Monotonicity of power in games with a priori unions

J.M Alonso-Meijide[1], C. Bowles[2], M. J. Holler[3], S. Napel[4]

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Abstract

Power indices are commonly required to assign at least as much power to a player endowed with some given voting weight as to any player of the same game with smaller weight. This local monotonicity and a related global property however are frequently and for good reasons violated when indices take account of a priori unions amongst subsets of players (reflecting, e.g., ideological proximity). This paper introduces adaptations of the conventional monotonicity notions that are suitable for voting games with an exogenous coalition structure. A taxonomy of old and new monotonicity concepts is provided, and different coalitional versions of the Banzhaf and Shapley-Shubik power indices are compared accordingly.

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[1] meijide@lugo.usc.es, Department of Statistics and Operations Research, University of Santiago de Compostela

[2] carlos.bowles@ecb.int, Economics Department, European University Institute

[3] Corresponding author. holler@econ.uni-hamburg.de, Department of Economics, University of Hamburg

[4] stefan.napel@uni-bayreuth.de, Department of Economics, University of Bayreuth

1 Introduction

Power indices measure players' abilities to influence outcomes in voting situations. They are valuable instruments to study power – arguably the most important concept in political science – because a player's voting power is generally *not* proportional to the voting weight at its source. On the one hand, this observation is obvious from considering, say, a 50% majority rule applied to an institution with three players and respective weights of (a) 51, 44, and 5 percent or (b) 49, 44, and 7 percent: Even though players 2 and 3 have a non-negligible share of voting weight in (a), they are in fact powerless. In contrast, all three players face a perfectly symmetric need (and opportunity) to find at least one coalition partner in order to pass a proposal in (b), i.e., a priori they have the same voting power. On the other hand, the actual translation of weights into power typically is not as straightforward as in (a) or (b), where it seems common sense to describe the situation by power vectors $(1, 0, 0)$ and $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$, respectively – at least if no further information about players' inclinations to form coalitions is available.

Several alternative indices have been proposed as suitable mappings from weights and decision quota to power. The most widely used ones are the Banzhaf and Shapley-Shubik power indices (Penrose, 1946, Banzhaf, 1965, Coleman, 1971; Shapley and Shubik, 1954), but there are many others.[1] In determining which of all these indices is most suitable in a given context, the respective axiomatic characterizations and probabilistic foundations play an important role. In addition, the monotonicity properties of a power index are commonly regarded as a major criterion. Some power indices will under certain circumstances indicate greater power for a player with a given voting weight than for another one who has greater weight. Other indices, in contrast, guarantee that any player endowed with voting weight w is identified as at least as powerful as any player who has a smaller weight w' in the given voting game. This property is known as *local monotonicity.* A related property which refers to comparisons across games is known as *global monotonicity.* Though this may not be obvious at first sight, there are good reasons why some indices are *not* monotonic in one, the other, or either sense (see Holler and Napel, 2004a and 2004b). This is true in particular when there exists information on the relations among players that is relevant for formation of a winning coalition (defined by jointly meeting the specified quota). Special relations among subsets of players can derive from their previous interaction, ideological proximity, geographic proximity, etc.

In this paper, we will specifically consider the case in which players' inclinations to form coalitions are captured by a so-called *coalition structure*: a partition of the set of players into pairwise disjoint *a priori unions.* This entails the assumption that either all members of any a priori union join a coalition, or none of them does. A priori unions can reflect the strict party

[1] See Felsenthal and Machover (2006) for a historical survey.

discipline which is prevailing in many national legislatures and electoral bodies such as the German Bundestag or the US Electoral College. They may also capture the implicit or explicit formation of blocks within supranational organizations – for instance by the Benelux or eurozone countries within the European Union (EU), or by EU members within the International Monetary Fund (IMF) or the World Trade Organization.

Games with coalition structures were first considered by Aumann and Drèze (1974), who extended the Shapley value to this new framework. A second approach was initiated by Owen (1977). Its main advantage is to allow for a transparent 'correction' of traditional power indices, while keeping track of their fundamental axiomatic properties with respect to the allocation of rewards and power both *within* and *between* unions. For example, Owen's (1977) coalitional power index twice invokes the Shapley value, while Owen (1982) applies the Banzhaf value within and between unions. In contrast, Alonso-Meijide and Fiestras-Janeiro (2002) use the Banzhaf value in the game between unions and apply the Shapley value to surplus sharing within unions.[2]

Coalitional power indices are a useful analytical tool with diverse real-world applications. Carreras and Owen (1988) explicitly apply the concept of a priori unions in an investigation of the distribution of power in the Catalonian Parliament (with a recent related study by Alonso-Meijide et al., 2005). Possible a priori unions formed, for example, by France and Germany, the Nordic countries, the Benelux countries, or the Mediterranean block play an important role in Widgrén's (1994) early evaluation of different EU enlargement scenarios. Vázquez et al. (1997) apply Owen's coalitional power index to aircraft landing fees. Alonso-Meijide and Bowles (2005) and Leech and Leech (2005, 2006) shed light on the distribution of voting power in the IMF in the context of a political controversy over the tacit a priori union formed by EU members. Kauppi and Widgrén (2007) have recently shown in voting power-based regression analysis that the treatment of the historical alliance between France and Germany plays a significant role for how well EU budget shares can be explained, indicating that a priori unions can have sizeable financial implications. Turnovec et al. (2007) point out that the European Parliament can be studied as a game between supranational party groups formed by national party representatives based on their shared political attitudes, but also as one between informal national a priori unions formed by the respective delegates from individual member states. There is scope for many more applied studies.

Since coalitional power indices which take a priori unions into account typically do not obey the conventional notion of local monotonicity, this property has naturally not played any role

[2]For related axiomatic characterizations see, e.g., Winter (1992), Vázquez et al. (1997), Amer et al. (2002), or Hamiache (1999). Procedures to compute coalitional values efficiently can be found in Owen and Winter (1992), Carreras and Magaña (1994), Alonso-Meijide et al. (2005), and Alonso-Meijide and Bowles (2005); the latter paper exploits generating functions while all others are based on multilinear extensions.

in discriminating between different coalitional power indices – let alone in their axiomatization. This is a main motivation for this paper. Its aim is to meaningfully extend local and global monotonicity to weighted majority games with a priori unions. It illustrates some distinguishing features of distinct coalitional power indices and clarifies their relation to traditional indices.

