most-preferred alternative or ideal point lies inside the convex hull of the ideal points of the other players. Then there exists no proposal that could make i the least or most enthusiastic player, in contrast to the players on the boundary of the convex hull. Considering random proposals, player i will therefore be the pivot player more often than players on the boundary who have the same weight. In fact, i will be in a powerful pivot position more often even than boundary players with greater weight, provided that the weight difference is not too big. This is formally captured by the *Owen-Shapley spatial power index* (see Owen, 1971, Shapley, 1977, and Shapley and Owen, 1989), which modifies the Shapley-Shubik index in a way that takes account of ideological proximity among players. For transparent and very good reasons, this index violates local monotonicity.

Braham and Steffen (2002) demonstrate that applications of Straffin's (1977) *partial homogeneity approach*, which concerns particular assumptions about probabilistic yes-no decisions in Owen's (1972) *multilinear extension* of weighted voting games, do not always produce results consistent with local monotonicity. This is because partial homogeneity treats players asymmetrically in a special way and, as mentioned, the power of a voter *i* by definition depends not only upon the number of coalitions for which *i* is critical but also upon the probabilities by which the various coalitions arise. Braham and Steffen argue that Straffin's partial homogeneity approach is not less a priori than the Banzhaf index and the Shapley-Shubik index. The partial homogeneity approach can, in fact, be interpreted as a combination of the Banzhaf index and the Shapley-Shubik. These two indices satisfy local monotonicity, although their original axiomatization does not include local monotonicity.

Originally, the Deegan-Packel index and the Public Good Index derive from an axiomatic approach; probabilistic arguments do not necessarily apply to these measures. However, if we generalize Owen's multilinear extension framework to allow for the Public Good Index by applying zero probabilities to winning coalitions with surplus players, then the a priori argument which Braham and Steffen (2002) provide for the partial homogeneity measure applies also to the Public Good Index.

Returning to the above-mentioned 'problem' of having a multitude of power measures to choose from – which can indeed assign players not only different index values for a given decision body, but also produce a different power ranking – it is noteworthy that the discriminating power of various monotonicity postulates is in any case questionable. Laruelle and Valenciano (2003) demonstrate that a *large class of power indices*, which measure the probability of players being decisive or pivotal given a weighted voting rule and probabilistic assumptions about their yes-or-no votes, passes the monotonicity 'tests' that are commonly advocated in order to screen good power indices from bad ones. Moreover, an *even larger class of success indices*, measuring the probability of agreeing with the collective decision no matter if the considered player has contributed to it or not, passes exactly the same tests. In other words, postulates designed to filter out the 'ideal index' apparently formalise monotonicity notions applying much more to success than to power.

5. REDISTRIBUTION OF VOTES AND NEW MEMBERS

When it comes to monotonicity of power with respect to voting weights, it is important to note that none of the existing measures guarantees that the power measure of player *i* will *not* decrease if his or her voting weight increases (also see Holler, 1998). Fischer and Schotter (1978) demonstrate this result (i.e., *the paradox of redistribution*) for the Shapley-Shubik index and the normalized Banzhaf index (see also Schotter, 1982). More specifically, take the voting game $v = (.70;.55,.35,.10)$. The corresponding values for the normalized Banzhaf and Shapley-Shubik indices are $\beta(v) = \Phi(v) = (1/2, 1/2, 0)$. Now let's assume that the vote distribution $w = (.55,.35,.10)$ changes to $w° = (.50,.25,.25)$ while the decision rule $d = .70$ remains unchanged. Then the resulting voting game $v° = (.70;.50,.25,.25)$ corresponds to the normalized Banzhaf index $\beta(v°) = (3/5, 1/5, 1/5)$ and to the Shapley-Shubik index $\Phi(v°) = (2/3, 1/3, 1/3)$. Both measures show that although the first voter's voting weight decreased from .55

to .50, his power increased, irrespective whether measured by Banzhaf or Shapley-Shubik.

