Aspiration Adaptation in the Ultimatum Minigame

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Version: January 14, 2002

Two agents recurrently play a 2×2 version of the ultimatum game. Each player sticks to his past action if it was satisfactory relative to an endogenous aspiration level and otherwise abandons it with positive probability. This type of satisficing behavior is shown to yield efficiency in the limit. It does not favor a specific distribution of surplus and can give an explanation for the incidence of equitable offers in practice. Numerical investigations link a player's character as captured by the model parameters to his average bargaining success. Results indicate that it is beneficial to be persistent and stubborn, i. e. slow in adapting aspirations and switching actions in response to major dissatisfaction. Also, it is an advantage to be capricious, i. e. to experience large and frequent perturbations of aspiration level and to discriminate only little between minor and major dissatisfaction.

Key Words: Bargaining, ultimatum game, bounded rationality, learning, satisficing, aspiration level, stochastic evolutionary games.

[1] I thank the editor in charge, B. Hehenkamp, L. Andreozzi, S. Berninghaus, W. Eichhorn, I. Dittmann, W. Güth, A. Haas, M.J. Holler, A. Mitropoulos, D. Mookherjee, A. Müller, A. Poulsen, W. Schnedler, S. Seifert, A. Tieman, and K.-H. Waldmann for constructive comments. The usual caveat applies.

1. INTRODUCTION

Non-cooperative game-theoretic models of bargaining, such as Rubinstein (1982), traditionally assume that players are perfectly rational and that this fact is common knowledge. In order to deduce sharp game-theoretic predictions, preferences over a player's share of surplus and time of agreement are frequently taken to be common knowledge, too.[2] Many experimental studies of human bargaining behavior have pointed out how demanding this set of assumptions is.[3] They indicate a need to investigate bargaining models based on more broadly defined preferences (e. g. incorporating aspects of fairness, aspirations, or history of play), and also ones dealing with boundedly rational, non-strategic players.

This paper considers repeated play of a 2×2 version of the ultimatum game, the *ultimatum minigame*, by two agents who use the satisficing heuristic investigated by Karandikar *et al.* (1998). Players stick to their past action if it performed 'well' relative to an endogenous aspiration level, and otherwise abandon it with positive probability. This particularly simple rule-of-thumb is analytically investigated concerning its long-run implications for *efficiency* and *distribution*. In computer simulations, the effects of soft factors—describing real-life agent characteristics like stubbornness or capriciousness—on average bargaining success are studied.

The model can be thought of as describing habitual bargaining involving e. g. two spouses or friends sharing a flat, boss and secretary, siblings, supervisor and student, or colleagues. These often trivial negotiations about the allocation of chores, how to work, what film to watch, at what time to meet, etc. may not even be recognized as a strategic game—prompting stimulus-response behavior rather than explicit calculations.

It is shown that aspiration-based satisficing behavior can provide an ex-

[2] See e. g. Binmore *et al.* (1992) and Muthoo (1999).

[3] Roth (1995) and Güth (1995) give good overviews.

planation for the observation that proposers in ultimatum bargaining frequently make equitable offers in practice. The exact way in which player's aspirations are occasionally perturbed determines whether an inequitable division of surplus, an equitable division, or—most commonly—both are in the support of the long run outcome as identified by techniques similar to Kandori *et al.* (1993) and Young (1993a, 1993b). Inefficient disagreement has vanishing probability. The simulations confirm that equitable offers are a prominent feature of the considered dynamics. Moreover, a player turns out to benefit from stubbornness and persistence concerning the adaptation of actions and aspirations, but also from capriciousness in the the sense of experiencing frequent and large perturbations of one's aspiration level.

Section 2 introduces the model. Section 3 presents and discusses the theoretical results. In Section 4, we compare different parameter scenarios in Monte-Carlo simulations. Section 5 concerns related literature, and Section 6 concludes.

2. THE MODEL

Two players can share an available surplus provided that they agree on how to split it. Both are assumed to care only about their own share. We consider the *ultimatum minigame* (UMG) shown in Fig. 1. First, player 1 makes either a high offer, H, or a low offer, L. After H, acceptance is assumed and the equitable payoff vector $(2, 2)$ results. After L, player 2 can either accept (action Y)—implying the inequitable surplus division $(3, 1)$—or reject (action N), implying that players receive $(0, 0)$. The game may represent an entire class of situations that from a player's point of view are equivalent in terms of payoffs and aspirations.

(L, Y) and (H, N) are the pure-strategy Nash equilibria of the UMG. The former is the unique perfect equilibrium. It corresponds to the unique asymptotically stable attractor of (unperturbed) two-population replicator dynamics (see Gale *et al.*, 1995) and is regarded as game theory's predicted

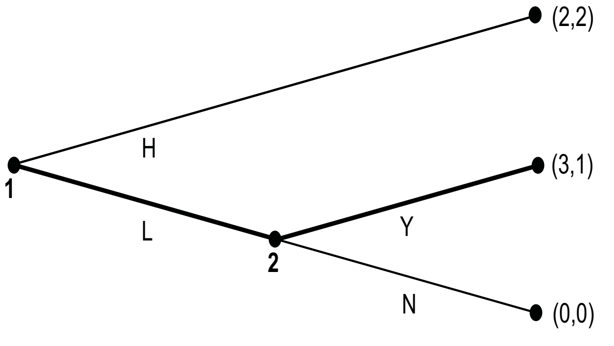

FIG. 1 The ultimatum minigame (UMG)

outcome. However, the equitable distribution implied by (H, N) and the maximin strategy profile (H, Y) is a focal point in laboratory experiments.[4]

We consider two agents who recurrently play the UMG in fixed roles. Each interaction is indexed by some $t \in \{0, 1, 2, \ldots\}$, referred to as *period* or *time*. Players are assumed to use a simple *satisficing heuristic*, which says:

1. Stick to your action if it was successful in the last period (compared to the aspiration level).

2. Otherwise change your action with positive probability, but not certainty.

This heuristic is myopic and involves neither backward induction by player 1 nor even simple payoff maximization by player 2.

Players' aspirations rise after positive feedback, i. e. a payoff above the aspiration level, and fall after negative feedback. Occasionally, aspirations are perturbed. One can imagine the random change of a player's aspiration level to result from positive or negative experience in a different (unmodeled) game, the observation of outcomes in a UMG played by dif-

[4]Yang *et al.* (1999), among others, confirm that behavior in the ultimatum minigame is qualitatively the same as in the ultimatum game.

ferent players, or simply an idiosyncratic optimistic or pessimistic shift in the player's perception of the world due to the weather, last night's sleep, etc.[5] This satisficing heuristic has first been investigated by Karandikar *et al.* (1998) in the context of symmetric 2×2-games.

In any period, both players recall only their own strategy in last period's interaction together with their personal payoff from that interaction. There is no individual memory apart from what has been 'condensed' into a player's aspiration level. Agents do neither observe their opponent's payoff, strategy, or aspiration level nor need they be aware of playing a 2-person game at all. Player 1's *state* at date t refers to her action $a_t \in A \equiv \{L, H\}$ chosen in t and her *aspiration level* $\alpha_t \in \mathsf{A} \equiv [0, 3]$ held in t.[6] Similarly, player 2's state in period t is given by $b_t \in B \equiv \{Y, N\}$ and his aspiration level $\beta_t \in \mathsf{B} \equiv [0, 2]$. The system's state in t is thus a 4-tuple $x_t = (a_t, \alpha_t, b_t, \beta_t)$, making $E \equiv A \times \mathsf{A} \times B \times \mathsf{B}$ the state space. The specification of A and B reflects that players are assumed to know their maximum and minimum feasible payoff, i. e. even a perturbation cannot make player 1 aspire to more than 3. The model is suited to study the gradual evolution of aspirations, although one could also consider jumps restricted e. g. to the set $\{0, 1, 2, 3\}$ by corresponding assumptions for aspiration adaptation and perturbations.