The paper is organized as follows. In Section 2 we introduce our notation, formally state the conventional notions of local and global monotonicity, and provide definitions of the most common power indices used for games with and without a given system of a priori unions. Then Sections 3 and 4 introduce two new notions of local monotonicity and two new notions of global monotonicity. We analyze their relationship and illustrate the new concepts by investigating the properties of three different coalitional power indices. Section 5 concludes.

2 Preliminaries

A finite *(cooperative) game* is a pair (N, v) where $N = \{1, 2, \ldots, n\}$ is the set of *players* and v, the *characteristic function*, is a real valued function defined on the subsets of N (referred to as *coalitions*) such that $v(\varnothing) = 0$. A *simple game* is a game (N, v) in which v only takes the values 0 and 1, is not identically 0, and satisfies the condition $v(T) \leq v(S)$ whenever $T \subseteq S$. A coalition S is *winning* if $v(S) = 1$, and *losing* if $v(S) = 0$. For given (N, v), the collection of all winning coalitions is referred to as W.

A simple game (N, v) is a *weighted majority game* if there exists a set of *weights* $w_1, w_2, \ldots, w_n$ for the players, with $w_i \geq 0$ $(i \in N)$ and $\sum_i w_i = 1$, and a *quota* $q \in (0, 1]$ such that $S \in W$ if and only if $w(S) \geq q$, where $w(S) = \sum_{i \in S} w_i$.[3] A weighted majority game is represented by $[q; w_1, w_2, \ldots, w_n]$ and we denote the set of all weighted majority games with player set N by $\mathcal{W}(N)$.

A *power index* for weighted majority games is a function $f \colon \mathcal{W}(N) \to \mathbb{R}^n$ (or, more precisely, a family of functions because N and n are not fixed) which assigns to any weighted majority game (N, v) a vector $f(N, v)$, where the real number $f_i(N, v)$ is the power of player i in the game (N, v) according to f.

The most important power indices are the *Banzhaf index* and the *Shapley-Shubik index* (hereafter BZ and SH index). These indices can be written as[4]

$$f_i(N, v) = \sum_{S \subseteq N \setminus i} p_S^i \cdot [v(S \cup i) - v(S)], \text{ for any } i \in N, \tag{1}$$

where $\left\{p_S^i : S \subseteq N \setminus i\right\}$ corresponds to a probability distribution over the collection of coalitions

[3]See Taylor and Zwicker (1999) for a characterization of the simple games that are weighted majority games.

[4]In line with the literature, we write $S \setminus i$ instead of $S \setminus \{i\}$ and $S \cup i$ instead of $S \cup \{i\}$. Also note that we consider the *non-normalized* version of the Banzhaf index.

not containing i. For the BZ index

$$p_S^i = \frac{1}{2^{n-1}}$$

and for the SH index

$$p_S^i = \frac{s!\,(n-s-1)!}{n!},$$

where s refers to the cardinality of S.

The difference $v(S \cup i) - v(S)$ is called the *marginal contribution* of player i to coalition S. Taking into account that for a simple game $v(T) = 1$ if $T \in W$ and $v(T) = 0$ otherwise, it holds that $v(S \cup i) - v(S) = 1$ if and only if S is losing and $S \cup i$ is winning. In this case, we say that the pair of coalitions $(S, S \cup i)$ is a *swing* for player i.

Given $(N, v) \in \mathcal{W}(N)$ two players $i, j \in N$ are called *symmetric* if their marginal contribution to any coalition which contains neither player is the same. A power index f is *symmetric* if the power of any two symmetric players is the same; in particular, a symmetric index must ascribe equal power to players that have equal weight.

An intuitively compelling property in the context of weighted majority games is local monotonicity:

Definition 1 *A power index $f\colon \mathcal{W}(N) \to \mathbb{R}^n$ is* locally monotonic *if for each weighted majority game $(N, v) = [q; w_1, w_2, \ldots, w_n] \in \mathcal{W}(N)$*

$$f_i(N, v) \geq f_j(N, v) \tag{2}$$

holds for every pair of players $i, j \in N$ such that $w_i \geq w_j$.

Note that one might impose inequality (2) only for players $i, j \in N$ such that $w_i > w_j$, i.e., not directly restricting power when $w_i = w_j$. This would result in an equivalent definition because (2) must hold for *each* weighted majority representation $[q; w_1, \ldots, w_n]$ of a simple game (N, v). If one such representation involves $w_i = w_j > 0$, there exist $\varepsilon_1 \geq 0$, $\varepsilon_2 > 0$ such that $[q - \varepsilon_1; w_1 + \frac{\varepsilon_2}{n-1}, \ldots, w_i + \frac{\varepsilon_2}{n-1}, \ldots, w_j - \varepsilon_2, \ldots, w_n + \frac{\varepsilon_2}{n-1}] \in \mathcal{W}(N)$ also represents (N, v) (cf. Felsenthal and Machover, 1998, p. 30); and therefore $f_i(N, v) \geq f_j(N, v)$ is required. So it is not essential that we refer to players' weights by weak inequalities; it has the advantage, however, that an index's violation of monotonicity is exhibited by *all* weighted voting representations of a given simple game. Note also that local monotonicity automatically restricts an index to be symmetric.

The marginal contribution of a player $i \in N$ weakly increases in weight w_i: if player $i \notin S$ with weight w_i can turn a losing coalition S into a winning one by joining, then any player $j \notin S$ with weight $w_j \geq w_i$ can necessarily do so, too. Moreover, the coefficients p_S^i in Eq. (1) only depend on the cardinality of N for the BZ index and on the cardinality of S and N for the SH index, i.e., they are constant in i. It follows that for given $[q; w_1, w_2, \ldots, w_n]$ with $w_i \geq w_j$, the BZ (SH) index of player i is at least as big as the BZ (SH) index of player j, i.e., both indices satisfy local monotonicity.