Of course, there are voting games which are robust against the paradox of redistribution which we just demonstrated. For example, for the voting game v' = (.70;.55,.25,.20) there is no alternative vote distribution so that the paradox of redistribution prevails if power is measured by Banzhaf or Shapley-Shubik. This raises the question how important the paradox of redistribution phenomenon is. Fischer and Schotter (1978) give some results which indicate that it has some substance for larger voting bodies. They prove the following propositions:

Prop1. For voting bodies with $n = 6$, a paradox of redistribution is always possible no matter what initial vote distribution exists, if power is measured by the Banzhaf index.

Prop2. For voting bodies with $n = 7$, a paradox of redistribution is always possible no matter what initial vote distribution exists, if power is measured by the Shapley-Shubik index.

Prop3. If $d = 1/2$, i.e., a simple majority decision rule applies and $n = 4$, then a paradox is always possible, irrespective of the initial distribution.

Given the popularity and widespread dissemination of simple majority voting, the result in Prop3 should be rather alarming for those who are worried about non-monotonicity of all kinds. Of course, none of the above proposition says that the paradox of redistribution is a frequently observed phenomenon. However, a comparison of Prop1 and Prop2 suggests that the paradox is more frequent when power is measured by the Banzhaf than by Shapley-Shubik index. In other words, there is some evidence that the Banzhaf index is more liable to non-monotonicities than the Shapley-Shubik index. However, there can be no proof of this proposition – mainly because the argument becomes circular if power relations in the voting body are expressed by the corresponding power measure only: power is then what the index measures, and if the index indicates monotonicity then power is monotonic.

The paradox of redistribution stresses the fact that power is a social concept: if we discuss the power of an individual member of a group in isolation from his or her social context, i.e. related only to his or her individual resources, we may experience all sorts of paradoxical results. It seems that sociologists are quite aware of this problem and non-monotonicity of an individual's power with respect to his or her individual resources does not come as a surprise to them (see, e.g., Caplow, 1968).

Political scientists, however, often see the non-monotonicity of power as a threat to the principle of democracy. To them it is hard to accept that increasing the number of votes a group has could decrease its power, although it seems that there is ample empirical evidence for it (see Brams and Fishburn, 1995, for references.) In general, economists also assume that controlling more resources is more likely to mean more power than less. However, they also deal with concepts like monopoly power, bargaining, and exploitation which stress the social context of power and the social value of resources (assets, money, property, etc.).

The *paradox of redistribution* is closely related to the *paradox of new member*. Felsenthal and Machover (1995) give an example in which the decision rule d is the same for the game before and after entry of a fifth voter j. They a djust the vote ratios so that the vote shares add up to one before and after entry of j. Thus, as a consequence the shares of the incumbent voters have to decrease. The games before entry, v', and after entry, v'', are

$$v' = (.51;.30,.30,.30,.10) \quad \text{and} \quad v'' = (.51;.15,.15,.15,.05,.50).$$

The *paradox of new members* is obvious. In the game v', having 10 per cents of the votes, player 4 is a dummy. In the game v'', now having only 5 per cents of the votes, player 4 can form a coalition with the entrant player 5 who controls 50 per cent of the votes. Felsenthal and Machover (1995, p. 222) argue that "any reasonable index of relative voting power" has to display the paradox of new members in this case. However, note that v' is equivalent to $v°$ which assigns a vote share of .00 to player 5 so that $v° = (.51;.30,.30,.30,.10,.00)$ alternatively describes the game before entry of player 5. Now if we compare $v°$ to v'' we clearly see that the *paradox of redistribution* prevails. Player 4 loses half of his vote share but is no longer a dummy in game v''.

Because of the adjustment of seat shares and by keeping the decision rule unchanged, the example of Felsenthal and Machover captures both the *paradox of redistribution* and the *paradox of new member*. Brams and Affuso (1976) originally discussed the *paradox of new member* for the addition of one or two players to the votes of the incumbents.