Player 1's actions are updated as follows. The probability that she sticks to a_t in $t + 1$ is a (weakly) decreasing function p_1 of her *disappointment*, $\Delta_1 \equiv \alpha_t - \pi_1(a_t, b_t)$, with a_t in period t's interaction as defined by actual and aspired payoffs (Δ_1 is non-positive if payoff was actually satisfactory). She repeats a_t with positive probability even if she is maximally dissatisfied, i. e. $p_1(\Delta_1)$ is bounded below by some $\tilde{p}_1 \in (0, 1)$. This reflects *inertia* in

[5] Under certain constraints, perturbations of aspirations can indirectly model perturbations of payoffs or inertia.

[6] Some may prefer to call a_t player 1's *behavioral mode* to stress that she does not 'choose' in the traditional economic way.

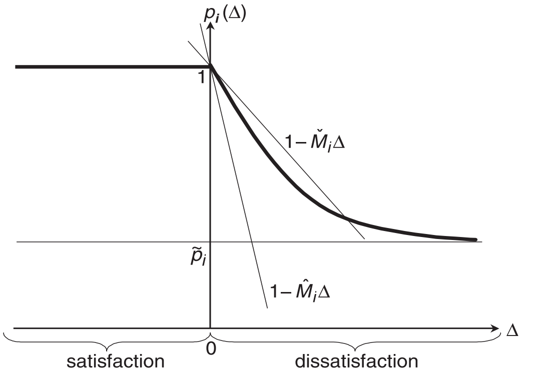

FIG. 2 A possible inertia function p_i

her behavior, also interpreted as *stubbornness*. Action a_t is with certainty repeated if she was satisfied, i. e. $p_1(\Delta_1) = 1$ for $\Delta_1 \leq 0$.

Function p_1 is assumed to be continuous. Moreover, the rate at which it falls is taken to be both bounded from above and away from zero.[7] These assumptions formalize that willingness to switch behavior is changing gradually and at moderate rate. Moreover, player 1's propensity to stick with the past action decreases visibly as soon as she is dissatisfied.

The assumptions for player 2 are analogous, with function p_2 bounded below by $\tilde{p}_2 \in (0, 1)$. We will refer to functions p_i as *inertia functions* and drop the player subscript on Δ. A possible inertia function p_i is illustrated in Fig. 2. Action updating is assumed to be stochastically independent across time and players for any given state.

Next, consider the updating of aspirations. We distinguish two update rules. The first defines a Markov process referred to as the *unperturbed satisficing process* Φ^0. In Φ^0, a player's aspiration level in $t + 1$ is simply a weighted average of the aspiration level in t and the payoff received in t.

[7]This means there is some $\hat{M}_1 < \infty$ for which $p_1(0) - p_1(\Delta) \leq \hat{M}_1\, \Delta$ for any $\Delta > 0$. Also, there exist $\check{M}_1 > 0$ and $\check{\Delta} > 0$ such that $p_1(0) - p_1(\Delta) \geq \check{M}_1\, \Delta$ for $\Delta \in (0, \check{\Delta})$.

More precisely,

$$\alpha_{t+1} = \alpha^*(x_t) \equiv \lambda\alpha_t + (1-\lambda)\pi_1(a_t, b_t) \tag{1}$$

with $\lambda \in [0, 1]$ for player 1. The larger λ, the slower aspirations change—for $\lambda = 1$ initial aspiration α_0 is kept forever, for $\lambda = 0$ aspirations equal the most recent payoff experience. Parameter λ measures the *persistence* of player 1. Player 2's aspirations are exponentially smoothed analogously and, for the time being, with identical parameter λ.

An alternative update rule defines the *perturbed satisficing process* Φ^η ($\eta > 0$). First, players' aspirations are deterministically updated as in Φ^0. Second, each player's aspiration level is independently perturbed with probability η. The new aspiration level is a random variable whose distribution can depend on the deterministically updated 'intermediate' aspiration level $\alpha^*(x_t)$ or $\beta^*(x_t)$, respectively.

To facilitate theoretical analysis, we assume both players to have the same state-independent probability $\eta > 0$ of experiencing a tremble. Post-perturbation aspirations for players 1 and 2 are assumed to have the distinct, state-dependent, and continuous conditional densities $g_1(\cdot|\alpha^*(x_t))$ and $g_2(\cdot|\beta^*(x_t))$ with supports contained in A and B, respectively. For any interval around $\alpha^*(x_t)$, let the probability that α_{t+1} lies in it be positive, i. e. shocks can be small. Also, it is convenient to assume that α_{t+1} lies in a neighborhood of α_t with positive probability, i. e. player 1 can 'forget' her most recent payoff experience as the result of a perturbation.[8] Similar assumptions apply for density g_2.

3. THEORETICAL RESULTS

We first investigate the unperturbed satisficing process Φ^0. Let us call a state in which both players' aspiration levels equal the respective payoffs

[8]These assumptions imply that Φ^η has the quite intuitive property of open-set irreducibility. ψ-irreducibility is, however, sufficient for the results.

a *convention.* This views a convention as a regularity of behavior to which both players conform, and which no player is prompted to abandon given that the other one conforms (cf. Lewis, 1969). We write c_{ab} for convention $(a, \pi_1(a,b), b, \pi_2(a,b))$. The UMG has four conventions, collected in the set C. As c_{HY} and c_{HN} yield the same surplus distribution, they are typically not distinguished; $c_{H\cdot}$ refers to their union. In the following, we examine whether the conventions are stable under the specified satisficing dynamics. A preliminary result is:

PROPOSITION 1. *State $x \in E$ is an absorbing state of Φ^0 if and only if it is a convention. For $\lambda < 1$ and any initial state $x_0 \in E$, Φ^0 converges almost surely to a convention $c \in C$.*

The first part follows from $p_i(0) = 1$ and the aspiration update rule (1). Proof of the second part amounts to confirming that from any state $x_t \notin C$ there is a positive probability $\varepsilon > 0$ of starting an infinite run on the present action pair in period t (see Karandikar *et al.*, 1998, Proposition 1).

The proposition states that players' adaptive interaction in Φ^0 will eventually establish a convention, but any of the three possible bargaining results—symmetric or equitable division, asymmetric or inequitable division, and break-down—can, depending on the initial state, be selected. In contrast, if players experience trembles in their aspiration levels, the influence of the initial state x_0 is gradually washed away. Φ^η is an *ergodic* Markov process:

PROPOSITION 2. *For $\lambda < 1$ and any given perturbation parameter $\eta \in (0, 1]$, Φ^η converges (strongly) to a unique limit distribution μ^η which is independent of the initial state $x_0 \in E$.*

The proof is given in the appendix. Proposition 2 establishes that Φ^η's long-run behavior is accurately described by a stationary distribution μ^η. In particular, the empirical frequency distribution over states up to a period t, sampled from an arbitrary process realization, converges to μ^η as $t \to \infty$.