While local monotonicity refers to the relation between power of two players in the *same* game, another intuitively desirable property looks at power of the same player in two *different* weighted majority games:

Definition 2 *A power index* $f\colon \mathcal{W}(N) \to \mathbb{R}^n$ *is* globally monotonic *if for every two weighted majority games* $(N, v) = [q; w_1, w_2, \ldots, w_n]$ *and* $(N, v') = [q; w'_1, w'_2, \ldots, w'_n] \in \mathcal{W}(N)$

$$f_i(N, v) \geq f_i(N, v')$$

holds for every player $i \in N$ *such that* $w_i \geq w'_i$ *and* $w_j \leq w'_j$ *for all* $j \neq i$.

Both BZ and SH indices satisfy global monotonicity (see Turnovec, 1998). Turnovec (1998) proves, too, that any power index f which is globally monotonic and symmetric, is also locally monotonic.[5]

We will next formalize an exogenous division of players $N = \{1, \ldots, n\}$ into *a priori unions* whose members will either join a coalition together or not at all. Denote by $\mathcal{P}(N)$ the set of all partitions of N. An element $P \in \mathcal{P}(N)$ is called a *coalition structure* or a *system of unions* of the set N. Whilst the two extreme coalition structures $P^n = \{\{1\}, \{2\}, \ldots, \{n\}\}$ and $P^N = \{\{N\}\}$ do not discriminate between the players, all others introduce an asymmetry amongst them which is generally unrelated to voting weights. A *weighted majority game with a coalition structure* is a triplet (N, v, P), where $(N, v) \in \mathcal{W}(N)$ and $P \in \mathcal{P}(N)$. The family of all weighted majority games with player set N and a coalition structure is denoted by $\mathcal{W}^{\mathcal{P}}(N)$.

If $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$, with $P = \{P_k\colon k \in M,\ M = \{1, \ldots, m\}\}$, the *quotient game* (M, v^P) induced by (N, v, P) is the weighted majority game played between the unions, i.e.,

$$(M, v^P) \in \mathcal{W}(M) \ \text{ and } \ v^P(R) = v(\bigcup_{k \in R} P_k) \text{ for all } R \subseteq M.$$

The game (M, v^P) can be represented by $[q; w(P_1), w(P_2), \ldots, w(P_m)]$ where $w(P_k) = \sum_{i \in P_k} w_i$ with $k \in M,\ M = \{1, \ldots, m\}$.

As in the case without an explicit coalition structure, we call two players $i, j \in P_k$ symmetric in $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$ if their marginal contribution to every coalition is the same. Analogously, two a priori unions P_k and P_l are called *symmetric* if their marginal contribution to any coalition in the quotient game (M, v^P) is the same.

As a straightforward extension of a power index defined on $\mathcal{W}(N)$, a *(coalitional) power index* for weighted majority games with a coalition structure is a function $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ which

[5] Global monotonicity is, however, not necessary for local monotonicity: e.g., the *normalized* BZ index is locally monotonic (and symmetric) but not globally monotonic.

assigns to each game (N, v, P) a vector $g(N, v, P)$, where the real number $g_i(N, v, P)$ is the power of player i in the game according to g. Coalitional power indices take a player's union membership into account, not only its marginal contributions. They therefore commonly fail to be symmetric in the conventional sense: players with equal weight may be assigned different power. We introduce two notions of symmetry which are better suited to games with a coalition structure:

Definition 3 *A coalitional power index* $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ *is* symmetric *within* unions *if for each weighted majority game with a coalition structure* $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$*, where* $(N, v) = [q; w_1, w_2, \ldots, w_n] \in \mathcal{W}(N)$ *and* $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$*,*

$$g_i(N, v, P) = g_j(N, v, P)$$

whenever the players $i, j \in P_k \in P$ *are symmetric. Moreover,* g *is* symmetric *between* unions *if*

$$\sum_{i \in P_k} g_i(N, v, P) = \sum_{j \in P_l} g_j(N, v, P)$$

whenever the unions $P_k, P_l \in P$ *are symmetric.*

Intuitively speaking, symmetry *within* unions guarantees that power differences for players in the same union must be based on differences in their weights (and not, e.g., different relations to other players inside or outside the union). Analogously, symmetry *between* unions formalizes that any difference in the aggregate power values of two a priori unions must be based on a difference in their respective total weights.

We will focus on three power indices defined on $\mathcal{W}^{\mathcal{P}}(N)$: the *Banzhaf-Owen index* (hereafter BO index), the *Symmetric Coalitional Banzhaf index* (SCB index) and the *Owen index* (OW index). In analogy to Eq. (1), and letting player i be contained in a priori union P_k, these indices can be written as

$$g_i(N, v, P) = \sum_{R \subseteq M \setminus k} \; \sum_{T \subseteq P_k \setminus i} p^i_{R,T} \cdot \big[v(Q_R \cup T \cup i) - v(Q_R \cup T) \big], \text{ for any } i \in N, \tag{3}$$

where $P = \{P_1, \ldots, P_m\}$ describes the coalition structure, $M = \{1, \ldots, m\}$ is P's index set, and $Q_R = \bigcup_{l \in R} P_l$ refers to the subset of players belonging to any union referred to by index subset $R \subseteq M$. The key difference to the standard evaluation of i's average marginal contribution (see Eq. (1)) is that a priori unions other than the union P_k which contains i are assumed to have either joined with all their members or not at all.

For the BO index (Owen, 1982) the weighting coefficients $p^i_{R,T}$, which are usually interpreted as probabilities,[6] are

$$p^i_{R,T} = \frac{1}{2^{m-1}} \cdot \frac{1}{2^{p_k - 1}},$$

[6] Also see Laruelle and Valenciano (2004) and Alonso-Meijide et al. (2007).

for the SCB index (Alonso-Meijide and Fiestras-Janeiro, 2002)

$$p^i_{R,T} = \frac{1}{2^{m-1}} \cdot \frac{t!(p_k - t - 1)!}{p_k!},$$

and for the OW index (Owen, 1977)

$$p^i_{R,T} = \frac{r!(m - r - 1)!}{m!} \cdot \frac{t!(p_k - t - 1)!}{p_k!}$$

where r, t, and p_k refer to the cardinality of sets R, T, and P_k, respectively.