Brams and Affuso (1976, p. 52) observe that the probabilities for occurrence of the paradox of new members "are high in relatively small weighted voting bodies." They argue that whether "there exists a voting body invulnerable to the paradox is of less practical import than the probability of occurrence of the paradox" (p. 50). The paradox of new members shows that the relative number of swings of a weighted voter i with a constant number of seats can increase if new weighted voters enter the voting body, and thus the relative vote share of i decreases. If power expresses the potential to form and to contribute to coalitions and thereby to control the outcome, then this paradox does not come as a surprise and an index which is not equipped to indicate the paradox seems inadequate to discuss the properties of the power relations in this case. In the end, an adequate power measure should clarify the properties of the game so that, for example, a disfavoured player (or unhappy designer) can change the game.

Power indices with a strong sensitivity to monotonicity can also be of help for a more abstract analysis of decision situations with respect to power. Myerson (1999, p. 1080) argues that "the task for economic theorists in the generations after Nash has been to identify the game models that yield the most useful insights into economic problems. The ultimate goal of this work will be to build a canon of some dozens of game models, such that a student who has worked through the analysis of these canonical examples should be prepared to understand the subtleties of competitive forces in the widest variety of real social situations." What can be said of understanding the subtleties of competitive forces also applies to power. The analysis of alternative social situations by means of power measures complements more direct approaches to enhance the understanding of power, its sources and its consequences.

6. CONCLUDING REMARKS

The fundamental problem that leads to non-monotonicity of power distributions for a given index and special decision rules should not be attributed per se to a supposed inappropriateness of the index. Rather, the inappropriateness lies in the description of a social or economic situation involving rational or boundedly rational agents who have to reach a collective decision by merely a vector of weights w and a quota d. All established indices are based on additional assumptions about the situation and agents either explicitly or, more often, implicitly (e.g., in the form of axioms about the behaviour of the index or probabilistic assumptions about the behaviour of players).

The high degree of abstraction entailed by the Spartan weight-quota framework therefore almost implies that indices will yield power distributions that can be considered 'paradoxical' from some point of view which is inconsistent with these assumptions, at least for situations in which the inconsistency is so pronounced that it is in fact desirable to observe the supposed 'paradox'.

What is 'paradoxical' under the implicit assumption of independent random votes on an exogenously given random proposal can be perfectly consistent with or directly called for by common sense in the context of a given coalition formation process, affinities between agents implied e.g. by certain positions in a policy space, or the necessity to form a priori unions – and *vice versa*! A claim that an index, such as the Public Good Index or the Owen-Shapley spatial power index, should be discarded because it violates local monotonicity or some monotonicity postulate amounts to a claim that the set of assumptions about the decision situation and players' behaviour which are underlying it are invalid. In our opinion, such strong assertions are unjustified in general. They may be substantiated in the analysis of *particular* real-world decision environments, which may be inconsistent with the assumptions underlying a given index. Ironically, it is the puristic weight-quota framework with stochastically independent yes-no decisions on unspecified proposals, for which local monotonicity is indeed an understandable concern, which has the biggest problems in finding real-world situation that 'fit' and in convincing decision makers of its relevance inside real-world institutions to which power indices are most commonly applied.

7. NOTES

[1] A different notion of dominance entails player j contributing to a coalition in the sense of turning it from losing into winning *only in the presence of* i while i contributes also to coalitions that do not involve j (see Napel and Widgrén, 2001).
[2] The probabilistic approach to power measurement has been generalized by Napel and Widgrén (2004) to a much wider class than commonly considered. They propose to calculate a priori power as expected a posteriori power, which in turn is inferred from the collective decision's sensitivity to action or preference trembles by individual players. This includes traditional indices as special cases, but can assess power derived by strategic behaviour in non-trivial decision-making procedures, too. Interaction between the general *decision behavior* of players and the *decision situation* described by, among other things, voting weights can explicitly be accounted for. Alternative decision situations and assumptions on expected behavior can imply monotonicity or non-monotonicity of power in voting weights.

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