It is generally not possible to give details on μ^η for arbitrary η and λ. Our analytical investigation concentrates on the benchmark case in which the probability of a tremble, η, is close to zero and in which, additionally, present payoff experience affects players' aspirations only marginally, i. e. λ is close to one.[9] Our first main result is

THEOREM 1. *Let the perturbed satisficing process Φ^η be defined as above and $\lambda < 1$. The limit stationary distribution of Φ^η for $\eta \to 0$, μ^*, can place positive weight only on the Pareto-efficient conventions c_{LY} and $c_{H\cdot}$ as $\lambda \to 1$.*

The proof is given in the appendix. Theorem 1 establishes that the players will (approximately) divide the available surplus *efficiently* if aspiration trembles are rare and aspirations are updated slowly. The intuition for this is the following: as the probability of a tremble approaches zero, Φ^η becomes more and more like the unperturbed process Φ^0. So by Proposition 1, Φ^η will spend most time in a convention.[10] However, the inefficient convention c_{LN} is unstable: A single perturbation of one player's aspiration level leads to convention $c_{H\cdot}$ or c_{LY} with positive probability because a likely action switch by the now dissatisfied player results in an efficient bargaining outcome that satisfies both players. The strategy combination is repeated forever unless another perturbation results in some player's dissatisfaction. Then, however, a move to c_{LN} is unlikely even if the perturbation results in the play of (L, N): For λ close to 1, an enormous number of periods has to pass before players' aspiration are close to $(0, 0)$. The odds are that at least one of the two dissatisfied players switches action before that happens, setting the course towards an efficient convention again. Our second main result is

THEOREM 2. *Let the perturbed satisficing process Φ^η be defined as above,*

[9] This investigation is the main reason for working with a continuous state space.

[10] Here, we implicitly refer to an arbitrary neighborhood of the respective state c_{ab}.

and let μ^* denote the limit stationary distribution of Φ^η for $\eta \to 0$. The supports of post-perturbation aspiration densities in conventions $c_{H\cdot}$ and c_{LY} can be chosen

i) such that μ^* places all weight on the asymmetric efficient convention c_{LY},

ii) such that μ^* places all weight on the symmetric efficient convention $c_{H\cdot}$, or

iii) such that μ^* places positive weight on both c_{LY} and $c_{H\cdot}$,

as $\lambda \to 1$. If, in particular, $g_i(\cdot|c_{LY})$ and $g_i(\cdot|c_{H\cdot})$ have full support in A and B, both c_{LY} and $c_{H\cdot}$ have positive weight.

The proof is given in the appendix. Theorem 2 implies that the average *surplus distribution* selected by the satisficing heuristic depends on the distribution of aspiration perturbations even for $\eta \to 0$ and $\lambda \to 1$. There is nothing in theory which discriminates in favor of the perfect equilibrium outcome. The intuition for this result again rests on the approximative description of Φ^η as the composition of the unperturbed satisficing process Φ^0 and rare interruptions by a perturbation of one player's aspiration level. By Theorem 1, only perturbations occurring in c_{LY} and $c_{H\cdot}$ matter. There exist small neighborhoods $U(c)$ of conventions $c \in \{c_{LY}, c_{H\cdot}\}$ such that a perturbation into $U(c)$ leads to c with arbitrarily higher probability than to convention $c' \neq c$. So, if perturbations from c_{LY} are never leading to aspirations outside $U(c_{LY})$ and if, in contrast, there is positive probability of a perturbation from $c_{H\cdot}$ into $U(c_{LY})$, then Φ^η will spend almost all time in the asymmetric convention c_{LY}. Similarly, the invariant distribution is as in ii) when perturbations from $c_{H\cdot}$ are very 'narrow' and those from c_{LY} are 'wide'. If perturbations have reasonably wide or even full support in both c_{LY} and $c_{H\cdot}$, iii) applies.

The stationary distribution of an ergodic stochastic process captures average behavior as time goes to infinity. Considering the limit of such a limit distribution as $\eta \to 0$ and as $\lambda \to 1$, i. e. trembles vanish and persistence of once formed aspirations becomes large, is a useful benchmark. Still, in actual process realizations with parameters close to the limit, the Pareto-inefficient outcome excluded by Theorem 1 will be observed. With this caveat, one may summarize above mathematical results as a quite robust prediction in terms of efficiency but not of a specific distribution of surplus.

4. SIMULATION RESULTS

It is worthwhile to investigate whether Theorems 1 and 2 in fact provide a good benchmark if parameters η and λ are not yet close to their respective limit. Is there a bias in favor of the perfect equilibrium (L, Y)? Moreover, how parameter-sensitive is the average surplus distribution and do monotonic trends exist? We look at a number of parameter scenarios using Monte-Carlo simulation.[11]

We assume piece-wise linear inertia functions

$$p_i(\Delta) = \begin{cases} 1; & \Delta \leq 0 \\ 1 - M_i\Delta; & \Delta \in (0, \frac{1-\tilde{p}_i}{M_i}) \\ \tilde{p}_i; & \Delta \geq \frac{1-\tilde{p}_i}{M_i} \end{cases}$$

with parameters $\tilde{p}_i, M_i \in \mathbb{R}_{++}$ $(i = 1, 2)$, and truncated normal perturbation distributions with means $\alpha^*(x_t)$ and $\beta^*(x_t)$ and player-specific standard deviations σ_1 and σ_2, respectively.[12] For more flexibility in modeling different agent characters we drop the requirement that both players have the same aspiration persistence and the same perturbation probability.

[11]Linear algebra methods were applied for control purposes, and Fig. 4 is actually based on a 31×21 grid approximation of E and the left-eigenvector of a sampled transition matrix.

[12]This implies that case iii) of Theorem 2 applies. Inertia functions which fall geometrically or uniform distributions with reasonably wide supports produce qualitatively the same observations.

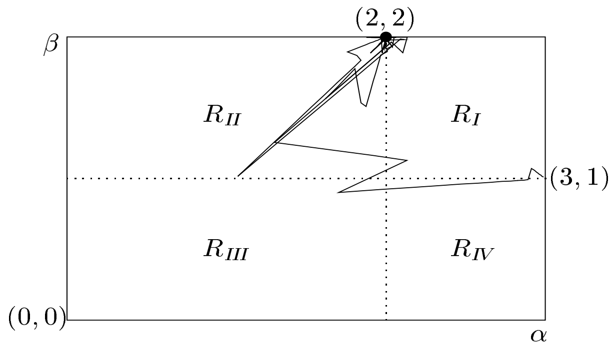

FIG. 3 Aspiration movements as $c_{H\cdot}$ is challenged and replaced by c_{LY}

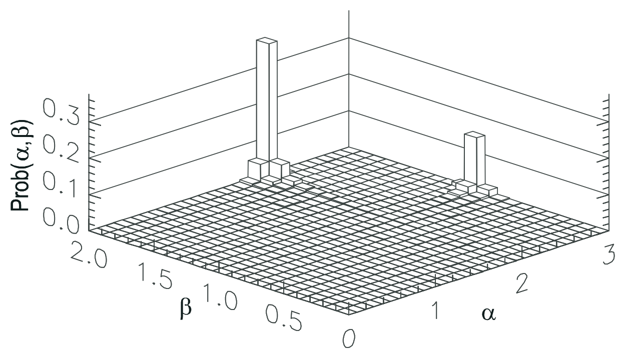

FIG. 4 Marginal stationary distribution over aspirations in scenario S0

The reference scenario S0 is based on the following parameter choices: $\eta = (0.05,\ 0.05)$, $\lambda = (0.8,\ 0.8)$, $\tilde{p} = (0.7,\ 0.7)$, $M = (1.0,\ 1.0)$, and $\sigma = (0.1,\ 0.1)$. In S0, both players on average experience one perturbation in 20 updates, lower their aspiration by 20% after one period of disagreement, and stick to their action with at least probability 0.7.