It follows from these weights that one can view all three indices as describing a two-level decision making process. First, the respective p_k members of the unions $P_k \in P$ take a decision amongst themselves – with influence on this decision measured using the probability model of either the SH or BZ index. Second, (representatives of) the m unions take an overall decision based on the respective bottom-level choices, where influence on the overall decision is again measured by either the SH or BZ index. Or one takes the perspective of the allocation of a surplus of transferable utility, corresponding to the worth of the grand coalition $v(N) = 1$. Then one may interpret above coefficients as formalizing that unions or union representatives first split the total amount between the unions, and thereafter each union internally allocates its share.[7] At each stage the respective opportunities for forming coalitions across and inside unions are taken into account in either the SH or BZ way. In particular for all games with coalition structures $P^n = \{\{1\}, \{2\}, \ldots, \{n\}\}$ or $P^N = \{\{N\}\}$, which respectively imply interaction only between or only within unions, the OW index coincides with the SH index of (N, v), and the BO index coincides with the BZ index of (N, v). The SCB index applies the BZ index to inter-union and the SH index to intra-union interaction, i.e., it coincides with the BZ index for (N, v, P^n) and the SH index for (N, v, P^N).

For illustration, consider the weighted majority game (N, v) represented by $\left[\frac{68}{135}; \frac{60}{135}, \frac{34}{135}, \frac{17}{135}, \frac{13}{135}, \frac{11}{135}\right]$ with coalition structure $P = \{\{1\}, \{2, 4, 5\}, \{3\}\}$.[8] Ignoring the information about the a priori union of players 2, 4, and 5, one can compute the SH and BZ power indices as

$$f^{SH}(N, v) = \left(\frac{6}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}, \frac{1}{10}\right)$$

and

$$f^{BZ}(N, v) = \left(\frac{7}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}\right).$$

[7] This interpretation implicitly assumes $\sum_{i \in N} g_i(N, v, P) \leq v(N)$, which does not generally hold if the (non-normalized) BZ index is invoked within or between unions.

[8] This weighted majority game actually reflects the Catalonian Parliament, a typical Western Europe parliamentary body, during Legislature 1995–1999 (see Alonso-Meijide et al. 2005). From an abstract point of view it is an instance of the so-called *apex game*: the apex player 1 can form a minimal winning coalition with *any* other player and in addition only the coalition involving *all* small players is minimal winning (meaning that each member makes a positive marginal contribution).

The respective power values are weakly increasing in voting weights, which, of course, must be the case for locally monotonic indices such as f^{SH} and f^{BZ}. In contrast, the BO, SCB, and OW coalitional power indices, which take coalition structure P into account, yield

$$g^{BO}(N,v,P) = \left(\frac{4}{8}, \frac{1}{8}, \frac{4}{8}, \frac{1}{8}, \frac{1}{8}\right),$$

$$g^{SCB}(N,v,P) = \left(\frac{3}{6}, \frac{1}{6}, \frac{3}{6}, \frac{1}{6}, \frac{1}{6}\right),$$

and

$$g^{OW}(N,v,P) = \left(\frac{3}{9}, \frac{1}{9}, \frac{3}{9}, \frac{1}{9}, \frac{1}{9}\right).$$

All clearly violate local monotonicity: $w_2 > w_3$ but each of the coalitional indices indicates greater power for player 3 than for player 2.

At an intuitive level, the a priori union between players 2, 4, and 5 means that feasible winning coalitions (or possible governments, in the political context that generated the example) never involve player 2 alone. Any spoils and policy influence which derive from a winning coalition will therefore have to be shared by player 2 with players 4 and 5. In contrast, player 3 (who is symmetric to player 2 ignoring P) only negotiates on its own behalf and by definition always is the unique swing player in its own bottom-level subgame. So player 3 keeps an undivided full share of spoils from the winning coalition potentially formed at the top level. Knowing the coalition structure formalized by P, we should thus expect the respective power allocation to violate local monotonicity: indices g^{OW}, g^{SCB}, and g^{BO} would have a problem if they did not (cf. Holler and Napel, 2004a and 2004b). Monotonicity concepts which take the behavioral constraints that are implied by a coalition structure into account therefore provide better benchmarks for the evaluation and comparison of these indices.

3 Local monotonicity and coalition structures

We will first consider adaptations of *local* monotonicity to weighted majority games with a system of a priori unions. The subsequent section will then address *global* monotonicity, and relate both notions of monotonicity to each other. In either case it is worthwhile to consider two separate monotonicity properties, within and between unions.

3.1 Local monotonicity within unions

Even when coalition formation in a given weighted majority game is restricted by a system of a priori unions, we would expect some kind of monotonicity at the 'very local' level, i.e., comparing players who belong to the same a priori union. This is naturally captured by

Definition 4 *A coalitional power index* $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ *is* locally monotonic within unions *if for each weighted majority game with a coalition structure* $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$*, where* $(N, v) = [q; w_1, w_2, \ldots, w_n] \in \mathcal{W}(N)$ *and* $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$*,*

$$g_i(N, v, P) \geq g_j(N, v, P)$$

holds for every pair of players $i, j \in P_k$ *such that* $w_i \geq w_j$ *with* $k \in M = \{1, \ldots, m\}$*.*

This property is weaker than conventional local monotonicity,[9] because it restricts power only for players i and j within the same union – not arbitrary pairs $i, j \in N$. If a coalitional power index satisfies local monotonicity then it also satisfies local monotonicity within unions. If it satisfies the latter but not the former, then examples of violations must, of course, involve coalition structures other than $P^N = \{\{N\}\}$.

Again note that the marginal contribution of player $i \in P_k$ to a given coalition, corresponding to the difference $v(Q_R \cup T \cup i) - v(Q_R \cup T)$ in Eq. (3), is not smaller than that of any player $j \in P_k$ with $w_j \leq w_i$ $(i, j \notin T)$. Moreover, the coefficients $p^i_{R,T}$ in Eq. (3) are identical for all $i \in N$ in case of the three considered coalitional indices, respectively. We thus obtain

Proposition 1 *The BO, SCB and OW indices satisfy local monotonicity within unions.*

3.2 Local monotonicity between unions

An additional and complementing notion of local monotonicity in games with a coalition structure refers to cross-union comparisons. It relates the aggregate weights of unions P_k and P_l to their aggregate power values. It is natural to use simple summation for the aggregation of weights, and this is arguably the best way for the aggregation of individual power values, too. We then say that a coalitional power index satisfies local monotonicity *between unions* if total power of a union cannot exceed total power of another union with greater total weight. Or, more formally:

Definition 5 *A coalitional power index* $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ *is* locally monotonic between unions *if for each weighted majority game with a coalition structure* $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$*, where* $(N, v) = [q; w_1, w_2, \ldots, w_n] \in \mathcal{W}(N)$ *and* $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$*,*

$$\sum_{i \in P_k} g_i(N, v, P) \geq \sum_{i \in P_l} g_i(N, v, P)$$

holds for every pair of a priori unions $P_k, P_l \in P$ *such that* $w(P_k) = \sum_{i \in P_k} w_i \geq w(P_l) = \sum_{i \in P_l} w_i$*.*

[9] Definition 1 refers to indices with domain $\mathcal{W}(N)$ and therefore, technically speaking, the two monotonicity notions cannot be compared. We here and later implicitly refer to the straightforward extension of local and global monotonicity to domain $\mathcal{W}^{\mathcal{P}}(N)$ (simply ignoring the given coalition structure).

Considering our earlier example $(N,v) = \left[\frac{68}{135}; \frac{60}{135}, \frac{34}{135}, \frac{17}{135}, \frac{13}{135}, \frac{11}{135}\right]$ with $P = \{\{1\},\{2,4,5\},\{3\}\}$ again, we can see that the BO index violates local monotonicity between unions:

$$w(P_2) = \frac{58}{135} \geq w(P_3) = \frac{17}{135}$$

but

$$\sum_{i \in P_2} g_i^{BO}(N,v,P) = \frac{3}{8} < \sum_{i \in P_3} g_i^{BO}(N,v,P) = \frac{4}{8}.$$

In contrast, we obtain

$$\sum_{i \in P_2} g_i^{SCB}(N,v,P) = \sum_{i \in P_3} g_i^{SCB}(N,v,P) = \frac{3}{6}$$

and

$$\sum_{i \in P_2} g_i^{OW}(N,v,P) = \sum_{i \in P_3} g_i^{OW}(N,v,P) = \frac{3}{9}$$

for the SCB and OW indices, i.e., these are candidates for coalitional power indices that satisfy local monotonicity between unions.

Local monotonicity between unions requires that unions' aggregate power values are weakly increasing in the respective total weights. This implies that players' individual power values are weakly increasing in the respective individual weights if all unions are singletons, i.e., if the game coincides with its quotient game (M, v^P). Therefore examples of a violation of local monotonicity by an index which is locally monotonic between unions must involve coalition structures other than $P^n = \{\{1\},\ldots,\{n\}\}$. One can relate local monotonicity between unions of a coalitional power index g to conventional local monotonicity of an underlying standard power index f (if it exists), provided that g treats the original game and its quotient game in a consistent fashion. In order to make this precise, we need to introduce two more concepts, namely the quotient game property (Winter, 1992) and coalitional extensions (Alonso-Meijide and Fiestras-Janeiro, 2002).

Definition 6 *A coalitional power index* $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ *satisfies the* quotient game property *if for all* $(N,v,P) \in \mathcal{W}^{\mathcal{P}}(N)$ *and all* $P_k \in P$

$$\sum_{i \in P_k} g_i(N,v,P) = g_k(M, v^P, P^m),$$

where $(M, v^P) \in \mathcal{W}(N)$ *is the quotient game induced by* (N,v,P) *and* $P^m = \{\{1\},\ldots,\{m\}\}$.

This requires that total power of any a priori union P_k in (N,v,P) is equivalent to the power of player k (representative of union P_k) in the quotient game v^P assuming the degenerate coalition structure $P^m = \{\{1\},\ldots,\{m\}\}$.

The property is satisfied by the OW and SCB indices but not by the BO index (Alonso-Meijide and Fiestras-Janeiro, 2002). The latter can be seen for the five player unanimity game

(N, v) represented by $\left[1; \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right]$ with coalition structure $P = \{\{1, 2, 3\}, \{4, 5\}\}$. The BO index for this game is

$$g^{BO}(N, v, P) = \left(\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{2}{8}, \frac{2}{8}\right).$$

However, by applying coalition structure $P = \{\{\mathit{1}\}, \{\mathit{2}\}\}$ in the quotient game (M, v^P), represented by $\left[1; \frac{3}{5}, \frac{2}{5}\right]$, one obtains

$$g^{BO}(M, v^P, P) = \left(\frac{1}{2}, \frac{1}{2}\right).$$

The notion that a coalitional power index g derives from an underlying conventional power index can be made precise as follows:

Definition 7 *Given a power index $f\colon \mathcal{W}(N) \to \mathbb{R}^n$, a coalitional power index $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is a* coalitional extension of f *if*

$$g(N, v, P^n) = f(N, v)$$

holds for every $(N, v) \in \mathcal{W}(N)$ with $P^n = \{\{1\}, \ldots, \{n\}\}$.

In particular, the OW index is a coalitional extension of the SH index, and the BO and SCB indices are coalitional extensions of the BZ index. Combining the notion of a coalitional extension with the quotient game property, we obtain:

Proposition 2 *If $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is a coalitional extension of $f\colon \mathcal{W}(N) \to \mathbb{R}^n$ and satisfies the quotient game property, then local monotonicity of f implies local monotonicity between unions of g.*

Proof. Let g be a coalitional extension of f that satisfies the quotient game property and consider an arbitrary game $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$, where $(N, v) = [q; w_1, w_2, \ldots, w_n] \in \mathcal{W}(N)$ and $P = \{P_k\colon k \in M,\ M = \{1, \ldots, m\}\} \in \mathcal{P}(N)$. For each $P_k \in P$ we have

$$\sum_{i \in P_k} g_i(N, v, P) = g_k(M, v^P, P^m) = f_k(M, v^P),$$

where the first equality uses the quotient game property and the second that g extends f. So

$$\sum_{i \in P_k} g_i(N, v, P) \geq \sum_{i \in P_l} g_i(N, v, P)$$

if and only if

$$f_k(M, v^P) \geq f_l(M, v^P).$$

The latter and hence the former must be true for any pair of a priori unions $P_k, P_l \in P$ such that $w(P_k) = \sum_{i \in P_k} w_i \geq w(P_l) = \sum_{i \in P_l} w_i$ whenever f is locally monotonic. $\square$