Typical dynamics of Φ^{η} are characterized by long stretches of time spent in one efficient convention, which is occasionally challenged and eventually replaced by the other efficient convention (cf. Fig. 3). So, the perturbed satisficing process is characterized by *punctuated equilibria.* An approximation of the marginal stationary distribution over aspirations for scenario S0 is depicted in Fig. 4, corresponding to the following approximate marginal distribution over action pairs:[13]

	LY	LN	HY	HN
Prob(a, b)	0.367	0.015	0.185	0.433

Aspirations are already very concentrated at levels corresponding to the efficient conventions. The inequitable perfect equilibrium (L, Y) is not favored over strategy combinations $(H, \cdot)$. The Nash equilibrium (H, N) is more than double as frequent as non-Nash equilibrium (H, Y). Latter observation is, however, not robust to parameter variations. In contrast, the observation of only little (L, N)-play even when η and λ are quite distant from 0 and 1, respectively, is robust. Therefore, one can concentrate on average payoff of player 1, denoted by $\bar{\pi}_1$, in the following sensitivity analysis; the average payoff of player 2 is slightly less than $4 - \bar{\pi}_1$ and comparative statics for $\bar{\pi}_2$ can be inferred from the $\bar{\pi}_1$-diagram.

We first vary $\tilde{p}_1$ and $\tilde{p}_2$. Given the moderately large values of slope M_i, these lower bounds on inertia are the value of p_i within region R_I of aspiration space, the open rectangle defined by corner points $(2, 2)$ and $(3, 1)$ (see Fig. 3), when the present action pair is (L, N). In this *conflict region* no strategy pair simultaneously satisfies players 1 and 2, so that inertia has the effect of *stamina* after (L, N)-play: Whoever loses patience and switches action first strongly increases the chances of eventual convergence to the less-favored efficient convention. With the caveat that Fig. 5 gives *ceteris paribus* information,[14] i. e. all parameters except $\tilde{p}_1$ and $\tilde{p}_2$ are as in scenario S0, we can state:

Observation 1. Player i's average bargaining success increases with his minimal level of action inertia, $\tilde{p}_i$.

[13] The approximation is obtained by a long-run simulation of 20m periods.

[14] For each ($\tilde{p}_1$,$\tilde{p}_2$)-combination, the stationary distribution over actions was approximated at least by 10m periods of Monte-Carlo simulation. The same holds for the numbers depicted in figures 6–9.

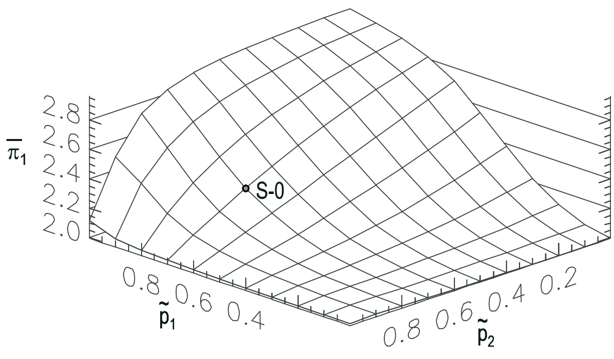

FIG. 5 Average payoff to player 1 for ($\tilde{p}_1$,$\tilde{p}_2$)-variation

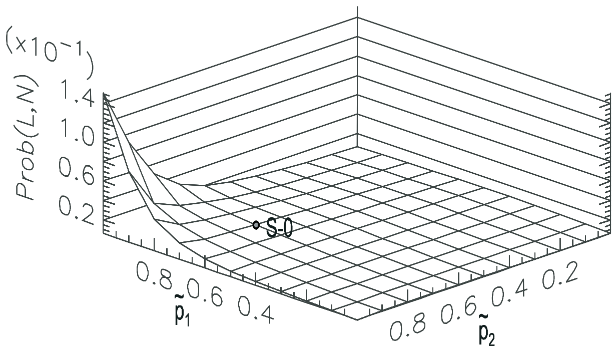

FIG. 6 Frequency of (L, N)-play for ($\tilde{p}_1$,$\tilde{p}_2$)-variation

Loosely speaking and everything else being equal, it pays to be stubborn or persistent after *major* dissatisfaction. However, the sum of both players' average payoffs decreases due to more (L, N)-play when *both* $\tilde{p}_1$ and $\tilde{p}_2$ are increased (see Fig. 6).

The second varied parameter, slope M_i, defines how drastic player i's response to *minor* dissatisfaction is. M_1 is relevant especially when α_t is slightly above 2 and (H, L) or (H, N) has been played. One may interpret M_i as a parameter representing a player's *irritability.* Figure 7 then indicates that from a boundedly-rational bargaining perspective it is (weakly) beneficial to be more irritable. More formally stated (with the caveat above):

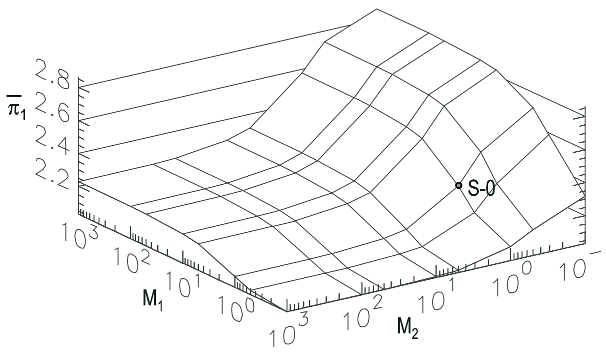

FIG. 7 Average payoff to player 1 for (M_1,M_2)-variation

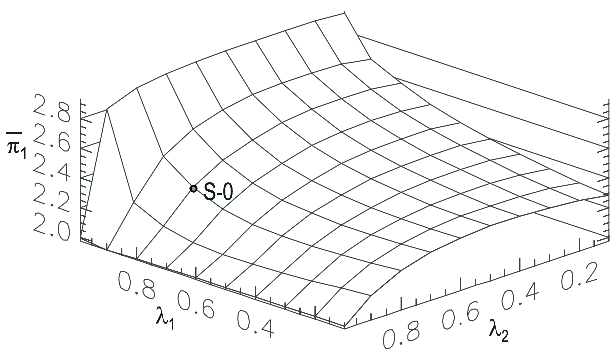

FIG. 8 Average payoff to player 1 for (λ_1,λ_2)-variation

Observation 2. Player i's average bargaining success increases with the slope M_i of her inertia function.

So, although it pays to be stubborn in response to major dissatisfaction, it is beneficial to be comparatively quickly agitated by minor frustration. Greater irritability translates into greater propensity to pick a 'fight' which challenges the less-preferred convention after a small upward tremble in aspiration. Also, it results in fiercer resistance when the latter is approached from inside R_I. Next, aspiration persistence parameter λ_i is varied. We summarize Fig. 8 (with the above caveat):

Observation 3. Player i's average bargaining success increases with the

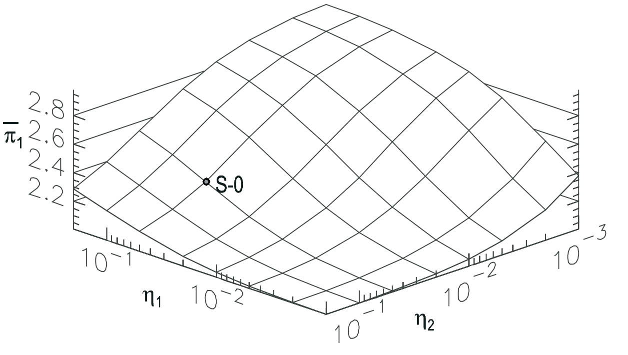

FIG. 9 Average payoff to player 1 for (η_1,η_2)-variation

persistence, λ_i, of his aspirations.

Again, one may in more colloquial terms infer that it pays to be stubborn or persistent—this time referring to aspiration rather than action updating. The intuition for this is that in the critical conflict region of aspiration space, R_I, a greater λ_1 decelerates player 1's moves towards 'surrender' (region R_{II}) but does not affect moves towards 'victory' (region R_{IV}). If one imagines an (unmodeled) encompassing biological or social evolution of agent characters, Observation 3 provides some justification for consideration of the limit case $\lambda \to 1$.

Figures 9 and 10 concern the frequency of perturbations, η_i, and the standard deviation, σ_i, of the normal distributions which define post-perturbation aspirations. Results can be summarized as follows:

Observation 4. Player i's average bargaining success increases with the probability, η_i, of trembles in his aspirations.

Observation 5. Player i's average bargaining success increases with the standard deviation, σ_i, of her aspiration perturbations.

A player obtains a bigger average share of the cake when she is capricious, i. e. has frequent and large variations in her mood or perception of the world. An intuition for this can be found in the asymmetric effect of upward

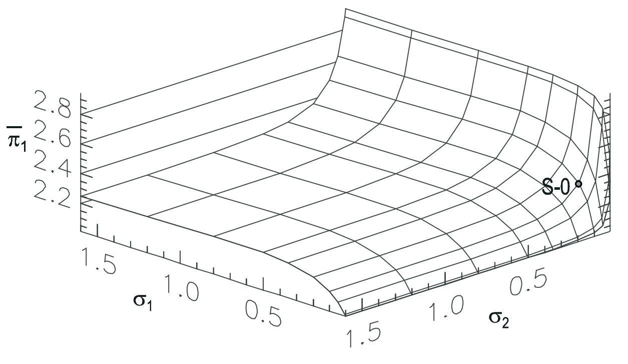

FIG. 10 Average payoff to player 1 for (σ_1,σ_2)-variation

and downward perturbations. An upward tremble in player 1's aspirations when her less-preferred convention $c_{H\cdot}$ is in place results in frustration, which will typically lead to an action switch to L. This bears the chance to establish 1's most-preferred convention c_{LY} for a long time. In contrast, a downward tremble in player 1's aspirations while c_{LY} is in place generally remains unnoticed and, hence, unexploited by player 2.[15]

Observation 4 raises a question if one again imagines an unmodeled encompassing social or biological evolution of player characteristics: If it is advantageous to have a high propensity for spontaneous shifts in aspiration, can the limit case $\eta \to 0$ be expected to have much practical relevance? A proper answer—related to the general relevance of the limit investigations in the tradition of Kandori *et al.* (1993) and Young (1993a)—cannot be given in this paper.

5. RELATION TO THE LITERATURE

To our knowledge, above model is the first to combine stochastic evolutionary methodology with satisficing behavior in a bargaining situation. We draw on the work of Karandikar *et al.* (1998), who consider symmetric

[15]This indicates that it is beneficial to experience asymmetric trembles.

2×2-games with only one efficient outcome. We find satisficing based on endogenous aspiration levels sufficient for approximate long-run efficiency even in the case of competing efficient conventions.

The assumption of satisficing with endogenous aspiration levels has a long tradition (Simon, 1955 and 1959; Sauermann and Selten, 1962; Selten, 1998). Gilboa and Schmeidler (1996) show that it can, in fact, be a viable optimization heuristic with very low information processing and calculation requirements, though this is put into a more sceptical perspective by Börgers and Sarin (2000). Different versions of satisficing based on endogenous aspiration levels in an interactive setting have been investigated by Dixon (2000), Pazgal (1997), and Kim (1999). They are concerned with common interest games, and findings are similar to those of Karandikar *et al.* (see Bendor *et al.*, 2001).

The investigation of satisficing players is still rather rare in the stochastic evolutionary literature. Most authors favor dynamics based on myopic best-replies (Kandori *et al.* 1993; Young, 1993a and 1993b), imitation (Robson and Vega-Redondo, 1996; Josephson and Matros 2000; Hehenkamp, 2002), or possibly a combination (e. g. Kaarbøe and Tieman, 1999). Myopic best-response behavior is already 'too rational' to support $(H, \cdot)$ as a stable outcome. This seems different when imitation dynamics are considered. For these and satisficing dynamics based on random exogenous aspiration levels, approximation e. g. by replicator dynamics is possible for finite time horizon (see Benaim and Weibull, 2000). Explicit satisficing models are, nevertheless, worthwhile since predictions concerning asymptotic behavior can differ significantly (cf. Börgers and Sarin, 1997).

Gale *et al.* (1995) study the UMG in a replicator framework.[16] (L, Y) is an asymptotically stable equilibrium. Only if noise in the responder population is sufficiently higher than in the proposer population (which can be

[16] They consider infinite populations and expected drift in deterministic replicator equations.

motivated), a mixed responder population combined with H-offers by—in the limit—all proposers also becomes asymptotically stable. In our model, the specification of noise is crucial, too. First, supports of perturbation densities in the efficient conventions qualitatively determine long-run behavior. Second, Gale *et al.*'s finding that more noisy behavior can benefit a population is confirmed by our comparative statics. In contrast to Gale *et al.*, a symmetric division can (in the limit) be the unique long-run outcome independent of the initial state in our model.

Other adaptive learning or evolutionary models of ultimatum bargaining which support the symmetric surplus division include Roth and Erev (1995), Harms (1997), Huck and Oechssler (1999), Peters (2000), and Poulsen (2000). A symmetric division also features prominently in Ellingsen's (1997) analysis of Nash demand bargaining. In contrast, the computer simulations by van Bragt *et al.* (2000) clearly favor the asymmetric perfect equilibrium division.

Shor (2000) considers experimental data on monopolistic price setting in a low-information environment. Among the several learning heuristics investigated in the literature, he finds greatest support for satisficing behavior as assumed by Karandikar *et al.* and in this paper. In experiments on continuous-time Cournot competition with limited information, Friedman *et al.* (2001) confirm the advantage of being a slow mover or, in our model interpretation, of being persistent and stubborn.

Bergin and Lipman (1996) warn that *any* invariant distribution of an unperturbed adaptation process can be 'selected' by the perturbed process if perturbation rates are *state-dependently* chosen in an appropriate way. State-dependent perturbation supports, in fact, drive Theorem 2, but a highly implausible type of state-dependent rates would be required to ensure positive weight on the inefficient outcome in the limit.

6. CONCLUDING REMARKS

The first of our main findings is that aspiration-based satisficing behavior—quite remote from traditional economic rationality at the individual level—suffices to reach approximate efficiency in the simple UMG bargaining situation, i. e. despite the conflict of interests regarding more than one efficient outcome. The predominance of either equitable or inequitable surplus division depends on player characteristics usually not considered in bargaining models. In particular, it turned out to be, ceteris paribus, beneficial to be persistent when it comes to updating actions and aspirations, but nevertheless to be capricious. These observations seem broadly consistent with anecdotal evidence about real-life bargaining situations e. g. among colleagues, friends, or family. Also, the incidence of equitable offers by the proposer is in line with observations in bargaining experiments.

For proper statements about the practical relevance of our very weak behavioral axioms and their implications, more experiments in the vein of Lant (1992), Mookherjee and Sopher (1994, 1997), Slembeck (1999), or Binmore *et al.* (2002), and simulations with actual experimental data like Roth and Erev (1995) and Shor (2000) seem worthwhile. The property of endogenous transitions between different adaptive equilibria or conventions could be tested in long-run experiments. The comparative statics observations could possibly be tested by letting human players interact repeatedly with each other and also machine players of well-specified characteristics. If the inherent exploitability of machine players' satisficing behavior were discovered by the human agents, or the comparative statics contradicted above results, the 'lower bound' on adaptive human bargaining behavior given in this paper would need to be raised.