Corollary 1 *The SCB and OW indices are locally monotonic between unions.*

The corollary demonstrates that local monotonicity between unions does not imply conventional local monotonicity, which is violated by the SCB and OW indices (see p. 8). The reverse is not true either: the trivial index defined by $g_i(N, v, P) = 1$ for all $i \in N$ is locally monotonic, but not locally monotonic between unions (consider, e.g., $P = \{\{1\}, \{2, 3\}\}$). However, if a coalitional power index g satisfies conventional local monotonicity *and* the quotient game property, then it is also locally monotonic between unions: g can be viewed as the coalitional extension of an index f defined by $f(N, v) \equiv g(N, v, P^n)$. This index is locally monotonic, and Prop. 2 can then be applied.

4 Global monotonicity and coalition structures

We now turn to *global* monotonicity, and show how above notions of local monotonicity within and between unions relate to their global analogues.

4.1 Global monotonicity within unions

Global monotonicity refers to the comparison of different games from a given player's perspective. We will require that the considered games are comparable in a sense that accounts for the coalition structure.

Definition 8 *A coalitional power index $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is* globally monotonic within unions *if for every two weighted majority games with the same coalition structure and quota, i.e., (N, v, P) and $(N, v', P) \in \mathcal{W}^{\mathcal{P}}(N)$ with $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $(N, v') = [q; w'_1, w'_2, \ldots, w'_n] \in \mathcal{W}(N)$ and $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$,*

$$g_i(N, v, P) \geq g_i(N, v', P)$$

holds for every player $i \in P_k$ such that $w_i \geq w'_i$, $w_j \leq w'_j$ for all $j \in P_k \setminus i$, and $w_j = w'_j$ for all $j \in P_l, l \neq k$.

This property is weaker than conventional global monotonicity because it restricts player i's power in (N, v, P) and (N, v', P) only when the weight of every player outside the a priori union P_k, which contains i, is the same in both games. If a coalitional power index satisfies global monotonicity then it also satisfies global monotonicity within unions. We have

Proposition 3 *The BO, SCB and OW indices satisfy global monotonicity, and hence global monotonicity within unions.*

Proof. Consider two weighted majority games (N, v, P) and (N, v', P) with $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $(N, v') = [q; w'_1, w'_2, \ldots, w'_n]$ satisfying $w_i \geq w'_i$ for some player i and $w_j \leq w'_j$ for all $j \neq i$.

Now consider an arbitrary coalition $S \subsetneq N$ which does *not* contain player i. We immediately have

$$\sum_{j \in S} w'_j \geq \sum_{j \in S} w_j. \tag{4}$$

And from $\sum_j w_j = \sum_j w'_j = 1$ we get

$$w_i - w'_i = \sum_{j \neq i} (w'_j - w_j) \geq \sum_{j \in S} (w'_j - w_j),$$

which implies

$$\sum_{j \in S} w_j + w_i \geq \sum_{j \in S} w'_j + w'_i. \tag{5}$$

Recalling that $v(S) = 0 \iff \sum_{j \in S} w_j < q$ and $v(S) = 1 \iff \sum_{j \in S} w_j \geq q$, Eq. (4) implies

$$v'(S) = 0 \implies v(S) = 0$$

and Eq. (5) implies

$$v'(S \cup i) = 1 \implies v(S \cup i) = 1.$$

So i's marginal contribution to *any* coalition is weakly greater in (N, v) than in (N, v'). This is true in particular for all coalitions $Q_R \cup T$ considered in Eq. (3), and so the claim follows from observing that the respective coefficients $p^i_{R,T}$ are unaffected by weight changes. □

It turns out that the link between global and local monotonicity via symmetry that exists for standard power indices (Turnovec, 1998) extends to the respective 'within-unions' notions of monotonicity for coalitional power indices:

Proposition 4 *If a coalitional power index $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is globally monotonic within unions and symmetric within unions, then it is also locally monotonic within unions.*

Proof. Let $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ satisfy global monotonicity within unions and symmetry within unions. Let game $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$, where $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$, be such that there exist two players $i, j \in P_k, k \in M = \{1, \ldots, m\}$ with $w_i \geq w_j$.

Now consider the game $(N, v', P) \in \mathcal{W}^{\mathcal{P}}(N)$ with $(N, v') = [q; w'_1, w'_2, \ldots, w'_n]$ defined by $w'_i = w_i - (w_i - w_j)/2$, $w'_j = w_j + (w_i - w_j)/2$, and $w_h = w'_h$ for all $h \neq i, j$. Global monotonicity within unions implies

$$g_i(N, v, P) \geq g_i(N, v', P)$$

$$g_j(N, v', P) \geq g_j(N, v, P).$$

And, given that $w'_i = w'_j$, symmetry within unions implies

$$g_i(N, v', P) = g_j(N, v', P).$$

So $g_i(N, v, P) \geq g_j(N, v, P)$. □

A (non-symmetric) index which is globally monotonic within unions but *not* locally monotonic within unions is, e.g., given by $g_i(N, v, P) \equiv g_i^{OW}(N, v, P)$ for players i outside a given union $P_l \in P$, whilst for every player $i \in P_l$ we have $g_i(N, v, P) \equiv p_i\, g_i^{OW}(N, v, P)$ with $p_i \geq 0$, $\sum_{j \in P_l} p_j = 1$, and $p_k \neq p_j$ for some $k, j \in P_l$.

4.2 Global monotonicity between unions

It remains to relate the aggregate power of a priori unions to their aggregate voting weights in different but (in order to be meaningful) closely related games:

Definition 9 *A coalitional power index $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is* globally monotonic between unions *if for every two weighted majority games with the same coalition structure and quota, i.e., (N, v, P) and $(N, v', P) \in \mathcal{W}^{\mathcal{P}}(N)$ with $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $(N, v') = [q; w_1', w_2', \ldots, w_n'] \in \mathcal{W}(N)$ and $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$,*

$$\sum_{i \in P_k} g_i(N, v, P) \geq \sum_{i \in P_k} g_i(N, v', P)$$

holds for every union $P_k \in P$ such that $w(P_k) = \sum_{i \in P_k} w_i \geq w'(P_k) = \sum_{i \in P_k} w_i'$ and $w(P_l) \leq w'(P_l)$ for all $l \neq k$.