Changes to above aspiration adaptation rule, incorporating e. g. overshooting, time lags, or different forms of randomness, would in most cases leave the analytical results in place. Similarly, little would change if agents 'trembled' in implementing their actions.[17]

Extending the model to bigger strategy sets—for example to improve the discrete approximation of the ultimatum game or to allow for alternating offers—is an interesting option. Unfortunately, analytical investigation gets considerably more complicated. It would probably be more worthwhile to rather consider more than two players (see Dixon 2000). This would allow for a shock model in which players' aspirations are also shifted by occasional observations of the success of other players. In a population-based model, it would then be natural to investigate the effects of different interaction structures (see e. g. Ellison, 1993; Berninghaus and Schwalbe, 1996; Tieman *et al.*, 2000).

APPENDIX: PROOFS

The proofs draw on techniques used by Karandikar *et al.* (1998), which are in turn based on Meyn and Tweedie (1993). Details are given in Napel (2000).

Proof of Proposition 2. Let $\sigma(E)$ denote the Borel σ-algebra on E. The unperturbed process Φ^0 is defined by a *transition kernel* $P : E \times \sigma(E) \to [0,1]$, where $P(x_t, S)$ is the probability to reach the set S from state x_t in period $t+1$. Four transitions to singleton sets can have positive probability for Φ^0—namely those to $x_{t+1} = (a, \alpha^*(x_t), b, \beta^*(x_t)\,)$, where feasible one-step transitions in action space are indicated in Fig. 11 (see Fig. 3 for the definition of the open rectangles R_I–R_{IV}).

Let Q_i denote the kernel of the perturbed satisficing process Φ^η conditioned on the event that *only* player i's aspirations are perturbed. $Q \equiv 1/2(Q_1+Q_2)$ denotes Φ^η's transition rule conditioned on exactly one player experiencing a tremble with both players equally likely to be the one. With

[17] Technically, perturbation of aspirations would still be needed to ensure Φ^η's ergodicity.

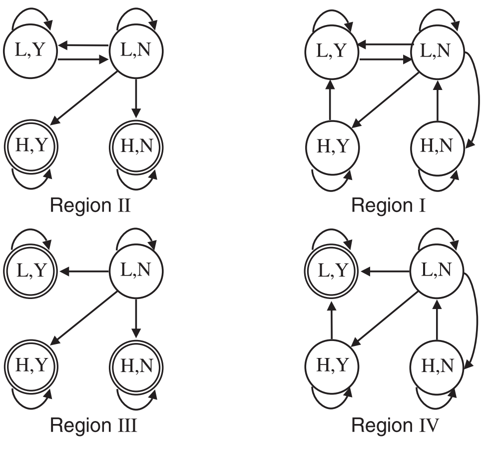

FIG. 11 Illustration of action updating in the four regions of aspiration space

Q_* as the transition probability kernel conditioned on *both* players' aspirations being perturbed, the complete kernel P^η for Φ^η can be written as

$$P^\eta = (1-\eta)^2 P + 2\eta(1-\eta)Q + \eta^2 Q_*.$$

Q_* is lower semi-continuous (l.s.c.) in x or a *strong Feller kernel.* For all $x \in E$, $S \in \sigma(E)$ we have $P^\eta(x,S) \geq \eta^2 Q_*(x,S)$. This establishes that Φ^η is a *T-chain* (see Meyn and Tweedie, 1993, p. 127).

P^η is *open set irreducible*, i. e. every open neighborhood U_x of every $x \in E$ can be reached from any $y \in E$ in a finite number of transitions. This follows from $\mathsf{A} \times \mathsf{B}$'s compactness, our assumptions on the supports of perturbation densities, g_1 and g_2, and the fact that any action profile can be reached with positive probability in at most three steps in R_I (and then be preserved by players' inertia).

Moreover, Φ^η is *strongly aperiodic*: First, given the inertia in action choices, the supports of perturbation densities g_1 and g_2 define, e. g. for $x^* \equiv c_{HN}$, a neighborhood U_{x^*} such that $P^\eta(x, U_{x^*}) \geq \nu_1(U_{x^*}) > 0$ for any $x \in U_{x^*}$ and some function ν_1. Second, from any $x \in E$ one can reach U_{x^*} with positive probability in a finite number of steps. The proposition then follows from Theorem 16.2.5 in Meyn and Tweedie (1993, p. 395). ∎

We consider a few lemmata before turning to Theorems 1 and 2. First, define the kernel $R : E \times \sigma(E) \to [0,1]$ with

$$R(x,S) \equiv \lim_{n\to\infty} P^n(x,S), \qquad x \in E,\ S \in \sigma(E),$$

where P^n denotes the n-step transition kernel of Φ^0 inductively defined from P. By Proposition 1 the limit exists. R defines a 'fast-forward' Markov process which moves from x directly to a convention with probabilities for each $c \in C$ defined by Φ^0's long-run behavior. Second, define an artificial cousin, Θ, of the perturbed process Φ^η by considering the result of just *one* perturbation by exactly one player—a transition according to Q—and of running the unperturbed process—described by R—for evermore afterwards. Θ has the transition kernel QR with

$$QR\ (x,S) \equiv Q(x,\cdot)R\ (S) = \int Q(x,ds)R(s,S)$$

for $x \in E$ and $S \in \sigma(E)$. The long-run behavior of Φ^η for η close to zero is very similar to that of Θ and, in fact, one has:

Lemma 1. *The sequence $\{\mu^\eta\}_{\eta\in(0,1]}$ of limit distributions of Φ^η converges weakly to a unique distribution μ^* on $(E, \sigma(E))$ as $\eta \to 0$. μ^* coincides with the unique invariant probability measure of Θ.*

Proof. The result follows directly from Theorem 2 in Karandikar *et al.* (1998), where it remains to check that QR has a unique invariant measure. By Proposition 1 any $c \in C$ is an absorbing state for transitions according to R and a transition according to R always results in some $c \in C$. Starting in c_{LN}, a transition according to Q leads into R_{III} with positive probability, and from there a transition to any $c \in C$ has positive probability according to R. Starting in $c_{H\cdot}$ or c_{LY}, a transition according to Q leads into R_I with positive probability, and from there a transition to any $c \in C$ has positive probability according to R. Hence, Θ is ψ-irreducible and recurrent. This implies that Θ has a unique invariant measure (cf. Meyn and Tweedie, pp. 133 and 182, and Theorem 10.4.9). ∎

First, we investigate unperturbed satisficing dynamics, Φ^0, in region R_{IV}. For $\hat{\alpha}, \hat{\beta} \in (0, \frac{1}{2})$, let $I(\hat{\alpha}, \hat{\beta}) \equiv [2+\hat{\alpha}, 3-\hat{\alpha}] \times [\hat{\beta}, 1-\hat{\beta}]$ be a rectangle in R_{IV}. One can establish a lower bound on the number of periods that are needed to exit the larger rectangle $I(\frac{\hat{\alpha}}{2}, \frac{\hat{\beta}}{2})$ from anywhere in $I(\hat{\alpha}, \hat{\beta})$. This number goes to infinity as λ approaches 1. Therefore the probability of *not* playing the only mutually satisfying action pair (L, Y) at least once before R_{IV} is left can be made arbitrarily small. However, once (L, Y) is played with aspirations in R_{IV}, Φ^0 converges to c_{LY}. This yields:

LEMMA 2. *Given $\hat{\alpha}, \hat{\beta} \in (0, \frac{1}{2})$ and any $\varepsilon > 0$, there exists $\lambda_1 \in (0,1)$ such that*

$$^{\lambda}\mathrm{Prob}(\, x_t \to c_{LY} \,|\, x_t = (a_t, \alpha_t, b_t, \beta_t) \;\wedge\; (\alpha_t, \beta_t) \in I(\hat{\alpha}, \hat{\beta})\,) > 1 - \varepsilon$$

for all $\lambda \in (\lambda_1, 1)$ and all $t \geq 1$ in unperturbed process Φ^0.

An analogous result holds for dynamics in region R_{II}, with rectangles $J(\hat{\alpha}, \hat{\beta}) \equiv [\hat{\alpha}, 2-\hat{\alpha}] \times [1+\hat{\beta}, 2-\hat{\beta}]$ for $\hat{\beta} \in (0, \frac{1}{2})$ and convergence to $c_{H\cdot}$.

Next, consider region R_{III}: For $\lambda \to 1$, the number of (L, N)-plays needed to lower a player's aspirations from given v to u increases without bound. The probability that player 1 (2) does not switch to the satisfying H (Y) for all these periods vanishes. This is the reasoning behind the following lemma, and an obvious analogue concerning player 2's aspirations:

LEMMA 3. *Given positive numbers u and v with $u < w \equiv \min\{2, v\}$ and any $\varepsilon > 0$, there exists $\lambda_2 \in (0,1)$ such that for arbitrary date T*

$$^{\lambda}\mathrm{Prob}(\alpha_t < u \textit{ for some } t \geq T \,|\, \alpha_T \geq v) < \varepsilon$$

for all $\lambda \in (\lambda_2, 1)$ in unperturbed process Φ^0.

Proof of Theorem 1. By Proposition 1, kernel R concentrates all weight on the conventions, and the same must be true for QR. Then, by Lemma 1, μ^* must place zero weight on all states except c_{LY}, c_{LN}, and $c_{H\cdot}$.

All aspects of QR relevant to asymptotic behavior can be captured by a 3×3-matrix $S = (s_{jk})$ where cell s_{jk} contains the probability of a transition from c_j to c_k ($j, k \in \{LY, LN, H\cdot\}$). This matrix depends on parameter λ.

Given an arbitrary $\lambda < 1$ one can find $\varepsilon(\lambda) > 0$ such that $s_{LY,LN} < \varepsilon(\lambda)$ and $s_{H\cdot,LN} < \varepsilon(\lambda)$ by Lemma 3 and its analogue for player 2, and $\varepsilon(\lambda) \to 0$ as $\lambda \to 1$. In contrast, given an arbitrary $\lambda < 1$ there exists $\delta(\lambda) > 0$ such that $s_{LN,LY} + s_{LN,H\cdot} \geq \delta(\lambda)$. This uses that perturbation densities $g_1(\cdot|x)$ and $g_2(\cdot|x)$ have support in a non-degenerate neighborhood of c_{LN}. Lemmata 2 and 3 (and their analogues for R_{II} and player 2, respectively) imply that $\delta(\lambda) \to 1$ as $\lambda \to 1$. This establishes Theorem 1. ∎

LEMMA 4. *Given arbitrary $(\alpha, \beta) \in R_I$ and $\varepsilon > 0$, there exists λ_3 such that for arbitrary date $T \geq 1$*

$$^{\lambda}\mathit{Prob}\left((\alpha_t, \beta_t) \in R_{II} \cup R_{IV} \text{ for some } t > T \mid x_T = (a_t, \alpha, b_t, \beta) \right) > 1 - \varepsilon$$

for all $\lambda \in (\lambda_3, 1)$ in unperturbed satisficing process Φ^0.

Proof. Consider arbitrary but fixed $(\alpha, \beta) \in R_I$. Any direct move from R_I into R_{III} will result in a move back to R_I or to $R_{II} \cup R_{IV}$ with arbitrarily high probability by Lemma 3 and its analogue for player 2. The probability of staying in R_I for ever without convergence to a convention is zero by Proposition 1. However, the probability of an infinite number of (L, Y)-plays despite dissatisfaction of player 2, which would be necessary to reach c_{LY} from inside R_I, is no more than

$$\prod_{t=1}^{\infty} p_2(\,\lambda^t(\beta - 1)\,),$$

which approaches zero for $\lambda \to 1$. In particular, we can find λ' such that above probability is less than $\varepsilon/2$. Similarly, there is λ'' such that the upper bound on the probability of an infinite number of $(H, \cdot)$-plays despite dissatisfaction of player 1 is smaller than $\varepsilon/2$. ∎

Now, for arbitrary $\lambda_4 \in (0,1)$ and a fixed aspiration level of player 1 in R_I, $\alpha \in (2,3)$, we define

$$\hat{p}_{(L,\cdot)}(\alpha) \equiv \sup_{\lambda \in (\lambda_4, 1)} \sup_{\beta \in (1,2)} \max_{(a,b) \in A \times B} {}^{\lambda}\text{Prob}(\, a_{t+2} = L \,|\, x_t = (a, \alpha, b, \beta)\,)$$

to establish an upper bound on the probability that player 1 will play L two periods ahead in time when present aspirations are in R_I. By definition, this bound is independent of λ. For $\alpha \in (2,3)$, $\hat{p}_{(L,\cdot)}(\alpha)$ is strictly positive because of player 1's inertia (consider $x_t = (L, \alpha, b, \beta)$). However, even when an action profile $(L, \cdot)$ was played in t there is a strictly positive probability for a 2-step transition away from it, namely first to (L, N) and then to $(H, \cdot)$ (cf. Fig. 11). This implies $\hat{p}_{(L,\cdot)}(\alpha) \leq \hat{p} < 1$ for some $\hat{p} \in (0,1)$ and all $\alpha \in (2,3)$.

Similarly, we define for $\alpha \in (2,3)$

$$\begin{aligned} \check{p}_{(L,N)}(\alpha) \quad \equiv \quad & \inf_{\lambda \in (\lambda_4, 1)} \inf_{\beta \in (1,2)} \min_{(a,b) \in A \times B} {}^{\lambda}\text{Prob}(\, (a_{t+2}, b_{t+2}) = (L, N) \\ & \qquad \qquad \qquad \qquad \qquad \qquad | \, x_t = (a, \alpha, b, \beta)\,) \end{aligned}$$

to yield a lower bound on the probability that action pair (L, N) will be played in two periods. Obviously, $\check{p}_{(L,N)}(\alpha) \leq \hat{p}_{(L,\cdot)}(\alpha)$. Moreover, $\check{p}_{(L,N)}(\alpha)$ approaches 0 as $\alpha \downarrow 2$. This is because the probability of player 1 switching away from H, $1 - p_1(\alpha - 2)$, converges to 0. Using our upper bounding assumption on players' inertia functions p_i (cf. Fig. 2), there must, however, exist $\check{M} > 0$ and $\check{\alpha} > 0$ with $\check{p}_{(L,N)}(\alpha) \geq (\alpha - 2)\check{M}$ for $\alpha \in (2, 2 + \check{\alpha})$.

For any $\hat{\alpha} \in (0,1)$, we now choose

$$m(\hat{\alpha}) \equiv \max \left\{ \frac{2 \ln \check{p}_{(L,N)}(2 + \hat{\alpha})}{\ln \hat{p}_{(L,\cdot)}(2 + \hat{\alpha})} \, , \, 3 \right\}.$$

Since $\check{p}_{(L,N)}(2 + \hat{\alpha})$ goes to zero but $\hat{p}_{(L,\cdot)}(2 + \hat{\alpha})$ is bounded away from both zero and one, $m(\hat{\alpha}) \to \infty$ as $\hat{\alpha} \to 0$.