Intuitively, global monotonicity between unions requires that a union's aggregate power is (weakly) bigger in a game in which it has more aggregate weight than in one with less, provided that other unions' respective total weights have not increased.

The lack of local monotonicity between unions, identified earlier, already suggests that the BO index also violates global monotonicity between unions. For example, the games $(N, v, P) = [\frac{17}{22}; \frac{6}{22}, \frac{16}{22}, 0, 0]$ and $(N, v', P) = [\frac{17}{22}; \frac{7}{22}, \frac{5}{22}, \frac{5}{22}, \frac{5}{22}]$ with $P = \{\{1\}, \{2, 3, 4\}\}$ satisfy

$$w(P_2) = \frac{16}{22} \geq w'(P_2) = \frac{15}{22} \;\text{ and }\; w(P_1) = \frac{6}{22} \leq w'(P_1) = \frac{7}{22}$$

but one obtains

$$\sum_{i \in P_2} g_i^{BO}(N, v, P) = \frac{1}{2} < \sum_{i \in P_2} g_i^{BO}(N, v', P) = \frac{3}{4}.$$

In contrast to the within-unions case, global monotonicity *between unions* and conventional global monotonicity are two independent properties of a coalitional index. For example, the index defined by

$$g_i(N, v, P) = \begin{cases} w_i & \text{if } i \in P_k \text{ and } i = \min P_k, \\ w_i/2 & \text{otherwise} \end{cases}$$

is globally monotonic but violates global monotonicity between unions (consider, e.g., a weight shift from the first player in P_k to the others). Similarly the index given by

$$g_i(N, v, P) = \begin{cases} 1 & \text{if } i \in P_k \text{ and } i = \min(\operatorname{argmin}_{j \in P_k} w_j), \\ 0 & \text{otherwise} \end{cases}$$

is globally monotonic between unions but violates conventional global monotonicity (increasing the weight of a union's smallest member can reduce its power).

As in the case of monotonicity within unions, the global version of monotonicity between unions implies the respective local property provided that the index under consideration satisfies an additional symmetry condition:[10]

Proposition 5 *If a coalitional power index $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is globally monotonic between unions and symmetric between unions, then it is also locally monotonic between unions.*

Proof. Let g satisfy global monotonicity between unions and symmetry between unions. Let game $(N, v, P) \in \mathcal{W}^{\mathcal{P}}(N)$, where $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\}$, be such that there exist two a priori unions $P_k, P_l \in P$ with $w(P_k) \geq w(P_l)$. Now consider the game $(N, v', P) \in \mathcal{W}^{\mathcal{P}}(N)$ with $(N, v') = [q; w_1', w_2', \ldots, w_n']$ defined by

$$w_j' = \begin{cases} (\sum_{i\in P_k} w_i - \frac{1}{2}(\sum_{i\in P_k} w_i - \sum_{i\in P_l} w_i))/|P_k| & \text{if } j \in P_k, \\ (\sum_{i\in P_l} w_i + \frac{1}{2}(\sum_{i\in P_k} w_i - \sum_{i\in P_l} w_i))/|P_l| & \text{if } j \in P_l, \\ w_j & \text{if } j \notin P_k \cup P_l. \end{cases}$$

Global monotonicity between unions implies

$$\sum_{i\in P_k} g_i(N, v, P) \geq \sum_{i\in P_k} g_i(N, v', P)$$

$$\sum_{i\in P_l} g_i(N, v', P) \geq \sum_{i\in P_l} g_i(N, v, P).$$

And, given that $\sum_{i\in P_k} w_i' = \sum_{i\in P_l} w_i'$, symmetry between unions implies

$$\sum_{i\in P_k} g_i(N, v', P) = \sum_{i\in P_l} g_i(N, v', P).$$

So $\sum_{i\in P_k} g_i(N, v, P) \geq \sum_{i\in P_l} g_i(N, v, P)$. □

As for the local analogue, it is possible to relate global monotonicity between unions of a coalitional power index to conventional global monotonicity of an underlying standard index:

Proposition 6 *If $g\colon \mathcal{W}^{\mathcal{P}}(N) \to \mathbb{R}^n$ is a coalitional extension of $f\colon \mathcal{W}(N) \to \mathbb{R}^n$ and satisfies the quotient game property, then global monotonicity of f implies global monotonicity between unions of g.*

Proof. Let g be a coalitional extension of f that satisfies the quotient game property and consider any arbitrary game (N, v, P), where $(N, v) = [q; w_1, w_2, \ldots, w_n]$ and $P = \{P_1, P_2, \ldots, P_m\} \in \mathcal{P}(N)$. The quotient game property and the fact that g extends f imply

$$\sum_{i\in P_k} g_i(N, v, P) = g_k(M, v^P, P^m) = f_k(M, v^P)$$

[10] An index which is globally but *not* locally monotonic between unions is, e.g., given by $g_i(N, v, P) \equiv p_k\, g_i^{OW}(N, v, P)$ for $i \in P_k$, where $p_k \geq 0$, $\sum_k p_k = 1$, and $p_k \neq p_l$ for some $k, l \in \{1, \ldots, m\}$.

for each $P_k \in P$. For any game (N, v', P) with the same coalition structure and quota, i.e., $(N, v') = [q; w'_1, w'_2, \ldots, w'_n] \in \mathcal{W}(N)$, one analogously has

$$\sum_{i \in P_k} g_i(N, v', P) = g_k(M, (v')^P, P^m) = f_k(M, (v')^P).$$

So

$$\sum_{i \in P_k} g_i(N, v, P) \geq \sum_{i \in P_k} g_i(N, v', P)$$

if and only if

$$f_k(M, v^P) \geq f_k(M, (v')^P).$$

The latter and hence the former must be true if $w(P_k) \geq w'(P_k)$ and $w(P_l) \leq w'(P_l)$ for all $l \neq k$ whenever f is globally monotonic. □

Corollary 2 *The SCB and OW indices are globally monotonic between unions.*

The Proposition also implies that even though we have shown conventional global monotonicity and global monotonicity between unions to be independent properties, they are close cousins: if a coalitional power index g satisfies conventional global monotonicity *and* the quotient game property, then it is also globally monotonic between unions. Namely, g is a coalitional extension of an (artificial) index f defined by $f(N, v) \equiv g(N, v, P^n)$, which inherits g's global monotonicity and to which Prop. 6 can be applied.