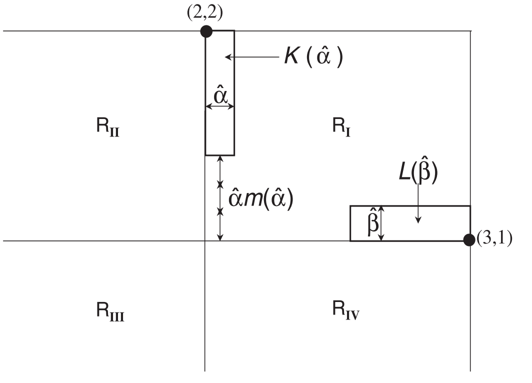

FIG. 12 Illustration of $K(\hat{\alpha})$ and $L(\hat{\beta})$

Finally, we define rectangles $K(\hat{\alpha}) \equiv (2, 2+\hat{\alpha}) \times (1+\hat{\alpha}\, m(\hat{\alpha}), 2)$ for all $\hat{\alpha}$ such that $\hat{\alpha}\, m(\hat{\alpha}) < 1$, also referred to as *permissable* $\hat{\alpha}$ (cf. Fig. 12). Using our bounds on $\check{p}_{(L,N)}(\alpha)$ and $\hat{p}_{(L,\cdot)}(\alpha)$ for $\alpha \in (2,3)$, we have

$$\frac{2\ln(\hat{\alpha}\,\check{M})}{\ln \hat{p}} + 3 \geq m(\hat{\alpha}) \geq 3.$$

From this follows $\hat{\alpha}\, m(\hat{\alpha}) \to 0$ as $\hat{\alpha} \to 0$, ensuring that there is always an $\hat{\alpha}^*(x)$ independent of λ such that all $\hat{\alpha} \in (0, \hat{\alpha}^*)$ are permissable.

Lemma 5. *From arbitrary state x_T with $(\alpha_T, \beta_T) \in K(\hat{\alpha})$ for an arbitrary permissable $\hat{\alpha}$, a move by unperturbed process Φ^0 into R_{II} becomes infinitely more likely as $\lambda \to 1$ than a move into R_{IV}.*

Proof. Consider an arbitrary but fixed state x_T with $(\alpha_T, \beta_T) \in K(\hat{\alpha})$ for some fixed permissable $\hat{\alpha}$, and let λ_4 in the definitions of $\check{p}_{(L,N)}$ and $\hat{p}_{(L,\cdot)}$ be large enough such that $\frac{\hat{\alpha}}{2(1-\lambda_4)} \geq 2$. One round of (L,N)-play lowers player 1's aspiration by at least $2\,(1-\lambda)$, and player 2's aspiration by at most the amount $2\,(1-\lambda)$. For player 2, the aspiration decrease caused by (L,N)-play is greater than that of (L,Y)-play.

The probability of a move to R_{II} is at least as high as that of observing

$$T^*(\hat{\alpha}, \lambda) \equiv \left\lceil \frac{\hat{\alpha}}{2\,(1-\lambda)} \right\rceil$$

periods of consecutive (L,N) play ($\lceil y \rceil$ denoting the smallest integer greater than y).

$$T^{**}(\hat{\alpha},\lambda) \equiv \left\lceil \frac{\hat{\alpha}\, m(\hat{\alpha})}{2\,(1-\lambda)} \right\rceil$$

is the minimal number of $(L,\cdot)$-plays which could decrease player 2's aspiration level below 1 and hence lead aspirations into R_{IV}.

Starting with x_T and considering periods $x_{T+2},\ldots,x_{T+T^*+2}$, we get $\check{p}_{(L,N)}(2+\hat{\alpha})^{T^*(\hat{\alpha},\lambda)}$ as a lower bound on the probability to move into R_{II}. Similarly, considering periods $x_{T+2},x_{T+3},\ldots$ we get $\hat{p}_{(L,\cdot)}(2+\hat{\alpha})^{T^{**}(\hat{\alpha},\lambda)}$ as an upper bound on the probability of moving to R_{IV}.

The ratio of the probabilities of 'escaping' from $K(\hat{\alpha})$ into region R_{II} and R_{IV} is

$$\begin{aligned} r(\lambda) \equiv & \frac{{}^{\lambda}\mathrm{Prob}(\,(\alpha_t,\beta_t)\in R_{II} \text{ for some } t\geq T\,|\,(\alpha_T,\beta_T)\in K(\hat{\alpha})\,)}{{}^{\lambda}\mathrm{Prob}(\,(\alpha_t,\beta_t)\in R_{IV} \text{ for some } t\geq T\,|\,(\alpha_T,\beta_T)\in K(\hat{\alpha})\,)} \\ \geq & \frac{\check{p}_{(L,N)}(2+\hat{\alpha})^{\left\lceil \frac{\hat{\alpha}}{2\,(1-\lambda)} \right\rceil}}{\hat{p}_{(L,\cdot)}(2+\hat{\alpha})^{\left\lceil \frac{\hat{\alpha}\, m(\hat{\alpha})}{2\,(1-\lambda)} \right\rceil}} \\ \geq & \frac{\check{p}_{(L,N)}(2+\hat{\alpha})^{2\frac{\hat{\alpha}}{2\,(1-\lambda)}}}{\hat{p}_{(L,\cdot)}(2+\hat{\alpha})^{\frac{\hat{\alpha}\, m(\hat{\alpha})}{2\,(1-\lambda)}}} \\ \geq & \left(\frac{\check{p}_{(L,N)}(2+\hat{\alpha})^2}{\hat{p}_{(L,\cdot)}(2+\hat{\alpha})^{m(\hat{\alpha})}}\right)^{\frac{\hat{\alpha}}{2\,(1-\lambda)}} . \end{aligned}$$

Given the choice of $m(\hat{\alpha})$, the base term is greater than 1. The exponent goes to infinity as λ approaches 1, implying $r(\lambda) \xrightarrow[\lambda\to\infty]{} \infty$. ∎

An analogous result holds for moves from appropriately defined rectangles $L(\hat{\beta})$ (see Fig. 12) into R_{IV} and R_{II}, respectively.

Proof of Theorem 2. Let s_{jk} refer to elements of the matrix defined in the proof of Theorem 1, where $s_{LY,LN}$ and $s_{H\cdot,LN}$ approach 0 for $\lambda \to 1$. Our assumptions on g_1 and g_2 ensure that a positive measure of perturbations from c_{LY} or $c_{H\cdot}$ stays in a neighborhood comprised in $L(\hat{\beta})$ or $K(\hat{\alpha})$ respectively for permissable $\hat{\beta}$ and $\hat{\alpha}$, implying an almost sure return to c_{LY} or $c_{H\cdot}$, respectively, as $\lambda \to 1$. So, $s_{LY,LY}, s_{H\cdot,H\cdot} \geq \nu > 0$.

For case i), choose the supports of $g_i(\cdot|c_{H\cdot})$ $(i = 1, 2)$ wide enough to place positive weight on some rectangle $L(\hat{\beta})$ for a permissable $\hat{\beta}$, but choose $g_i(\cdot|c_{LY})$ such that they place all weight on some rectangle $L(\hat{\beta}')$ for a permissable $\hat{\beta}'$ and R_{IV}. Then, $s_{LY,H\cdot} < \varepsilon(\lambda)$ but $s_{H\cdot,LY} \geq \delta > 0$. As $\lambda \to 1$, $\varepsilon(\lambda) \to 0$ by Lemma 2 and the analogue of Lemma 5. In the limit c_{LY} is the only absorbing state of Φ^{η}, and μ^* places all weight on it. Case ii) is analogous. For case iii) choose the supports of $g_i(\cdot|c_{H\cdot})$ $(i = 1, 2)$ wide enough to place positive weight on some rectangle $L(\hat{\beta})$ for a permissable $\hat{\beta}$, and choose those of $g_i(\cdot|c_{LY})$ wide enough to place positive weight on some rectangle $K(\hat{\alpha})$ for a permissable $\hat{\alpha}$. Now, both $s_{LY,H\cdot}$ and $s_{H\cdot,LY}$ are bounded away from zero. Consequently, μ^* will place positive weight on both c_{LY} and $c_{H\cdot}$ even in the limit $\lambda \to 1$. ∎

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