5 Concluding Remarks

Coalitional power indices which take a priori unions into account fundamentally differ from standard power indices as they can introduce an asymmetry among the players on top of the one created by differences in voting weights. Both types of asymmetry interact in a way that makes conventional notions of monotonicity – being merely a reflection of weight asymmetry – incomplete or even meaningless.

This paper adapted the conventional local and global monotonicity concepts to take account of a priori unions. First, we restricted power comparisons to players that differ in weight but belong to the same a priori union; only they are comparable in a straightforward sense. Second, we extended the notion of power monotonicity from individuals to groups of individuals. The former amounted to a comparison of power among players within a given union, the latter to comparisons between the a priori unions as actors themselves. The relationships identified in this paper between new and old concepts are illustrated in Figure 1.[11] Other useful monotonicity

[11] It is straightforward that all considered indices are symmetric within unions. However, only the OW and SCB indices are also symmetric between unions (see the example after Definition 6). Note also that the index

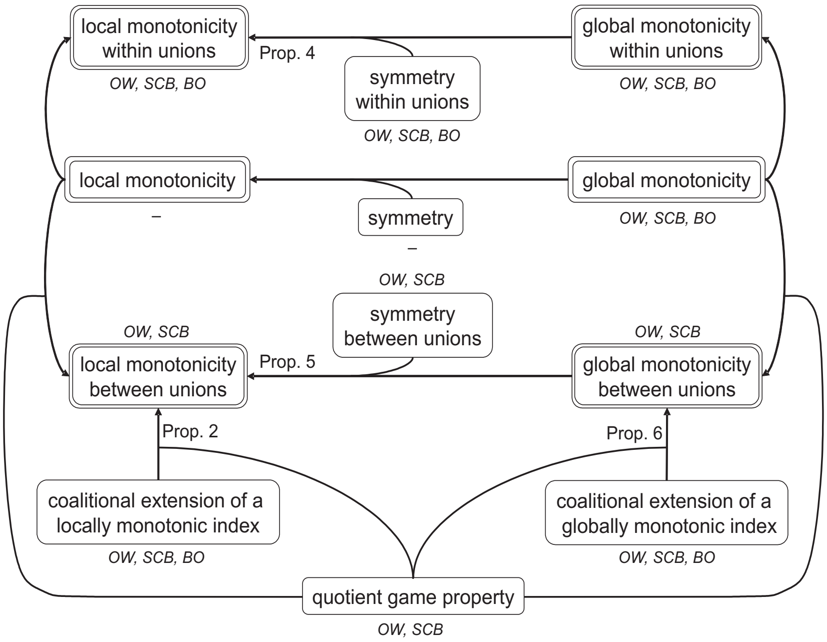

Figure 1: Taxonomy of monotonicity concepts for coalitional indices

notions might be defined (e.g., relating players of distinct but still in some sense 'comparable' unions); we believe to have singled out the four most natural ones.

Our respective local monotonicity properties are as in the standard case implied by the related global monotonicity property in conjunction with a rather compelling symmetry requirement. One can, therefore, focus on inter-player or inter-union comparisons within the same game, i.e., local monotonicity. Amongst the two local concepts which we considered, within-union monotonicity failed to discriminate between the major coalitional indices – the Owen index, the Banzhaf-Owen index and the Symmetric Coalitional Banzhaf index. On the one hand, it is good news that all three satisfy the intuitively appealing requirement that if two individual players differ in nothing but weight, the one with smaller weight cannot have more power than the other. On the other hand, it could be regarded as dissatisfying: local monotonicity within unions is of limited use for selecting between the main coalitional indices. The discriminatory power of the popular local monotonicity requirement for conventional indices, however, has the same limitation: *all* indices based on Eq. (1) with $p_S^i \equiv p_S$ are locally monotonic, including the Banzhaf and Shapley-Shubik power indices.

The between-union version of local monotonicity does discriminate between the major coalitional indices, but it is a less straightforward requirement. It is not obvious why an a priori union

considered in fn. 10 is not symmetric between unions but satisfies the quotient game property, and that $g_i(N, v, P) \equiv \max_{r \in M} |P_r|/|P_k|$ for $i \in P_k$ is symmetric between unions but does not satisfy the quotient game property; hence these two attributes are independent.

with smaller weight should not wield more power in aggregate than one with greater weight: the respective numbers of members and internal weight distributions are possibly very different. One plausible argument is that unions with fewer members or ones with more concentrated weights are more influential because they suffer from fewer internal divisions.[12] In fact, the Banzhaf-Owen index tends, ceteris paribus, to indicate more power for smaller unions. It fails the 'between-unions' local monotonicity test, whilst the Owen and Symmetric Coalitional Banzhaf indices pass it.

So within-union monotonicity can be considered as a general adequacy criterion for coalitional power indices (assuming that all aspects relevant to players' interaction are captured by weights and coalition structure). In contrast, between-union monotonicity is a more divisive property, which is compelling to some but will seem artificial to others. The same can also be said of the quotient game property, introduced by Winter (1992). Our analysis has identified it as being crucial for monotonicity between unions.

The multitude of indices proposed for games without a priori unions allows for an even bigger number of coalitional power indices. So there is ample scope for application of the monotonicity concepts proposed in this paper. Assessing whether the basic requirement of monotonicity within unions is met seems a good first test. Next, it is in our view worthwhile to distinguish indices that rule out effects of intra-union details (in particular, effects of a union's cardinality or its internal weight distribution) and that are hence monotonic between unions – and those that do not. Together with other adequacy requirements, such as efficiency or zero power for null players (with zero weight), monotonicity within and between unions can thus help to determine which of many possible indices is, in a given context, the most suitable one.

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[12] Conceivably, the opposite could be true: more internal divisions might make it less likely that (members of) other unions are decisive.

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